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Thermal Instability of Compressible and Rotating Viscoelastic Rivlin-Ericksen Fluid Permeated with Suspended Particles Saturating Porous Media in Hydromagnetics

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Abstract: This paper deals with the instability of the plane interface between two uniform, superposed, electrically conducting and counter-streaming viscoelastic fluids saturating porous media in the presence of horizontal magnetic field. The rheology of the viscoelastic fluid is described by Walters' (model B'). The effects of medium porosity, surface tension and square of the Alfvén velocity, on the growth rate (both the real and the imaginary) of the most unstable mode have been investigated numerically. In the absence of surface tension, perturbations transverse to the direction of streaming are found to be unaffected by the presence of streaming if the perturbations in the direction of streaming are ignored. For perturbations in all other directions, there exists instability for a certain wave number range. The simultaneous presence of the magnetic field and the surface tension are able to suppress this Kelvin-Helmholtz instability for small wavelength perturbations and the medium porosity has critical strength to suppress the instability on the real growth rates of the most unstable mode. However, in case of imaginary growth rates of the most unstable modes remain uninfluenced by the increase in surface tension, the square of the Alfvén velocity and medium porosity. All these results have been computed numerically and depicted graphically.

Keywords: Thermal instability, Rivlin-Ericksen fluids, Suspended particles, Porous medium.

Mathematics Subject Classification: 76S05

1. Introduction

A detailed account of the onset of thermal instability (Bénard convection) of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics have been given by Chandrasekhar[1]. Motivation for the study of certain effects of particles immersed in the fluid such as: particle heat capacity, particle mass fraction and thermal force is due to the fact that the knowledge concerning fluid particle mixture is not commensurate with their industrial and scientific importance. Further motivation is provided by recalling decades-old concentration between the theory for the onset of convection and experiment. The theory agrees with experimental determinations of the onset of convection in liquid layers confined between two horizontal rigid surfaces. Chandra[2] observed a contradiction between the theory and his experiments for the onset of convection in fluids heated from below. He performed his experiment in an air layer and found that the instability depended on the depth of the layer. A Bénard type cellular convection with fluid descending at the cell centre was observed when the predicted gradients were imposed, for layers deeper than 10mm. A convection which was different in character from that in deeper layer occurred at much lower gradients than predicted if the layer depth was less than 7mm and Chandra called this motion columnar instability. He added an aerosol to mark the flow pattern. A complete survey of subsequent experimental studies, which confirms Chandra’s result, can be found in report by Jones[3] on effect of different aerosols on stability. Those effects which he felt may be important are thermal forces, electrostatic charges, evaporation condensation and buoyancy forces. Jones concluded that columnar instability is not an example of simple phase natural convection and that it is moist likely due to the unique properties of aerosol suspensions. There has been no analysis to determine the effect of the aerosol itself on stability and experiments have shown effects to be important.

Theoretically, discussions of columnar instability have given by Sutton[4] and Segel and Stuart[5]. Motivated by interest in fluid particles mixtures generally and columnar instability in particular, Scanlon and Segel[6] investigated some of the continuum effects of particles on Benard convection. They have found that the critical Rayleigh number was reduced solely because the heat capacity of pure gas was supplemented by that of the particles. The effect of suspended particles was found to destabilize the layer, i.e. to lower the critical temperature gradient. Sharma and Rani[7] have studied the double-diffusive convection with fine dust and have found that the suspended particles (fine dust) have destabilizing influence on the system.

Lapwood[8] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding[9]. Scanlon and Segel[6] have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The suspended particles were thus found to destabilize the layer. Sharma[10] has studied the effect of rotation on thermal instability of a viscoelastic fluid. Sharma[11] studied the thermal instability in compressible fluids in the presence of rotation and magnetic field.

There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations or Walters' (model B') constitutive equations. One such class of fluids is Rivlin-Ericksen elastico-viscous fluids we are interested therein. Rivlin and Ericksen[12] have proposed a theoretical model for such elastic-viscous fluids. Such and other class of polymers are used in the manufacture of parts of space-crafts, aeroplanes, tires, belt conveyers, ropes, cushions, foam, plastics, engineering equipments etc. Recently, polymers are used in agriculture, communication appliances and biomedical applications.

The fluid is often not pure but contains suspended particles. On the other hand, the multiphase fluid systems are concerned with the motions of a liquid or gas containing immiscible inert identical particles. Of all multiphase fluid systems observed in nature, blood flow in arteries, flow in rocket tubes, moment of inert solid particles in atmosphere, sand or other particles in sea or ocean beaches are the most common examples of multiphase fluid systems.

When the fluids are compressible, the equations governing the system become quite complicated. Spiegel and Veronis[13] simplified the set of equations governing the flow of compressible fluids under the assumption that the depth of the fluid layer is much smaller than the scale height as defined by them, and the motions of infinitesimal amplitude are considered. The Boussinesq approximation can be best summarized by two statements:

(1) The fluctuations in density which appear with the advent of motion result principally from thermal effects.

(2) In the equations for the rate of change of momentum and mass, density variation may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force.

The flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous and viscoelastic terms in the equations of motion of Rivlin-Ericksen fluid is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\right]q$, where μ and μ' are the viscosity and viscoelasticity of the incompressible Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid.

Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. The problem of thermal instability of a fluid in a porous medium is of importance in geophysics, soil sciences, ground water hydrology and astrophysics. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust [Lister[14]].

