

Threshold N-Policy for $M^X/H_2/1$ Queueing System with Un-reliable Server and Vacations*

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Abstract: In this paper, we consider a $M^X/H_2/1$ queueing system under N-policy with vacation and un-reliable server. Whenever the system becomes empty, the server may take vacation. After returning from vacation, if the server finds at least N customers waiting in the queue, it immediately provides service to all the customers who are waiting in the queue. If the number of the customers waiting in the queue is less than N, then the server may take another vacation and this process is continued. Some performance measures of the system are derived, which are further evaluated by using Adaptive Network-based Fuzzy Interference Systems (ANFIS). Numerical results and sensitivity analysis are carried out by taking an illustration.

Keywords: Batch arrival, N-policy, $M^X/H_2/1$ queue, Un-reliable server, Generating Function, Queue size.

1. Introduction

In queueing literature when the server does not provide service to the customer until some specified number of customers, say N are accumulated in the system is referred as threshold N-policy. N-policy queueing models have wide applicability in many areas such as computer and telecommunication system, production and manufacturing system, etc.. For some notable contributions in this area we refer the works of Ke¹, Wang *et al.*², Choudhury *et al.*³ and many others.

In recent past, many researchers have analyzed queueing models with vacation. A vacation policy for queueing model was studied by Choudhury and Madan⁴, Sikdar *et al.*⁵, Ke *et al.*⁶, respectively in different frameworks.

Queueing system with an un-reliable server has received a significant amount of attention of the researchers working in the area of queueing and reliability theory. Mokaddis *et al.*⁷, Jain and Agrawal⁸, Choudhury and Tadj⁹ have analyzed the steady-state behavior of unreliable server.

In this paper, we study an unreliable $M^X/H_2/1$ queue under N-policy by incorporating vacation and state dependent rates. The organization of rest of

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the paper is as follows. We outline some assumptions and notations in section 2. Section 3 is devoted to the analysis of the queueing model. Various performance measures are derived in section 4. In section 5 and 6, we determine the minimum cost and special cases, respectively. Numerical results and concluding remarks have been given in section 7 and 8, respectively.

2. Model Discription

An un-reliable $M^X/H_2/1$ queueing system with vacation under N-policy is considered in our analysis. To formulate the mathematical model, we made the following assumptions:

- ❖ The system states are described by triplet i.e. (n,i,j) , where n ($n=0,1,\dots$) represents the number of the customers in the system; i (1,2) represents that the customer is in i^{th} type of service; j represents the states of the server which is given:

$$j = \begin{cases} V, & \text{represents that the server is on vacation} \\ B, & \text{represents that the server is in busy state} \\ D, & \text{represents that the server is in brokendown state} \end{cases}$$

- ❖ The customers arrive in batches with arrival rates λ_j depending upon the server states. Let X be the random variable denoting the batch size, then the generating function for the batch size distribution is given by

$$X(z) = \sum_{k=1}^{\infty} z^k c_k \cdot$$

$$\lambda_j = \begin{cases} \lambda_0, & \text{when the server is on vacation} \\ \lambda_1, & \text{when the server is in busy state} \\ \lambda_2, & \text{when the server is in brokendown state} \end{cases}$$

- ❖ The server provides service according to 2-hyper exponential distribution with rate μ_i of i^{th} type service. Let q_i ($i=1,2$) denote the probabilities of the next customer to enter in the service of type i , where $q_1+q_2=1$.
- ❖ Whenever the system becomes empty the server goes for a sequence of vacations; the vacation times follow the exponential distribution with mean $1/\theta$.
- ❖ In busy state, the server may break down with a Poisson rate α and the failed server immediately sent for repairing with repair rate β .

3. The Analysis

The probabilities of different states of the system are given below:

$P_{0,V}(n)$: Probability that there are n customers in the system when the server is on vacation.

$P_{i,B}(n)$: Probability that there are n customers in the system when the server is in busy state and the customer is in i^{th} ($i=1, 2$) type of service.

