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Availability Analysis of Embedded Computer System with Two Types of Failure and Common Cause Failure*

M. Jain

Department of Mathematics, Indian Institute of Technology, Roorkee, India

Email: madhufma@iitr.ernet.in, madhujain@sancharnet.in

S. C. Agrawal and Sulekha Rani

School of Basic and Applied Sciences, Shobhit University, Meerut, India.

Email: scagrawal7@rediffmail.com; sulekha.math@gmail.com

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Abstract: Redundancy of hardware components is generally required to design highly reliable embedded computer systems. A common form of redundancy is a K-out-of-N: G system in which at least K out of N components must be good for the system to be good. This investigation is concerned with a Markov model for K-out-of-N: G system with common cause failure. The hardware system consists of N non- identical components and Y warm standby components. There is a single repairman who repairs the failed components on a first-come-first-served basis. The developed probabilistic model represents the redundant computer system with one software component. The software/hardware system along with human error and hardware error has been investigated in order to obtain reliability indices under the assumption that each components may fail due to two types of failures (hardware and human) or common cause or software failure. Numerical results have been facilitated with the help of Runge-Kutta method of fourth order to validate the analytical results. The sensitivity of parameters on system availability has also been carried out.

Keywords:- Reliability, Availability, K-out-of-N: G system, Human error, Hardware/ Software failure, Common cause failure.

1. Introduction

In various situations, the systems are sometime affected by environmental factors such as human errors or common cause failure. Human errors are important while predicting the reliability and safety measures of any engineering system. In a real life situation, many faults are caused directly or indirectly due to human errors such as wrong action, poor communication, wrong interpretation, poor handling, poor maintenance and operation procedure, etc. Further, common cause failure is also key factor that should be incorporated to predict the system reliability in different

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frameworks. The common cause failure may occur due to equipment design deficiency, power supply, humidity, temperature, etc.. An example of a human error is the fire in a room where the redundant units are located. In this case the entire redundant system will fail, irrespective of whether one or more units were operating. Hardware failures occur due to flaws in the design and manufacturing processes, faulty operations, poor quality control, poor maintenance, etc. Hence realistic reliability model must include the occurrence of human errors, hardware failure and common cause failure. The system reliability/availability can be quantified more accurately by the use of these concepts.

In recent years significant attention of researchers has been focused on reliability issues by considering the common cause failure. Jain[1] and Vaurio[2,3] discussed the reliability analysis of two units system with common cause failure. Whittaker et al.[4] considered a Markov chain model for predicting the reliability of multi-build software. Kuo et al.[5] presented the framework for modeling software reliability using various testing-effort and fault detection rates. Yadavalli et al.[6] analyzed the asymptotic confidence limits for the steady state availability of a two unit parallel system with preparation time for the repair facility. Ou and Bechta-Dugan[7] considered the approximate sensitivity analysis for acyclic Markov reliability models. Azaron et al.[8] studied the reliability function of a class of time dependent systems with standby redundancy. Blokus[9] presented the reliability analysis of large systems with dependent components. Levitin[10] considered the block diagram method for analyzing multistate systems with uncovered failures. Hall and Mosleh[11] analyzed the framework for reliability growth of one-shot systems. El-Damcese[12] examined a warm standby system subject to common cause failures with time varying failure and repair rates.

It is common knowledge that redundancy can be used to increase the reliability of a system without changing the reliability of the individual units that form the system. k-out-of-n:G warm standby systems have found applications in various fields including power plant, network design, redundant system testing, medical diagnosis, etc.. In a K-out-of-n: G system, K is the minimum number of components that must work if the whole system consisting of total N components is to work. Dutuit and Rauzy[13] and Smidt-Destombesa et al.[14] considered the assessment of K-out-of-N and related systems. Zhang et al.[15] obtained availability and reliability of K-out-of (M+N): G warm standby systems. Lu and Lewis[16] studied the configuration determination for K-out-of-N partially redundant systems. Chakravarthy et al.[17] considered the influence of delivery times on repairable K-out-of-N systems with spares.

In the present investigation, we develop Markov model for K-out-of-N: G system by incorporating the failures caused by human error and hardware problem for a multi-component system which is initially considered to be.