Magnetic field plays important roles in astrophysical situation, chemical engineering etc. Sharma[10] have studied the effect suspended particles on the onset of Bénard convection in hydromagnetics and have found that the magnetic field has an inhibiting effect on the onset of Bénard convection, whereas the influence of the suspended particles is to destabilize the layer. Another application of the result of flow through a porous medium in the presence of magnetic field is in the geothermal region. Also, the rotation of the earth distorts the boundaries of a hexagonal convection cell in a fluid through a porous medium and the distortion plays an important role in the extraction of energy in the geothermal regions. Rana and Kumar[15] have studied the incompressible Rivlin-Ericksen rotating fluid permeated with suspended particles and variable gravity field in porous medium.

Keeping in mind the importance and applications of uniform magnetic field, the present paper deals with the effect of uniform vertical magnetic field on the thermal instability of compressible and rotating viscoelastic Rivlin-Ericksen fluids permeated with suspended particles saturating homogeneous porous media. The present problem also finds its usefulness in thermal instability of such electrically conducting colloidal suspensions in the presence of magnetic field especially in ground water hydrology and astrophysics (interstellar atmospheres).

2. Formulation of the Problem and Perturbation Equations

An infinite horizontal layer of compressible, electrically conducting Rivlin-Ericksen viscoelastic fluid layer of thickness d permeated with suspended particles is considered bounded by the planes $z=0$ and $z=d$ in an isotropic and homogeneous medium of porosity ε and medium permeability k_1. This layer is heated from below so that, a uniform temperature gradient $\beta(=|dt/dz|)$ is maintained. The fluid-particle is acted on by a uniform vertical rotation $\Omega=(0,0,\Omega)$, a gravity force $g=(0,0,-g)$ and a uniform vertical magnetic field $H=(0,0,-H)$. The equations of motion and continuity governing the flow, using Boussinesq approximation are

$$\text{(1)}\qquad \frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t}+(q.\nabla)q\right]=\frac{1}{\rho}\nabla p-g\left(1+\frac{\delta\rho}{\rho}\right)-\frac{1}{k_1}\left(\nu+\nu'\frac{\partial}{\partial t}\right)q+\frac{\mu_e}{4\pi\rho}(\nabla\times H)\times H$$
$$+\frac{K'N}{\rho\varepsilon}(q_d-q)+\frac{2}{\varepsilon}(q\times\Omega)\ ,$$

$$\text{(2)}\qquad \nabla.q=0\,.$$

where $p,\rho,T,q,q_d(x,t),N(x,t),\nu$ and ν' denote fluid pressure, density, temperature, filter (seepage) of fluid velocity (initially zero), suspended particles velocity, suspended particles number density, kinematic viscosity and kinematic viscoelasticity, respectively. $K'=6\pi\mu\eta'$, η' being particle radius, is the Stoke's drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equations of motion (1), proportional to the velocity difference between the particles and the fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The effects due to pressure, gravity, Darcy's force and magnetic field on the particles are small and so are ignored.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$\text{(3)}\qquad mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\varepsilon}(q_d.\nabla)q_d\right]=K'N(q-q_d)\,,$$

$$\text{(4)}\qquad \varepsilon\left[\frac{\partial N}{\partial t}+\nabla.(Nq_d)\right]=0\,.$$

If c_v, c_{pt}, T and q' denote the heat capacity of fluid at constant volume, heat capacity of the particles, temperature and thermal conductivity of the viscoelastic fluid, respectively and assuming that the particles and the fluid are in thermal equilibrium, then the equation of heat conduction gives

(5) $$[\rho_0 c_v \varepsilon + \rho_s c_s (1-\varepsilon)]\frac{\partial T}{\partial t} + \rho_0 c_v (q.\nabla)T + mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t} + q_d.\nabla\right)T = q'\nabla^2 T .$$

The Maxwell's equations yield

(6) $$\varepsilon\frac{\partial H}{\partial t} = (H.\nabla)q + \varepsilon\eta\nabla^2 H ,$$

(7) $$\nabla.H = 0 ,$$

where η is the electrical resistivity.

The equation of state for the fluid is

(8) $$\rho = \rho_0[1-\alpha(T-T_0)] ,$$

where ρ_0, T_0 and α are the density, temperature and the coefficient of thermal expansion; the subscript zero refers to values at the reference level $z=0$. The kinematic viscosity ν, kinematic viscoelasticity ν', electrical resistivity η and coefficient of thermal expansion α are all assumed to be constants. The initial stationary state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution N_0 and is therefore, a state in which the velocity, particle velocity, temperature, density, pressure and particle number density at a point in the fluid are given by

(9) $$q=(0,0,0),\ q_d=(0,0,0),\ T=T_0-\beta z,\ \rho=\rho_0(1+\alpha\beta z),$$

$$p = p_0 - g\rho_0\left(z + \frac{\alpha\beta z^2}{z}\right),\ N = N_0 \text{ (constant).}$$

The character of the equilibrium of this stationary state can be determined by disturbing the system slightly and then, following its further evolution. Let $q(u,v,w)$, $q_d(l,r,s)$, $\delta\rho$, δp, θ, N and $h(h_x,h_y,h_z)$ denote the perturbations in fluid velocity (zero initially), particle velocity (zero initially), density $\rho(z)$, pressure $p(z)$, temperature T, number density N_0 and vertical magnetic field $H=(0,0,H)$ respectively.