$P_{i,D}(n)$: Probability that there are n customers in the system when the server is in broken down state and the customer is in i^{th} ($i=1, 2$) type of service.

The Chapman-Kolmogorov equations governing the model are:

$$(3.1) \quad \lambda_0 P_{0,V}(0) = \mu_1 P_{1,B}(1) + \mu_2 P_{2,B}(1),$$

$$(3.2) \quad P_{0,V}(0) = P_{0,V}(n) = \sum_{k=1}^n P_{0,V}(n-k)c_k, \quad 1 \leq n \leq N-1$$

$$(3.3) \quad (\lambda_0 + \theta)P_{0,V}(n) = \lambda_0 \sum_{k=1}^n P_{0,V}(n-k)c_k, \quad n \geq N$$

$$(3.4) \quad (\lambda_1 + \alpha + \mu_i)P_{i,B}(1) = q_i[\mu_1 P_{1,B}(1) + \mu_2 P_{2,B}(2)] + \beta P_{i,D}(1), \quad \forall i=1,2$$

$$(3.5) \quad (\lambda_1 + \alpha + \mu_i)P_{i,B}(n) = q_i[\mu_1 P_{1,B}(1) + \mu_2 P_{2,B}(2)] + \lambda_1 \sum_{k=1}^{N-1} P_{i,B}(n-k)c_k + \beta P_{i,D}(n), \\ 2 \leq n \leq N-1, \quad \forall i=1,2$$

$$(3.6) \quad (\lambda_1 + \alpha + \mu_i)P_{i,B}(n) = q_i[\mu_1 P_{1,B}(1) + \mu_2 P_{2,B}(2)] + \lambda_1 \sum_{k=1}^{N-1} P_{i,B}(n-k)c_k \\ + \beta P_{i,D}(n) + q_i \theta P_{0,V}(n), \quad n \geq N, \quad \forall i=1,2$$

$$(3.7) \quad (\lambda_2 + \beta)P_{i,D}(1) = \alpha P_{i,B}(1), \quad \forall i=1,2$$

$$(3.8) \quad (\lambda_2 + \beta)P_{i,D}(n) = \alpha P_{i,B}(n) + \lambda_2 \sum_{k=1}^{n-1} P_{i,D}(n-k)c_k, \quad n \geq 2, \quad \forall i=1,2$$

Define the probability generating functions (PGF) as

$$H_{0,V}(z) = \sum_{n=0}^{\infty} z^n P_{0,V}(n), \quad |z| \leq 1,$$

$$H_{i,B}(z) = \sum_{n=1}^{\infty} z^n P_{i,B}(n), \quad |z| \leq 1, \quad \forall i=1,2$$

$$H_{i,D}(z) = \sum_{n=1}^{\infty} z^n P_{i,D}(n), \quad |z| \leq 1, \quad \forall i=1,2$$

Lemma 1: *The partial generating functions of the system are given by:*

$$(3.9) \quad H_{0,v} = \left[\frac{1 - z^{N-1}X(z)}{1 - X(z)} + \frac{\lambda_0 z^{N-1}X(z)}{(\lambda_0 + \theta - \lambda_0 X(z))} \right] P_{0,v}(0),$$

$$(3.10) \quad H_{i,B} = \frac{N_i(z)}{D(z)} P_{0,v}(0), \quad \forall i = 1, 2$$

$$(3.11) \quad H_{i,D}(z) = \frac{\alpha}{(\lambda_2 + \beta - \lambda_2 X(z))} H_{i,B}(z), \quad \forall i = 1, 2$$

where

$$N_1(z) = \lambda_0 z^2 q_1 \left[1 - \frac{\theta z^{N-1}}{(\lambda_0 + \theta - \lambda_0 X(z))} \left[\lambda_1 X(z) - (\lambda_1 + \alpha + \mu_2 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)}) \right] \right],$$

$$N_2(z) = \lambda_0 z^2 q_2 \left[1 - \frac{\theta z^{N-1}}{(\lambda_0 + \theta - \lambda_0 X(z))} \left[\lambda_1 X(z) - (\lambda_1 + \alpha + \mu_1 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)}) \right] \right],$$

$$D(z) = \left[\lambda_1 z X(z) - (\lambda_1 + \alpha + \mu_1 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)}) z + q_1 \mu_1 \right] \times \\ \left[\lambda_1 z X(z) - (\lambda_1 + \alpha + \mu_2 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)}) z + q_2 \mu_2 \right] - q_1 q_2 \mu_1 \mu_2,$$

