

MHD Flow of a Dusty Viscous Incompressible Fluid Confined between Two Vertical Walls with Volume Fraction of Dust*

R. K. Khare and Neetu Singh

Department of Mathematics & Statistics
SHIATS, Allahabad (Deemed-To-Be-University)
(Formerly Allahabad Agricultural Institute)
E-mail: drrajeevkhare@gmail.com

(Received February 20, 2010)

Abstract: This study is concentrated on MHD flow of a dusty viscous incompressible fluid having significant amount of dust particles and the flow is occurring between two vertical walls under uniform magnetic field applied perpendicular to the direction of the flow. After forming the equations required for the concerned flow based on the existing conditions and assuming the suitable parameters, the equations have been solved to derive the expressions for velocities of both fluid and dust particle. Using simulation technique, the results have been discussed both graphically and numerically which have been found in consistent with the physical nature of the problem. It has been found that the maximum mean velocity occurs for the value of magnetic parameter ($B_0=2$) for all values of volume fraction of dust particles.

Keywords: MHD flow, magnetic parameter, incompressible fluid.

2000 Mathematics Subject Classification No.: 76W05

1. Introduction

The flow of a viscous fluid having volume fraction of dust particle confined between two vertical walls, in presence of magnetic field has an important place in different sectors therefore, it has attracted the attention of many researchers in recent years. In view of growing applications, Shankar and Sinha¹ studied the Rayleigh problem for a wavy wall. Gourela and Katock² discussed an unsteady free convection flow through the vertical parallel plates in the presence of uniform magnetic field. Takhar³ considered the effect of radiation on free convection flow along semi-infinite vertical plate in presence of transverse magnetic field. Patidar and Purohit⁴ studied free convection flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Borkakati and Chakraborty⁵

*Presented at CONIAPS XI, University of Allahabad, Feb. 20-22, 2010.

investigated the nature and behavior of a viscous incompressible, electrically conducting fluid over a flat plate moving with a uniform speed in a fluid in the presence of a uniform magnetic field. In their study, they have found that for an incompressible fluid, velocity decreases with increasing viscosity parameter. Rao & Radhkrishnamacharya⁶ studied this flow of magnetic fluid through a non uniform wavy tube. Recently, Sinha⁷ investigated the effect of heat transfer on unsteady hydromagnetic flow in a parallel plate channel of an electrically conducting viscous, incompressible fluid. He found that velocity distribution increased near the plate and then decreases very slowly at the central portion between the two plates.

In this paper, we have studied the flow of a dusty viscous incompressible fluid with volume fraction of dust such that the flow occurs between two vertical and parallel walls in presence of a magnetic field. The equations governing the flow have been solved numerically using dimensionless quantities for both phases but we have presented our findings about mean velocity of the flow.

2. Formulation of the problem

Consider a steady laminar flow of a dusty incompressible fluid occurring in presence of a transverse magnetic field between two vertical and parallel walls. The distance between the flat wall parallel (y-axis) is d . The equations governing this two-dimensional flow are:

$$(2.1) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{f}{\tau} (u - v) - \frac{\sigma B_0^2 u}{\rho},$$

$$(2.2) \quad u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right),$$

where u and v are velocities of fluid and dust particles, respectively and

B_0 : is the uniform magnetic field,

p : is the pressure,

σ : is the electrical conductivity,

ρ : is the density of the dust particles,

ν : Kinematic coefficient of viscosity,

f : (mN_0/ρ) is the mass-concentration parameter of the dust –particles,

τ : Relaxation- parameter equal to (m/K) .

To solve these equations, consider the following dimensionless quantities:

$$(2.3) \quad \bar{x} = \frac{x}{d}, \quad \bar{y} = \frac{y}{d}, \quad \bar{u} = \frac{ud}{\nu}, \quad \bar{v} = \frac{vd}{\nu}, \quad \bar{p} = \frac{p}{\rho \left(\frac{\nu}{d}\right)^2},$$

which on substitution in the equations (2.1) & (2.2); modify these equations and conditions to give them as (after dropping bars)

$$(2.4) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{fd^2}{\tau\nu} (u - v) - M^2 u,$$

$$(2.5) \quad u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right),$$

where $M^2 = \frac{\sigma B_0^2 d^2}{\rho\nu}.$

According to the condition of the flow of fluid the solution will consist of a mean part and a perturbed part of velocity in the form of equation (2.6):

$$(2.6) \quad \begin{aligned} u(x, y) &= u_0(y) + \varepsilon u_1(x, y), \\ v(x, y) &= \varepsilon v_1(x, y), \\ p(x, y) &= p_0(x) + p_1(x, y). \end{aligned}$$