The change in density δp caused mainly by the perturbation θ in temperature is given by

(10) $$\delta\rho = -\alpha\rho_0\theta \,.$$

Using the linear theory by neglecting the products and higher order of perturbations; retaining only linear terms and Boussinesq approximation, the equations (1)-(7) in the linearized perturbation form become

(11) $$\frac{1}{\varepsilon}\frac{\partial q}{\partial t} = \frac{1}{\rho_0}\nabla\delta p - g\alpha\theta - \frac{1}{k_1}\left(\upsilon + \upsilon'\frac{\partial}{\partial t}\right)q + \frac{\mu_e}{4\pi\rho_0}(\nabla\times h)\times H + \frac{K' N_0}{\rho_0}(q_d - q) + \frac{2}{\varepsilon}(q\times\Omega)\,,$$

(12) $$\nabla . q = 0\,,$$

(13) $$(E + h\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w - hs) + \kappa\nabla^2\theta\,,$$

(14) $$\varepsilon\frac{\partial h}{\partial t} = (H.\nabla)\upsilon + \varepsilon\eta\nabla^2 h\,,$$

(15) $$\nabla . h = 0\,,$$

(16) $$mN_0\frac{\partial q_d}{\partial t} = K' N_0(q - q_d)\,,$$

where $E = \varepsilon + (1-\varepsilon)\rho_s c_s / \rho_0 c_f$ is a constant. $\rho_s c_s$ and ρ_0, c_f stand for density and specific heat of solid (porous matrix) material and fluid, respectively.

$$h = f\frac{c_{pt}}{c_f},\quad f = \frac{mN}{\rho_0},\quad \kappa = \frac{q}{\rho_0 c_f}\,.$$

Eliminating z-component of particle velocity i.e., s from equations (13) and (16), we obtain

(17) $$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\left[(E + h\varepsilon)\frac{\partial}{\partial t} - \kappa\nabla^2\right]\theta = \left(\beta - \frac{g}{c_p}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t} + 1 + h\right)w\,.$$

Applying the curl operator twice to equation (11), we get

(18) $$\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)\frac{1}{\varepsilon}\frac{\partial\zeta}{\partial t}=\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)\left[\eta\nabla^2\zeta+H\frac{\partial\xi}{\partial z}+\frac{2\Omega}{\varepsilon}\frac{\partial w}{\partial z}\right]+\frac{K^{'}N_0}{\varepsilon\rho_0}\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}\right)\zeta,$$

and

(19) $$\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)\frac{1}{\varepsilon}\frac{\partial\nabla^2 w}{\partial t}=\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)\left[g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(-\frac{\delta\rho_0}{\rho_0}\right)-\frac{\upsilon}{k_1}\nabla^2 w\right.$$
$$\left.-\frac{2\Omega}{\varepsilon}\frac{\partial\xi}{\partial z}\right]+\frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial x}\nabla^2 h_z\frac{KN_0}{\varepsilon\rho_0}\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}\right)\zeta,$$

where ξ and ζ are the z-component of vorticity and the current density, respectively. Eliminating q_d, u, v, h_x, h_y and δp from equations (11)-(16) and after a little algebra, we obtain

(20) $$\frac{\partial}{\partial t}\left[\frac{m}{K^{'}}\frac{\partial}{\partial t}+1+mN_0K^{'}/\rho_0\right]\left(\nabla^2 w\right)+\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)\left[\frac{\varepsilon}{k_1}\left(\upsilon+\upsilon^{'}\frac{\partial}{\partial t}\right)\nabla^2 w\right.$$
$$\left.-\varepsilon g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\frac{\mu_e\varepsilon H}{4\pi\rho_0}\frac{\partial(\nabla^2 h_z)}{\partial x}+2\Omega\frac{\partial\xi}{\partial z}\right]=0.$$

The z-component of equation (14) yields

(21) $$\varepsilon\left(\frac{\partial}{\partial t}-\eta\nabla^2 h_z\right)=H\frac{\partial w}{\partial z}.$$

Since the fluid under consideration is confined between two horizontal planes $z=0$ and $z=d$, the fluid quantities must satisfy certain boundary conditions. Further, because the bounding surfaces are fixed and are maintained at fixed temperature, we must have

(22) $$w=0=\theta \text{ at } z=0 \text{ and } z=d.$$

The boundary conditions (22) are independent of the nature of the surfaces. Let's take the boundaries as free surfaces, though little artificial. It is the most appropriate to find the exact solutions for stellar atmospheres; on which tangential stresses do not act, i.e.

(23) $$T_{xz}=0=T_{yz},$$

where T_{ij} denote the stress tensor acting in the direction of x_j per unit area on the element to surface normal to x_i. The conditions (23) are equivalent to

(24) $$T_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0,$$

and

(25) $$T_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0.$$

Now, as w vanishes for all x and y on the boundaries, it follows from equation (24) and (25) that

(26) $$\frac{\partial u}{\partial z} = 0 = \frac{\partial v}{\partial z}.$$

Differentiating equation (12) with respect to z and using (26), implies

(27) $$\frac{\partial^2 w}{\partial z^2} = 0.$$

Since medium adjoining the fluid is a perfect conductor,

(28) $$\frac{\partial h_z}{\partial z} = 0.$$

Thus the boundary conditions appropriate to the problem are

(29) $$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial h_z}{\partial z} = \theta = 0 \text{ at } z = 0 \text{ and } z = d.$$

3. Dispersion Relation

Now an arbitrary perturbation is analyzed into a complete set of normal modes and then the stability of these modes is examined individually. For the system of equations (17)-(21), analysis can be made in terms of two-dimensional periodic waves of assigned wave number. Thus we ascribe all the quantities describing the perturbation dependence on x, y, z and t of the form

(30) $$[w, \theta, \xi, \zeta, h_z] = [W(z), \Theta(z), Z(z), X(x), K(z)]\exp\left(ik_x x + ik_y y + nt\right),$$

where k_x and k_y are wave numbers along x and y directions respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number of the disturbances and n is the growth rate, which is in general a complex constant.