Proof: The algebraic manipulation of equations (3.2) and (3.3) yields

$$(3.12) \quad H_{0,v} = \left[\frac{1 - z^{N-1}X(z)}{1 - X(z)} + \frac{\lambda_0 z^{N-1}X(z)}{(\lambda_0 + \theta - \lambda_0 X(z))} \right] P_{0,v}(0),$$

Multiplying equations (3.1), (3.4), (3.5), (3.6) by $q_i z$, z^2 , z^{n+1} ($2 \leq n \leq N-1$), z^{n+1} ($n \geq N$), respectively and summing over n , we obtain

$$(3.13) \quad [\lambda_1 z X(z) - (\lambda_1 + \alpha + \mu_i) z] H_{i,B}(z) + q_i [\mu_1 H_{1,B}(z) + \mu_2 H_{2,B}(z)] + \beta z H_{i,D}(z) \\ = \lambda_0 q_i z \times \left[1 - \frac{\theta z^{N-1}}{(\lambda_0 + \theta - \lambda_0 X(z))} \right] P_{0,v}(0), \quad \forall i = 1, 2$$

Again multiplying equations (3.7) and (3.8) by z and z^n then summing over n , we obtain

$$(3.14) \quad (\lambda_2 + \beta) H_{i,D}(z) = \alpha H_{i,B}(z) + \lambda_2 X(z) H_{i,D}(z), \quad \forall i = 1, 2$$

$$(3.15) \quad H_{i,D}(z) = \frac{\alpha}{(\lambda_2 + \beta - \lambda_2 X(z))} H_{i,B}(z), \quad \forall i = 1, 2$$

Now put $i=1$ and 2 , respectively in equation (3.13) and using equation (3.15), we get

$$(3.16) \quad H_{1,B}(z)[\lambda_1 zX(z) - (\lambda_1 + \alpha + \mu_1 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)})z + q_1\mu_1] \\ + q_1\mu_2 H_{2,B}(z) = q_1 z\lambda_0 P_{0,V}(0) [1 - \frac{\theta z^{N-1}}{(\lambda_0 + \theta - \lambda_0 X(z))}],$$

$$(3.17) \quad q_2\mu_1 H_{1,B}(z) + [\lambda_1 zX(z) - (\lambda_1 + \alpha + \mu_2 + \frac{\alpha\beta}{(\lambda_2 X(z) - \lambda_2 - \beta)})z + q_2\mu_2] H_{2,B}(z) \\ = q_2 z\lambda_0 P_{0,V}(0) [1 - \frac{\theta z^{N-1}}{(\lambda_0 + \theta - \lambda_0 X(z))}] P_{0,V}(0),$$

With the help of Cramer's rule we solve equations (3.16)-(3.17) and obtain equations (3.9)-(3.11).

Theorem 1: Let $H(z)$ denotes the probability generating function of the number of the customers in the system which is given by

$$(3.18) \quad H(z) = [\frac{1-z^{N-1}X(z)}{1-X(z)} + \frac{\lambda_0 z^{N-1}X(z)}{(\lambda_0 + \theta - \lambda_0 X(z))}] + \sum_{i=1}^2 [1 + \frac{\alpha}{(\lambda_2 + \beta - \lambda_2 X(z))}] \frac{N_i(z)}{D(z)}] P_{0,V}(0).$$

Proof: The p.g.f. of the number of the customers in the system is

$$(3.19) \quad H(z) = H_{0,V}(z) + \sum_{i=1}^2 [H_{i,B}(z) + H_{i,D}(z)],$$

Substituting the values of $H_{0,V}(z)$, $H_{i,B}(z)$ and $H_{i,D}(z)$ from equations (3.9)-(3.11) into equation (3.19), we obtain the equation (3.18).