2. Model Description

We develop the Markov model for the multicomponent system which is initially considered to be in good state. The system or the components may fail due to hardware failure and human error. In addition to this the system is subject to failure due to some common cause as well as due to software failure. The provision of warm standbys hardware components is also taken to be consideration. The following assumptions are made to formulate the model:

- The system consists of M operating and Y warm standby hardware components. The system functions successfully with at least K components.
- The failed components are repaired in the order of their failure.
- There is single repairman and he is always available to repair the failed components.
- The life time and repair time of the hardware components are exponentially distributed.
- The switching time from standby to operating component is assumed to be negligible.
- The system may also fail due to common cause failure or software failure according to exponential distribution.

Notations

N: Total number of hardware components in the system i.e. N=M+Y.

λ : Failure rate of an operating hardware component due to hardware failure.

λ' : Failure rate of an standby hardware component due to hardware failure.

λ_h : Failure rate of an operating hardware component due to human failure.

λ'_h : Failure rate of a standby hardware component due to human failure.

λ_C : Failure rate of an operating hardware component due to common cause failure.

λ_S : Failure rate of an operating hardware component due to software failure.

μ : Repair rate of a component failed due to hardware faults when at least one standby is available.

μ_h : Repair rate of a failed component due to human failure.

μ_C : Repair rate of a failed component due to common cause.

μ_S : Repair rate of a component failed due to software failure.

$P_{(0,0)}(t)$: Probability that there is no failed component at time t.

$P_{(i,j)}(t)$: Probability that there are i $(0 \le i \le N)$ and j $(0 \le j \le N)$, where $1 \le i + j \le N$, components failed due to hardware failure and human failure respectively, at time t.

$P_{(Sf)}(t)$: Probability that the system fails due to software failure at time t.

$P_{(cf)}(t)$: Probability that the system fails due to common cause at time t.

3. The Analysis

Now we construct the differential difference equations governing the model as follows:

(1) $$\frac{dP_{(0,0)}(t)}{dt} = -\left[(M\lambda + Y\lambda') + (M\lambda_h + Y\lambda'_h) + \lambda_C + \lambda_S\right]P_{(0,0)}(t) + \mu P_{(1,0)}(t) + \mu_h P_{(0,1)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf.)}(t)$$

(2) $$\frac{dP_{(i,0)}(t)}{dt} = -[(M\lambda) + (Y-i)\lambda' + (M\lambda_h + (Y-i)\lambda'_h) + \lambda_C + \lambda_S + \mu]P_{(i,0)}(t) + \left[M\lambda + (Y-i+1)\lambda'\right]P_{(i-1,0)}(t) + \mu P_{(i+1,0)}(t) + \mu_h P_{(i,1)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf.)}(t), \quad 1 \le i \le Y$$

(3) $$\frac{dP_{(i,0)}(t)}{dt} = -\left[(M+Y-i)\lambda + (M+Y-i)\lambda_h + \lambda_S + \lambda_C + \mu\right]P_{(i,0)}(t) + M\lambda P_{(i-1,0)}(t) + \mu_h P_{(i,1)}(t) + \mu P_{(i+1,0)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t), \qquad Y+1 \le i \le N-1$$

(4) $$\frac{dP_{(N,0)}(t)}{dt} = -\mu P_{(N,0)}(t) + \lambda P_{(N-1,0)}(t),$$

(5) $$\frac{dP_{(0,j)}(t)}{dt} = -[\{M\lambda + (Y-j)\lambda'\} + \{M\lambda_h + (Y-j)\lambda'_h\} + \lambda_C + \lambda_S + \mu_h + \mu]P_{(0,j)}(t) + (M\lambda_h + (Y-j+1)\lambda'_h)P_{(0,j-1)}(t) + \mu P_{(1,j)}(t) + \mu_h P_{(0,j+1)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t), \quad 1 \le j \le Y$$

(6) $$\frac{dP_{(0,j)}(t)}{dt} = -\left[\left(M+Y-j\right)\lambda + \left(M+Y-j\right)\lambda_h + \lambda_C + \lambda_S + \mu + \mu_h\right]P_{(0,j)}(t)$$
$$+ M\lambda_h P_{(0,j-1)}(t)$$
$$+ \mu_h P_{(0,j+1)}(t) + \mu P_{(1,j)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t),$$
$$Y+1 \le j \le N-1$$