Using expression (30), equations (17) - (21) in the non-dimensional form Become

(31) $$(D^2-a^2)\left[\frac{\sigma}{\varepsilon}\left(1+\frac{f}{1+p_1\sigma\tau}\right)-\frac{(1-F\sigma)}{p_l}(D^2-a^2)\right]W-\frac{\mu_e Hd}{4\pi\rho_0 v}(D^2-a^2)DK$$
$$+\frac{9\alpha d^2}{v}a^2\theta+\frac{2\Omega d^3}{v\varepsilon}DZ=0,$$

(32) $$\left[\frac{(1-F\sigma)}{p_l}(D^2-a^2)-\sigma\left(1+\frac{f}{1+p_1\sigma\tau}\right)\right]Z=\frac{\mu_e Hd}{4\pi\rho_0 v}DX+\frac{2\Omega d}{v\varepsilon}Dw,$$

(33) $$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW,$$

(34) $$(D^2-a^2-p_2\sigma)X=-\left(\frac{Hd}{\varepsilon\eta}\right)DZ,$$

(35) $$(1+p_1\sigma\tau)(D^2-a^2-Ep_1\sigma)\Theta=-\frac{\beta d^2}{\kappa}\frac{(G-1)}{G}(H_1+p_1\sigma\tau)W,$$

where $a=kd$, $\sigma=nd^2$, $p_1=v/\kappa$ is the thermal Prandtl number, $p_2=(v/\eta)$ is the magnetic Prandtl number, $F=v'/d^2$ is the dimensionless kinematic viscoelasticity, $G=c_p\beta/g$ is the dimensionless compressibility, $p_l=k_1/d^2$ is the dimensionless medium permeability, $h=mN_0c_{pt}/\rho_0c_v$, $f=mN_0/\rho_0$, $\tau=m\kappa/kd^2$. We have expressed the coordinators x, y, z in new units of length d, time t in the new unit of length d^2/κ and let $H_1=1+h$, $\tau_1=\tau v/d^2$, $x^*=x/d$, $y^*=y/d$, $z^*=z/d$ and $d=d/dz^*$. Stars have been omitted hereafter, for convenience.

The appropriate boundary conditions (29) using the expression (30), for which equations (31) - (35) must be solved, transform to

(36) $$W=D^2W=0,\ DZ=0,\ \Theta=0,\ DX=0,\ K=0 \text{ at } z=0 \text{ and } z=1.$$

Applying the operator
$$(D^2-a^2-Ep_1\sigma)(D^2-a^2-p_2\sigma)^2\left[\frac{(1-F\sigma)}{p_l}(D^2-a^2)-\sigma\left(1+\frac{f}{1+p_1\sigma\tau}\right)\right]$$
to equation (31) and then eliminating Θ, Z, X and K by using equations (31) - (35), we obtain

(37) $$(D^2-a^2)\left[\frac{\sigma}{\varepsilon}\left(1+\frac{f}{1+p_1\sigma\tau}\right)-\frac{(1-F\sigma)}{p_l}(D^2-a^2)\right]W+\frac{Q(D^2-a^2)}{(D^2-a^2-p_2\sigma)}D^2W$$
$$+T_A\left[\frac{(1-p\sigma)}{p_l}(D^2-a^2)-\sigma\left(1+\frac{f}{1+p_1\sigma\tau}\right)+Q\frac{D^2}{(D^2-a^2-p_2\sigma)}\right]^{-1}D^2W$$
$$-Ra^2\left(\frac{G-1}{G}\right)\left[\frac{H_1+p_1\sigma\tau}{(1+p_1\sigma\tau)(D^2-a^2-Ep_1\sigma)}\right]W=0,$$

where $Q=\dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number and $R=\dfrac{g\alpha\beta d^4}{\nu\kappa}$ is the Rayleigh number.

Making use of boundary conditions (36) in equation (31), we obtain

(38) $$D^4W=0 \text{ at } z=0 \text{ and } z=1.$$

Differentiating equations (31) respectively w.r.t z, using the boundary conditions (38), it can be shown that all even order derivatives of W must vanish for $z=0$ and $z=1$ and hence, the proper solution of W characterizing the lowest mode is

(39) $$W=W_0\sin\pi z,$$

where W_0 is a constant.

Substituting this solution in equation (36), required charactestic equation is

(40) $$(1+x)\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{f}{1+ip_1\sigma\tau}+\frac{1-i\pi^2F\sigma_1}{p}\right)\right]W$$
$$-R_1x\left(\frac{G-1}{G}\right)\left[\frac{H_1+i\pi^2p_1\sigma_1\tau}{(1+i\pi^2p_1\sigma\tau)(1+x+i\pi^2p_1\sigma)}\right]W-Q_1\frac{(1+x)}{(1+x+ip_2\sigma)}W$$
$$+T_A\left[\frac{1-i\pi^2F\sigma_1}{p}-\frac{i\sigma_1}{\varepsilon}\left(1+\frac{f}{1+i\pi^2p_1\sigma_1\tau}\right)+\frac{Q_1}{(1+x+i\pi^2p_2\sigma_1)}\right]^{-1}W=0,$$

where $Q_1=\dfrac{Q}{\pi^2}$, $R_1=\dfrac{R}{\pi^4}$, $T_{A_1}=\dfrac{T_A}{\pi^2}$, $i\sigma_1=\dfrac{\sigma}{\pi^2}$, $x=\dfrac{a^2}{\pi^2}$, $P=\pi^2P_l$.