4. Performance Measures

In this section, objective is to provide explicit expressions for the system state probabilities and some other performance measures using results established in previous section.

❖ The long run probabilities of the server being on vacation, busy and breakdown states are denoted by P_V , P_B and P_D , respectively and are given by

$$(4.1) \quad P_V = H_{0,V}(1) = [1 + \frac{N-1}{X'(1)} + \frac{\lambda_0}{\theta}] P_{0,V}(0),$$

$$(4.2) \quad P_B = H_{i,B}(1) = \frac{\xi_i \lambda_0 \beta [((N-1) + X'(1)) + \frac{\lambda_0 X'(1)}{\theta}]}{X'(1) [\beta - X'(1) (\lambda_1 \beta + \lambda_2 \alpha) (\frac{q_1}{\mu_1} + \frac{q_2}{\mu_2})]} P_{0,V}(0), \quad i=1,2$$

$$(4.3) \quad P_D = H_{i,D}(1) = \frac{\xi_i \lambda_0 \alpha [((N-1) + X'(1)) + \frac{\lambda_0 X'(1)}{\theta}]}{X'(1) [\beta - X'(1) (\lambda_1 \beta + \lambda_2 \alpha) (\frac{q_1}{\mu_1} + \frac{q_2}{\mu_2})]} P_{0,v}(0), \quad i = 1, 2$$

where

$$P_{0,v}(0) = \frac{\theta X'(1) \Omega}{[\theta (X'(1) + (N-1)) + \lambda_0 X'(1)] [\Omega + \lambda_0 (\alpha + \beta)]},$$

$$\Omega = [\beta - (\xi_1 + \xi_2) (\lambda_1 \beta + \lambda_2 \alpha)], \quad \xi_i = \frac{X'(1) q_i}{\mu_i}.$$

Theorem 2: The expected number of customers in the system ($E[NS_1]$) is given by

$$(4.4) \quad E[NS_1] = [(N-1) + \frac{(N-1)}{2(X'(1))^2} [X'(1)(N-2) - X''(1)] + \frac{\lambda_0}{\theta^2} [((N-1) + X'(1))\theta + \lambda_0 X'(1)]] P_{0,v}(0) + P_{0,v}(0) \sum_{i=1}^2 [(\frac{N_i'' D' - N_i' D''}{2D'^2}) + \alpha (\frac{\beta N_i'' D' - N_i' (\beta D'' - 2\lambda_2 D' X'(1))}{2\beta^2 D'^2})],$$

where $N_1' = q_1 \lambda_0 \mu_2 [((N-1) + X'(1)) + \lambda_0 X'(1) / \theta]$,

$$N_2' = q_2 \lambda_0 \mu_1 [((N-1) + X'(1)) + \lambda_0 X'(1) / \theta],$$

$$N_1'' = 2q_1 \lambda_0 [((N-1) + X'(1)) + \lambda_0 X'(1) / \theta] [2\mu_2 + X'(1) / \beta (\lambda_2 \alpha - \lambda_1 \beta)] + \frac{q_1 \lambda_0 \mu_2}{\theta^2} [(N-1)(N-2) + 2(N-1)X'(1) + X''(1) + \frac{\lambda_0 X''(1)}{\theta} + 2\lambda_0 X'(1)]$$

$$N_2'' = 2q_2 \lambda_0 [((N-1) + X'(1)) + \lambda_0 X'(1) / \theta] [2\mu_1 + X'(1) / \beta (\lambda_2 \alpha - \lambda_1 \beta)] + \frac{q_2 \lambda_0 \mu_1}{\theta^2} [(N-1)(N-2) + 2(N-1)X'(1) + X''(1) + \frac{\lambda_0 X''(1)}{\theta} + 2\lambda_0 X'(1)]$$