(7) $$\frac{dP_{(0,N)}(t)}{dt} = -\mu_h P_{(0,N)}(t) + \lambda_h P_{(0,N-1)}(t),$$

(8) $$\frac{dP_{(i,j)}(t)}{dt} = -\begin{bmatrix}\left\{M\lambda + \left(Y-i-j\right)\lambda'\right\} \\ +\left\{M\lambda_h + \left(Y-i-j\right)\lambda'_h\right\} + \lambda_C + \lambda_S + \mu + \mu_h\end{bmatrix}P_{(i,j)}(t)$$
$$+\left\{M\lambda + \left(Y-i-j+1\right)\lambda_h\right\}P_{(i-1,j)}(t)$$
$$+\left\{M\lambda_h + \left(Y-i-j\right)\lambda'_h\right\}P_{(i,j-1)}(t)$$
$$+\mu P_{(i+1,1)}(t) + \mu_h P_{(i,j+1)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t),$$
$$i, j \ne 0,\ 2 \le i+j \le Y$$

(9) $$\frac{dP_{(i,j)}(t)}{dt} = -[\{M+Y-i-j\}\lambda + \{M+Y-i-j\}\lambda_h + \lambda_C$$
$$+\lambda_S + \mu + \mu_h]P_{(i,j)}(t) + M\lambda\lambda_{(i-1,j)}(t) + M'_h P_{(i,j-1)}(t)$$
$$+\mu P_{(i+1,0)}(t) + \mu_h P_{(i,j+1)}(t) + \mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t),$$
$$i, j \ne 0,\ Y+1 \le i+j \le N-1$$

(10) $$\frac{dP_{(i,j)}(t)}{dt} = \left(\mu + \mu_h + \lambda_C + \lambda_S\right)P_{(i,j)}(t) + \lambda P_{(i-1,j)}(t) + \lambda_h P_{(i,j-1)}(t)$$
$$+\mu_S P_{(Sf)}(t) + \mu_C P_{(Cf)}(t),$$
$$i, j \ne 0,\ i+j = N$$

4. Performance Indices

We obtain the some performance indices by using probabilities obtained in previous section as follows:

- Expected number of failed components at time t due to hardware error is

(11) $$E\{N_{hard}(t)\} = \sum_{i=1}^{N} i \sum_{j=0}^{N-i} P_{(i,j)}(t).$$

- Expected number of failed components at time t due to human error is

(12) $$E\{N_{human}(t)\} = \sum_{j=1}^{N} j \sum_{i=0}^{N-j} P_{(i,j)}(t).$$

- Expected number of working components in the system at time t is

(13) $$E\{N_{working}(t)\} = M - \sum_{i+j=Y+1}^{N} (i+j-Y) P_{(i,j)}(t).$$

- Expected number of standby components in the system at time t is

(14) $$E\{N_{standby}(t)\} = \sum_{i+j=0}^{Y} (Y - i - j) P_{(i,j)}(t).$$

- Component availability at time t is

(15) $$A_{comp}(t) = 1 - \left[\frac{E\{N_{human}(t)\} + E\{N_{hard}(t)\}}{N} \right].$$

- System unavailability at time t is

(16) $$UA_{system}(t) = 1 - A_{comp}(t)$$

5. Numerical Results

Runge-Kutta Technique (RKT) of fourth order is used to calculate the system of differential equations, which is implemented by exploiting the software MATLAB's 'ode 45' function. A time span is taken with equal intervals. The numerical results are displayed in tables 1(a)-1(b). The graphical presentation of reliability R(t) has been done in figs 2(a)-2(d) for different varying parameters and default parameters choosen as follows

$$\lambda = 0.1, \lambda' = 0.01, \lambda_h = 0.01, \lambda_h' = 0.15, \lambda_S = 0.14, \lambda_C = 0.29, \mu = 0.001,$$
$$\mu_h = 0.002, \mu_S = 0.001, \mu_C = 0.001.$$

From table 1(a) we notice the patterns of various performance indices namely $E\{N_{hard}(t)\}$, $E\{N_{human}(t)\}$, $E\{N_{working}(t)\}$ and $E\{N_{s\tan dby}(t)\}$ by varying the repair rates. It is observed that there is a decreasing trend in the values of the $E\{N_{hard}(t)\}, E\{N_{human}(t)\}$, $E\{N_{working}(t)\}$, $E\{N_{s\tan dby}(t)\}$ with increasing values of μ, μ_h, μ_C, μ_S. In the table 1(b), we demonstrate the system availability for different values of failure rates at fix time t = 5.