4. Stability of the System and Oscillatory Modes

Here we examine the possibility of oscillatory modes, if any, on the Rivlin-Ericksen elastico-viscous fluid due to the presence of suspended particles, compressibility, viscoelasticity, medium permeability, rotation and magnetic field. Multiplying equation (31) by W^* (the complex conjugate of W and integrating over the vertical range of z and using equations (32)-(35) together with the boundary conditions (36), we get

$$\frac{\sigma}{\varepsilon}\left(1+\frac{f}{1+p_1\sigma^*\tau}\right)I_1+\frac{(1-F\sigma)}{p_l}I_2-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0 v}(I_3+p_2\sigma^* I_4)$$
$$-\frac{g\alpha\kappa a^2}{\beta v}\left(\frac{G}{G-1}\right)\frac{(1+p_1\sigma^*\tau)}{(H_1+p_1\sigma^*\tau)}\left[I_5+Ep_1\sigma^* I_6\right]+d^2\left[\frac{(1+F\sigma^*)}{P_l}I_7\right.$$
$$\left.+\sigma^*\left(1+\frac{f}{1+p_1\sigma^*\tau}\right)I_8+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0 v}\left[I_9+p_2\sigma^* I_{10}\right]\right]=0, \tag{41}$$

where $I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz,\ I_2=\int_0^1(|D^2W|^2+2a^2|DW|^2+a^4|W|^2)\,dz,$

$$I_3=\int_0^1(|D^2K|^2+2a^2|DK|^2+a^4|K|^2)\,dz,\ I_4=\int_0^1(|DK|^2+a^2|K|^2)\,dz,$$

$$I_5=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz,\ I_6=\int_0^1|\Theta|^2\,dz,\ I_7=\int_0^1(|DZ|^2+a^2|Z|^2)\,dz,$$

$$I_8=\int_0^1|Z|^2\,dz,\ I_9=\int_0^1(|DX|^2+a^2|X|^2)\,dz,\ I_{10}=\int_0^1|X|^2\,dz\,.$$

The integrals I_1-I_{10} are all positive definite. Putting $\sigma=i\sigma_i^*$ in the imaginary part of equation (41), we obtain

$$\sigma_i\left[\frac{1}{\varepsilon}\left(1+\frac{f}{1+p_1^2\sigma_i^2\tau}\right)I_1+\frac{F}{p_l}I_2-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0 v}p_2 I_4\right.$$
$$-\frac{g\alpha\kappa a^2}{\beta v}\left(\frac{G}{G-1}\right)\frac{\tau(H_1-1)I_5(H_1+p_1^2\sigma_i^2\tau^2)H_1p_1I_6}{H_1+p_1^2\sigma_i^2\tau^2}$$
$$\left.+d^2\left\{\frac{F}{p_l}I_7+\sigma^*\left(1+\frac{f}{1+p_1^2\sigma_i^2\tau}\right)I_9+\frac{\mu_e\varepsilon\eta d^2}{4\pi\rho_0 v}p_2I_{10}\right\}\right]=0\,. \tag{42}$$

Equation (42) implies that σ_i may be zero or non-zero, which means that the modes may be non-oscillatory or oscillatory. In the absence of magnetic field and suspended particles, equation (42) reduces to

$$\sigma_i\left[\frac{1}{\varepsilon}\left(1+\frac{f}{1+p_1^2\sigma_i^2\tau}\right)I_1+\frac{F}{p_l}I_2+d^2\left\{\frac{F}{p_l}I_7+\sigma^*\left(1+\frac{f}{1+p_1^2\sigma_i^2\tau}\right)I_8\right\}\right]=0. \tag{43}$$

The term inside the bracket is positive definite. Thus $\sigma_i=0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The magnetic field and suspended particles introduce oscillatory modes into the systems which were non-existent in their absence.

5. The Stationary Convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma_1=0$. Putting $\sigma_1=0$, equation (40) reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{xH_1}\right)\left[Q_1+\frac{(1+x)^2}{P}+T_A\left(\frac{P(1+x)}{(1+x)^2+PQ_1}\right)\right], \tag{44}$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters G, H_1, Q_1, P and T_{A_1}. The cases $G<1$ and $G=1$ are irrelevant here as they correspond to negative and infinite values of critical Rayleigh numbers in the presence of compressibility. The viscoelastic parameter F vanishes with σ_1 and therefore, the visco-elastic Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid.

In the absence of magnetic field, the equation (44) reduces to

$$R_1=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{xH_1}\right)\left[\frac{(1+x)^2}{P}+T_A\left(\frac{P}{(1+x)}\right)\right]. \tag{45}$$

To study the effects of suspended particles, magnetic field, rotation, medium permeability and compressibility on the system, we examine the nature of dR_1/dH_1, dR_1/dQ_1, dR_1/dT_{A_1}, dR_1/dP and dR_1/dG analytically.

Equation (44) yields

(46) $$\frac{dR_1}{dH_1}=-\left(\frac{G}{G-1}\right)\left(\frac{1+x}{xH_1^2}\right)\left[Q_1+\frac{(1+x)^2}{P}+T_{A_1}\left(\frac{P(1+x)}{(1+x)^2+PQ_1}\right)\right],$$

the negative sign implies that, for a stationary convection, the suspended particles have destabilizing effect on the system.
Equation (44) depicts that

(47) $$\frac{dR_1}{dQ_1}=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{xH_1}\right)\left[1-T_{A_1}\left(\frac{P^2(1+x)}{(1+x)^2+PQ_1^2}\right)\right],$$

which shows that magnetic field has stabilizing effect in the absence of rotation.
In the presence of rotation, magnetic field will have a destabilizing effect if $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}$, and stabilizing effect if $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}$.