$$D' = \mu_1 \mu_2 - \frac{X'(1)}{\beta} (\lambda_1 \beta + \lambda_2 \alpha) (\mu_1 q_2 + \mu_2 q_1),$$

$$D'' = [2\lambda_1 X'(1) + \lambda_1 X''(1) + \frac{\alpha}{\beta^2} \lambda_2 (\beta X'(1) + X''(1)) + 2\lambda_1 (X'(1))^2] (\mu_1 q_2 + \mu_2 q_1) + 2[(\frac{X'(1)}{\beta} (\lambda_1 \beta + \lambda_2 \alpha))^2 - \frac{X'(1)}{\beta} (\lambda_1 \beta + \lambda_2 \alpha) (\mu_1 + \mu_2) + \mu_1 \mu_2],$$

Proof: Using, $E[NS_1] = \lim_{z \rightarrow 1} H'(z)$ and applying L Hospital rule twice, we obtain equation (4.4).

5. Optimal Design of N-Policy

Using the memoryless property, the expected length of idle period is given by

$$(5.1) \quad E[I] = \frac{N}{\lambda X'(1)}$$

Lemma 4: *The expected length of cyclic period, expected busy period, broken down period and completion period, respectively are given by*

$$(5.2) \quad E[C] = \frac{N\theta}{\lambda[\theta X'(1) + (N-1)\theta + \lambda_0 X'(1)]} P_{0,v}(0),$$

$$(5.3) \quad E[B] = E[C] \sum_{i=1}^2 \frac{q_i \lambda_0 \beta [((N-1) + X'(1)) + \frac{\lambda_0 X'(1)}{\theta}]}{\mu_i [\beta - X'(1)(\lambda_1 \beta + \lambda_2 \alpha) (\frac{q_1}{\mu_1} + \frac{q_2}{\mu_2})]} P_{0,v}(0),$$

$$(5.4) \quad E[D] = E[C] \sum_{i=1}^2 \frac{q_i \lambda_0 \alpha [((N-1) + X'(1)) + \frac{\lambda_0 X'(1)}{\theta}]}{\mu_i [\beta - X'(1)(\lambda_1 \beta + \lambda_2 \alpha) (\frac{q_1}{\mu_1} + \frac{q_2}{\mu_2})]} P_{0,v}(0),$$

$$(5.5) \quad E[H] = E[C] \sum_{i=1}^2 \frac{q_i \lambda_0 (\alpha + \beta) [((N-1) + X'(1)) + \frac{\lambda_0 X'(1)}{\theta}]}{\mu_i [\beta - X'(1)(\lambda_1 \beta + \lambda_2 \alpha) (\frac{q_1}{\mu_1} + \frac{q_2}{\mu_2})]} P_{0,v}(0),$$

Proof: Using,

$$(5.6) \quad E[C] = \frac{E[I]}{P_V}$$

and substituting the value of P_V from equation (4.1), we obtain the value of $E[C]$. Substituting the values of $E[C]$ and P_B from equations (5.2) and (4.3), respectively in $E[B] = E[C]P_B$, we obtain the expected length of busy period given in equation (5.3). Using relation,

$$(5.7) \quad E[D] = E[C]P_D$$

$$(5.8) \quad E[H] = E[B] + E[D]$$

Substituting the values of $E[C]$, P_D , $E[B]$ and $E[D]$ from equations (5.2), (4.3), (5.3) and (5.4), in above results, we obtain the values of $E[D]$ and $E[H]$, respectively.

Theorem 3: *The expected cost total per unit time is given by*

$$(5.9) \quad T_c(N_1) = (c_s + c_t) \frac{1}{E[C]} + c_v P_V + c_b P_B + c_d P_D + c_h E[NS_1],$$

where c_s (c_t) startup (shutdown) cost when the server is in turned on (off) state, c_v (c_b) cost incurred per unit time for keeping the server is on vacation (busy), c_d breakdown cost per unit time incurred on a broken down server, c_h holding cost per unit time for each customer present in the system.