Table 1(a): Performance indices for different values of (μ , μ_h) and (μ_C , μ_S)

μ	μ_h	μ_C	μ_S	t	E{N$_{hard}$(t)}	E{N$_{human}$(t)}	E{N$_{work}$(t)}	E{N$_{stand}$(t)}
0.45	0.4	0.5	0.5	0	0	0	1	1
				2	0.19999	0.134955	0.094451	0.006999
				4	0.070383	0.043826	0.030845	0.000878
				6	0.037462	0.021731	0.015971	0.00041
				8	0.020852	0.011232	0.008658	0.000211
0.9	0.8	0.5	0.5	0	0	0	1	1
				2	0.227216	0.151455	0.115752	0.019112
				4	0.063309	0.037373	0.029252	0.002943
				6	0.022243	0.011735	0.009802	0.000886
				8	0.008314	0.003991	0.003536	0.000302
0.45	0.4	0.7	0.9	0	0	0	1	1
				2	0.071508	0.047969	0.033341	0.002169
				4	0.020423	0.012539	0.00882	0.000192
				6	0.009112	0.005135	0.003813	0.000076
				8	0.004276	0.002188	0.001731	0.000032

Table 1(b): System Availability for different values of (λ_h , λ_h')

System Availability $A(t)$						
λ	λ'	λ_s	λ_C	t	$(\lambda_h = 0.3, \lambda_h' = 0.$	$(\lambda_h = 0.9, \lambda_h' = 0.$
0.4	0.5	0.5	0.5	5	0.988739	0.977129
0.8	0.7	0.5	0.5	5	0.981654	0.969133
0.4	0.5	0.9	0.9	5	0.997062	1.000000

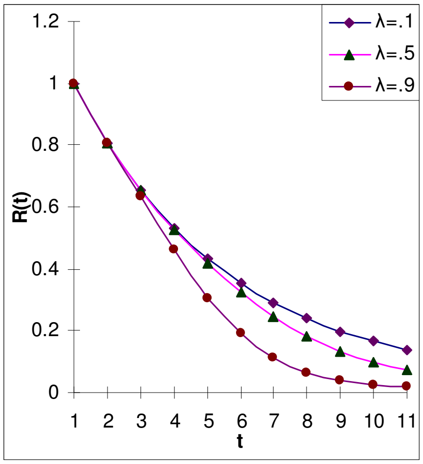

Fig. 2(a): Reliability vs time by varying λ

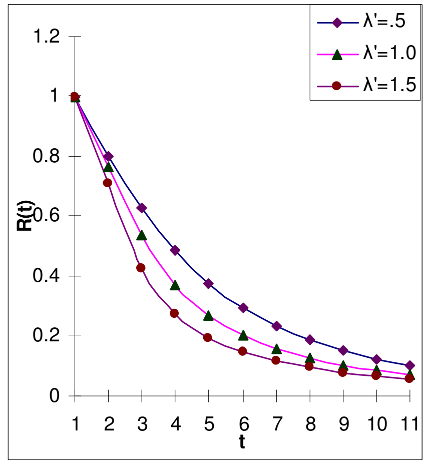

Fig. 2(b): Reliability vs time by varying λ'