To study the effect of rotation, we examine the nature of dR_1/dT_{A_1}. From equation (44), we obtain

(48) $$\frac{dR_1}{dT_{A_1}}=\left(\frac{G}{G-1}\right)\left(\frac{1+x}{xH_1}\right)\left[\left(\frac{P(1+x)}{(1+x)^2+PQ_1}\right)\right].$$

For analyzing the effect of permeability, we examine the nature of $\frac{dR_1}{dP}$.
Equation (44) yields

(49) $$\frac{dR_1}{dP}=-\frac{1}{P^2}\left(\frac{G}{G-1}\right)\frac{(1+x)^2}{xH_1}\left[1-T_{A_1}\left(\frac{P^2(1+x)}{(1+x)^2+PQ_1^2}\right)\right].$$

It is clear from equation (48) that, for stationary convection, the medium permeability has destabilizing effect on the system in the absence of rotation.
In the presence of rotation, it has stabilizing effect if $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}$ and destabilizing effect if $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}$.
Equation (44) also yields

(50) $$\frac{dR_1}{dG}=-\frac{1}{(G-1)^2}\left(\frac{1+x}{xH_1}\right)\left[Q_1+\frac{(1+x)^2}{P}+T_{A_1}\left(\frac{P^2(1+x)}{(1+x)^2+PQ_1^2}\right)\right],$$

which is always negative implying thereby the destabilizing effect of compressibility on the system.

It is not feasible to find the critical non-dimensional wave numbers from the dispersion relation given by equation (44). Therefore, to find the critical thermal Rayleigh number in the presence of parameters G, H_1, Q_1, P and T_{A_1}, equation (44) has been examined numerically using the software Mathematica version 5.2.

6. Numerical Results and Discussion

We have plotted the variation of Rayleigh number for stationary convection with respect to wave number using equation (44), for fixed permissible values of the dimensionless parameters G=10, T_{A_1}=30, Q_1=30, $P=0.2$, $\varepsilon=0.5$ and $H_1=1.01$. These are the permissible values for the respective parameters and are in good agreement with the corresponding values used by Chandrasekhar[1] and many authors while discussing various hydrodynamic and hydromagnetic stability problems.

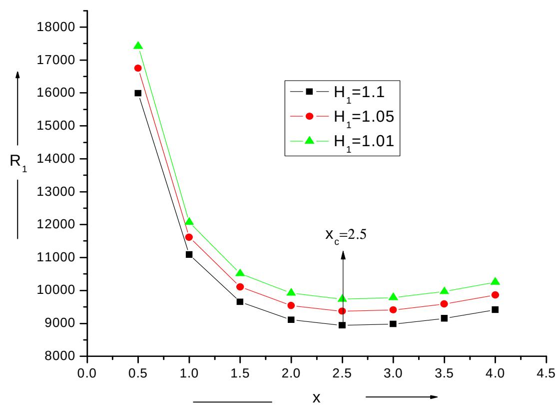

Figure 1: The variation of Rayleigh number R_1 with wave number x for three values of suspended particles H_1 =1.01, 1.05, 1.1 for fixed permissible values of other parameters G=10, T_{A_1}=30, Q_1=30, $P=0.2$, $\varepsilon=0.5$.

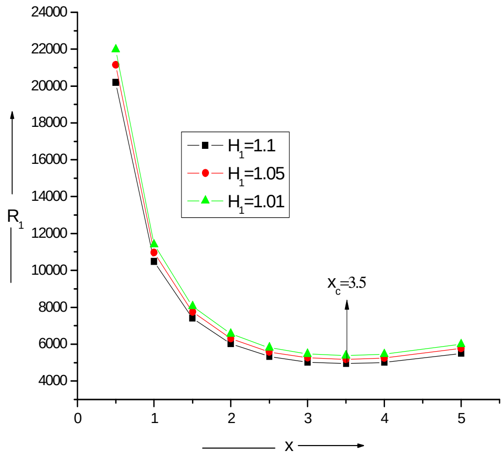

Figure 2: The variation of Rayleigh number R_1 with wave number x for three values of suspended particles $H_1 = 1.01, 1.05, 1.1$ for fixed permissible values of other parameters G=10, T_{A_1} =30, $P = 0.2$, $\varepsilon = 0.5$ in the absence of magnetic field.

Figures 1 and 2 correspond to three values of the suspended particles $H_1 = 1.01, 1.05, 1.1$, in the presence and absence of magnetic field, respectively. The graphs show that Rayleigh number decreases with the increase in suspended particles for a fixed wave number depicting thereby the destabilizing effect on the system. It is also clear from the graphs that the critical wavenumber and critical Rayleigh number decrease with the increase in suspended particles.

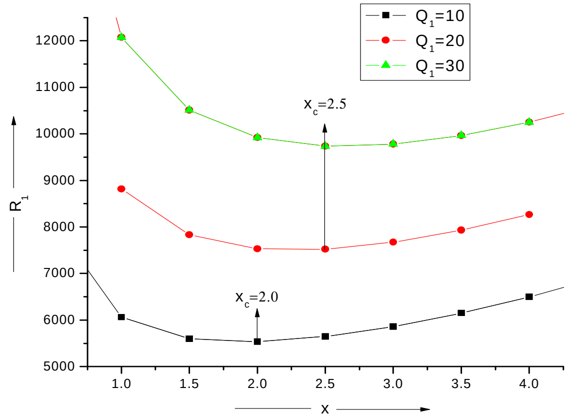

Figure 3: The variation of Rayleigh number R_1 with wave number x for three values of magnetic field Q_1=10, 20, 30 for fixed permissible values of other parameters G=10, T_{A_1} =30, H_1=1.01, $P = 0.2$, $\varepsilon = 0.5$.

Figures 3 corresponds to three values of the Chandrasekhar number Q_1=10, 20, 30 accounting for magnetic field satisfying the condition $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}$, respectively. The graph shows that Rayleigh number increases with the increase in Q_1 for a fixed wave number. The critical wave number and critical Rayleigh number also increase with the increase in Q_1. Thus magnetic field has large enough stabilizing effect on the system.