In view of the form of equations (2.6), the governing equations (2.4) and (2.5) assume the form:

$$(2.7) \quad \frac{\partial^2 u_0}{\partial y^2} - N^2 u_0 = -C,$$

$$(2.8) \quad u_0 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_0}{\partial y} = -\frac{\partial p_1}{\partial x} + \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right) - \frac{fd^2}{\tau\nu} v_1 - N^2 u_1,$$

$$(2.9) \quad u_0 \frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \right),$$

where $C = -\frac{\partial p_0}{\partial x}, \quad N^2 = \left[M^2 - \frac{fd^2}{\nu\tau} \right].$

The boundary conditions are:

$$\begin{aligned} u_0 &= 0, & \text{at } y &= 0, \\ u_0 &= h_1 u'_0, & \text{at } y &= 1, \end{aligned}$$

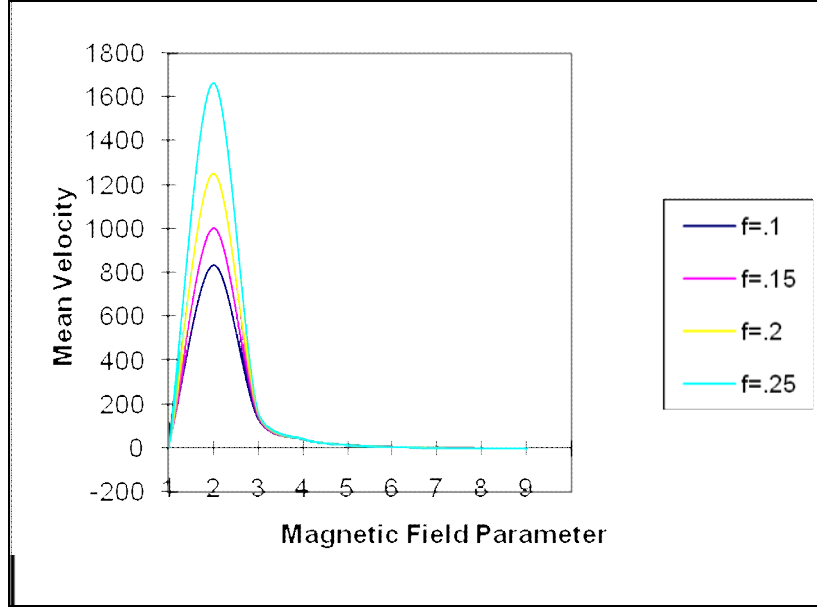
where h_1 is a constant.

The differential equation (2.7) is solved with above boundary conditions, to obtain the mean velocity

$$u_0 = Ae^{-Ny} + Be^{Ny} + \frac{C}{N^2},$$

where

$$A = -B = \frac{C}{N^2 \left[e^{-N} - e^N + h_1 N e^{-N} + e^N h_1 N \right]}.$$



3. Result & Discussion

Mean Velocity (U_0) Vs Magnetic-Field (B_0):

The graph shows that as the magnetic field increases, for different values of the volume fraction (f), the mean velocity also increases upto a maximum value and comes down to zero level on positive side with large amplitudes showing a wavy – form with a resonance character which is obvious due to the fact that this velocity is dominated by the magnetic field. As f increases, the peak occurs at greater heights i.e. the mean velocity gains maximum value for large values of f . Moreover the increasing value of f accelerates this condition to occur for low values of B_0 . It is observed that the maximum mean velocity occurs for low values of f (near $B_0=2$). The increasing concentration of dust particles, seem to be more effective in this case.

Therefore, it is observed that the motion of a dusty fluid having magnetic sensitive particles (two phase system) can be controlled effectively by the

application of magnetic field as expected. Hence, it gets an important role in the flow of dusty fluids between parallel walls.

References

1. P. N. Shankar & U. N. Sinha, The Rayleigh problem for a wavy wall., *J. Fluid Mech.*, **77** (1976) 243-56.
2. M. G. Gurla and I. Suaham Katoch, Unsteady free convection MHD Flow between heated vertical plate, *Ganita*, **42(2)** (1991) 143-153.
3. H. S. Takhar, R. S. R. Garal and V. M. Soundagekar., Short communication radiation effect on MHD free convection flow a gas past a semi-infinite vertical plate, *Int. J. Num. Methods for Heat and Fluid*, **6 (2)** (1996) 77-83.
4. R. P. Patidar & G. N. Purohit, Free convection flow a viscous incompressible fluid in a porous medium between two long vertical wavy walls., *Indian J. Math.*, **40**(1998) 76-86.
5. A. S. Borkakati and S. Chakrabarty Effect of variable viscosity on laminar convection flow an electrically conducting fluid on a uniform magnetic field, *Theoretical and Applied Mechanics*, **27** (2002) 49-61.
6. G. Radhakrishnamacharya and P. Srinivasa Rao, Flow of a magnetic fluid through a non uniform wavy tube, *Proc. Nat. Acad. Sci. India*, **76(A) III** (2007) 241-245.
7. K. G. Singha, The effect of heat transfer on unsteady hydromagnetic flow in a parallel plate channel of an electrically conducting, *issue 2* (2008) 172-186.