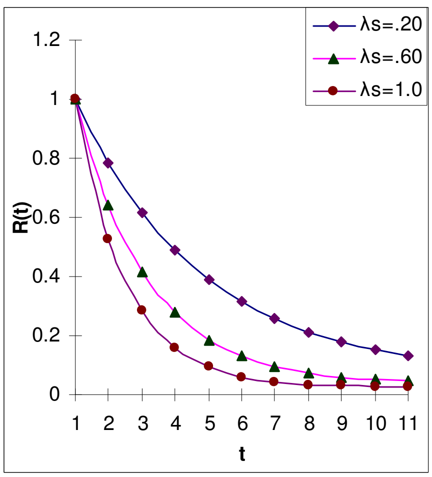

Fig. 2(c): Reliability vs time by varying λ_S

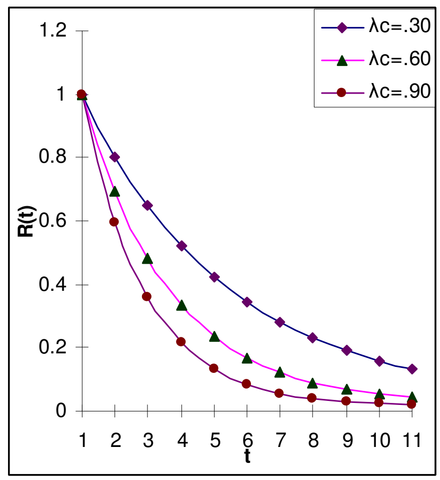

Fig. 2(d): Reliability vs time by varying λ_C

In figs 2(a)-2(d), we show the variation of reliability with time for different values of $\lambda, \lambda', \lambda_S$ and λ_C, respectively. Fig. 2(a) reveals the behavior of reliability with respect to time t and failure rate λ. It is found that reliability decreases sharply for the initial values of t but shows a smooth decreasing pattern for the further increased values of t. Then after in figs 2(b)-2(d), we illustrate the behavior of reliability R(t) with respect to time t by varying the parameters λ', λ_S and λ_C, respectively. It is noticed that as the values of failure rates $(\lambda', \lambda_S \text{ and } \lambda_C)$ increase, the reliability decreases in each case.

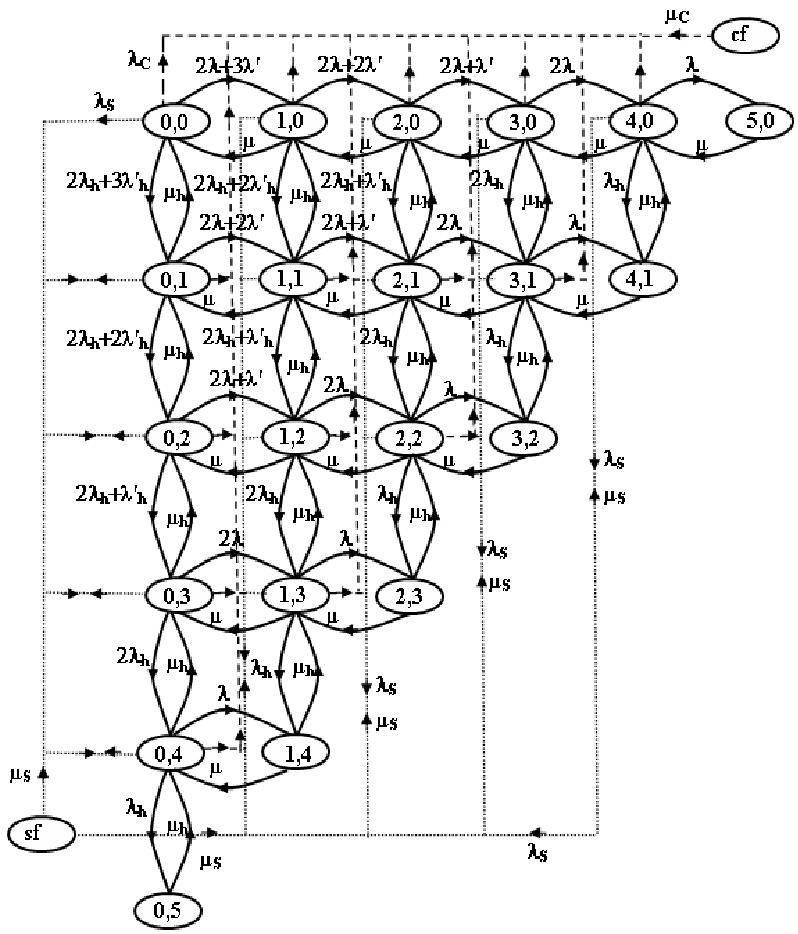

Fig. 1: State transition digram

Conclusion

The reliability of a system without assuming human error and common cause failure may not depict a real picture of the actual reliability/availability modeling. Therefore the real time system reliability modeling must include the occurrence of common cause failures, hardware error and human error. A K-out-of-N: G system with warm standby components is studied in this paper. The transient availability and other performance indices obtained may be helpful to improve the system availability in particular when occurrence of common cause failure and human errors are involved. Our investigation in the present study facilitates an insight to the system designers and developers to produce more reliable embedded computer systems by judging correct measure of fault generation.

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