Figure 4 corresponds to three values of the Chandrasekhar number Q_1=10, 20, 30 accounting for magnetic field satisfying the condition $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}$, respectively. The graph shows that Rayleigh number decreases with the increase in Q_1 for a fixed wave number. The critical wave number and critical Rayleigh number also increase with the increase in Q_1. Thus magnetic field has destabilizing effect on the system.

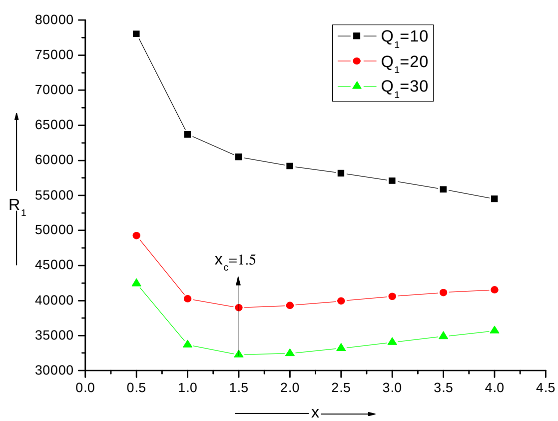

Figure 4: The variation of Rayleigh number R_1 with wave number x for three values of magnetic field $Q_1 = 10, 20, 30$ for fixed permissible values of other parameters G=10, T_{A_1}=1000, H_1=1.01, $P = 0.2$, $\varepsilon = 0.5$.

Figures 5 and 6 correspond to three values of the rotation parameter T_{A_1}=10, 20, 30, in the presence and absence of magnetic field, respectively. The graphs show that Rayleigh number increases with the increase in rotation parameter for a fixed wave number depicting thereby the stabilizing effect on the system in the presence of magnetic field as well as in the absence of magnetic field. However, the critical wave number and Rayleigh numbers increase with the increase in rotation parameter in the absence of magnetic field.

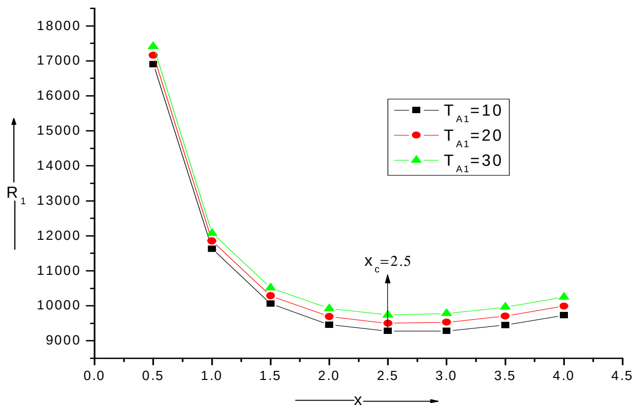

Figure 5: The variation of Rayleigh number R_1 with wave number x for three values of rotation parameter $T_{A_1}=10,\ 20,\ 30$ for fixed permissible values of other parameters $G=10,\ \ Q_1=30,\ \ H_1=1.01,\ \ P=0.2,\ \ \varepsilon=0.5$.

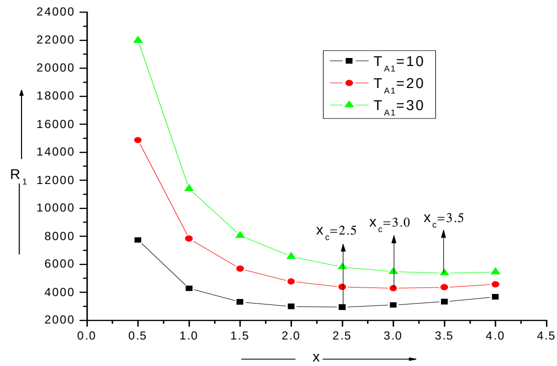

Figure 6: The variation of Rayleigh number R_1 with wave number x for three values of rotation parameter $T_{A_1}=10,\ 20,\ 30$ for fixed permissible values of other parameters $G=10,\ \ H_1=1.01,\ \ P=0.2,\ \ \varepsilon=0.5$ in the absence of magnetic field.

Figures 7-9 correspond to three different values of the medium permeability parameter $P=1.5,\ 2,\ 2.5$, in the presence and absence of magnetic field respectively. The graph shows that Rayleigh number decreases/increases with the increase in medium permeability for a fixed wave number depicting thereby the destabilizing / stabilizing effect on the system in the presence/absence of magnetic field. In the absence of magnetic field, the reverse effect may also occur for large wave numbers, as has been depicted in figure 4, implying thereby that the medium permeability has a stabilizing effect on the short wavelengths of the perturbations.

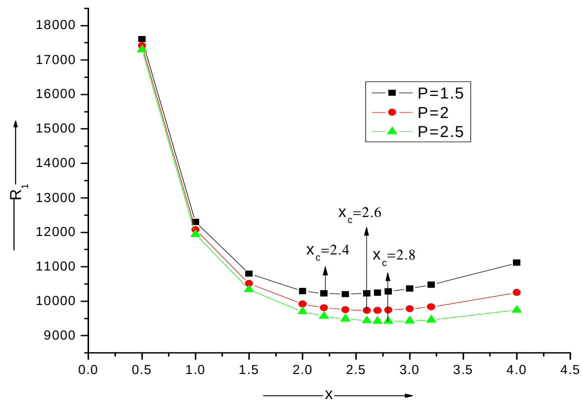

Figure 7: The variation of Rayleigh number R_1 with wave number x for three values of medium permeability $P=1.5,\ 2,\ 2.5$ for fixed permissible values of other parameters $G=10,\ T_{A_1}=30,\ Q_1=30,\ H_1=1.01,\ P=0.2,\ \varepsilon=0.5$.

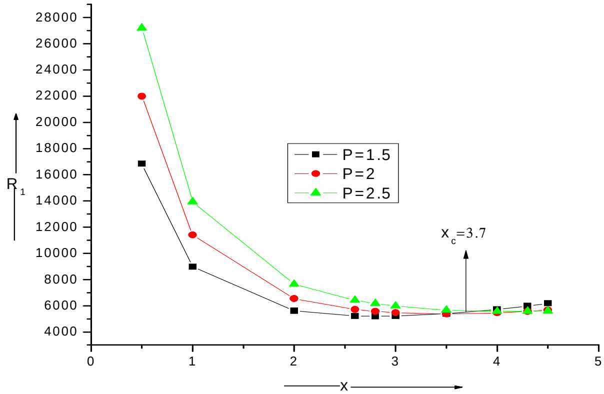

Figure 8: The variation of Rayleigh number R_1 with wave number x for three values of medium permeability $P=1.5,\ 2,\ 2.5$ for fixed permissible values of other parameters $G=10,\ T_{A_1}=30,\ H_1=1.01,\ P=0.2,\ \varepsilon=0.5$.

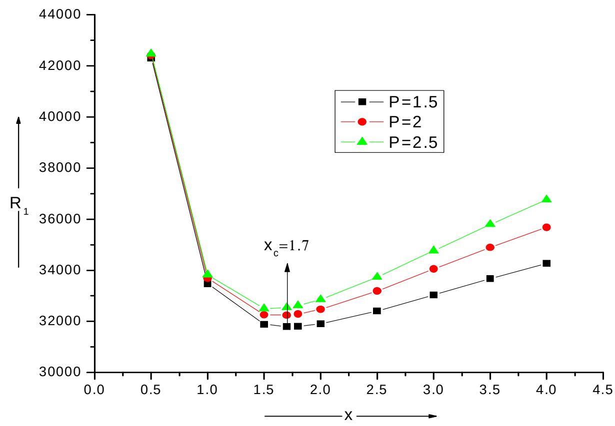

Figure 9: The variation of Rayleigh number R_1 with wave number x for three values of medium permeability $P = 1.5,\ 2,\ 2.5$ for fixed permissible values of other parameters $G=10,\ T_{A_1}=1000,\ H_1=1.01,\ P=0.2,\ \varepsilon=0.5$.

It is clear from the graphs that the Rayleigh number decreases with the increase in medium permeability in the absence of magnetic field satisfying the condition $T_{A_1} < \dfrac{(1+x+PQ_1)^2}{P^2(1+x)}$.

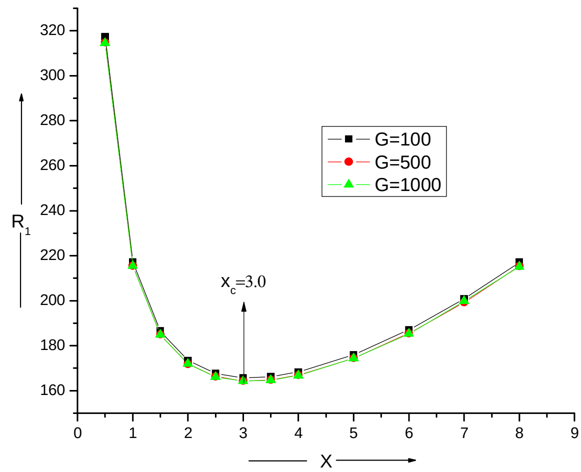

Figure 10: The variation of Rayleigh number R_1 with wave number x for three values of compressibility parameter $G=100,\ 500,\ 1000$ for fixed permissible values of other parameters $T_{A_1}=60,\ Q_1=50,\ H_1=1.01,\ P=2,\ \varepsilon=0.5$.

Figure 10 corresponds to three values of the compressibility parameter $G=100,\ 500,\ 1000$, respectively. The graph shows that Rayleigh number decreases slightly with the increase in the compressibility parameter for a fixed wave number depicting thereby very little destabilizing effect of the compressibility parameter on the system.

Conclusions

The thermal instability of compressible Rivlin-Ericksen elastico-viscous rotating fluid permeated with suspended particles saturating porous media in the presence of uniform magnetic field has been investigated analytically and numerically. The dispersion relation, including the effects of rotation, suspended particles, compressibility, medium permeability, magnetic field and viscoelasticity on the thermal instability of a Rivlin-Ericksen fluid is derived. From the analysis of the results, the principal conclusions drawn are as follow:

(i) For the case of stationary convection, Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid as elastico-viscous parameter F vanishes with σ_1.

(ii) The effect of compressibility and the suspended particles is to destabilize the system, thereby postponing onset of thermal instability.

(iii) Rotation parameter has always stabilizing effect on the system.

(iv) Magnetic field has stabilizing effect on the system in the absence of rotation whereas in the presence of rotation, it has destabilizing effect if $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}$ and stabilizing effect if $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}$ which is supported by figures 3 and 4.

(v) The medium permeability has always destabilizing effect on the system in the absence of rotation whereas in the presence of rotation it has stabilizing effect if $T_{A_1} < \frac{(1+x+PQ_1)^2}{P^2(1+x)}$ and destabilizing effect if $T_{A_1} > \frac{(1+x+PQ_1)^2}{P^2(1+x)}$.

(vi) The presence of rotation, compressibility, medium permeability, magnetic field and viscoelasticity introduce oscillatory modes.

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