Proof: We multiply each cost element with the corresponding probabilities to construct the cost function. To determine the optimum value N^* , we differentiate equation (5.9) with respect to N and then set the results equal to zero. Using the classical method of maxima and minima, we can obtain the best positive integer value of N which covers N^* and provides the smaller cost of $T_c(N_1)$. The suitable condition for satisfying the solution of N is $\frac{\partial^2 T_c(N^*)}{\partial N^2} > 0$. However, it is difficult to represent the result in explicit form. For this purpose, numerical experiment is done to illustrate that the function is indeed convex and the solution must provide a minimum value.

6. Special Cases

By setting appropriate parameters, we can deduce results for some existing models. Some specific cases are as follows:

Case 1: When $\lambda_0 = \lambda_1 = \lambda_2 = \lambda$, $X=1$, $K=2$ and $\theta=1$, then our model reduces to the model considered by Wang *et al.* (2004).

Case 2: When $\lambda_0 = \lambda_1 = \lambda_2 = \lambda$, $X=1$, $K=1$ and $\theta=1$, our model facilitates results for removable and non-reliable server model which was studied by Wang (1995).

7. Numerical Results and Sensitivity Analysis

In this section, we are interested in sensitivity analysis by taking the numerical illustrations. Mathematical software ‘MATLAB’ is used to develop a computational program. The numerical results are summarized in tables 2-3 and are visualized in figs 1(a)-2(b). The default parameters for tables 2-3 and graphs 1(a)-2(b) are chosen as $\lambda=.7$ ($\lambda_0=.9\lambda$, $\lambda_1=.8\lambda$, $\lambda_2=.7\lambda$), $\mu_1=2$, $\mu_2=1$, $\alpha=.01$, $\beta=1$, $\theta=.9$, $N=2$, $q_1=.5$ and $E(X)=2$.

Table 1: The different cost sets

Cost sets	Set 1	Set 2	Set 3	Set 4
c_s	50	100	50	50
c_t	50	50	50	25
c_v	2	2	2	2
c_b	25	25	25	50
c_d	10	10	10	10
c_h	10	15	10	10

In table 2, we examine the effect of (μ_1, μ_2) and α on various system characteristics. As we increase μ_1 and μ_2 (α), the decreasing (increasing) pattern is followed by P_B , P_D , $E[B]$, $E[D]$, $E[H]$ and $E[C]$ while P_V increases (decreases). $E[I]$ remains constant for table 2. Table 3 displays the effect of

α and β for the different values of λ and q_1 on N^* and $T_c(N_1)$ for the different sets of cost elements given in table 1. We observed from table 3 that $T_c(N_1)$ increases as α and β increase and decreases with the increasing values of λ and q for all the sets of costs. We also observe that the value of threshold parameter N^* slightly increases on increasing values of α and β .

Table 2: The effect of μ_1, μ_2, α and β on various system characteristics

(μ_1, μ_2)	P_V	P_B	P_D	$E[I]$	$E[B]$	$E[D]$	$E[H]$	$E[C]$
(1,2)	0.418	0.660	0.006	2.211	0.022	0.022	2.232	3.63
(2,2)	0.782	0.521	0.005	0.933	0.009	0.009	0.943	2.34
(3,2)	0.923	0.457	0.004	0.693	0.006	0.006	0.700	2.10
(4,2)	0.997	0.421	0.004	0.591	0.005	0.005	0.597	1.99
(α, β)	P_V	P_B	P_D	$E[I]$	$E[B]$	$E[D]$	$E[H]$	$E[C]$
(.01,1)	0.418	0.660	0.006	1.400	2.210	0.022	2.232	3.632
(.02,1)	0.414	0.662	0.013	1.400	2.235	0.044	2.280	3.680
(.03,1)	0.411	0.664	0.019	1.400	2.262	0.067	2.330	3.730
(.04,1)	0.407	0.666	0.026	1.400	2.289	0.091	2.380	3.780

Table 3: The effect of α and β on N^* and $T_c(N_1)$ for different cost sets

	(α, β)	$\{N^*, T_c(N_1)\}$			
		$\lambda=.5$	$\lambda=.6$	$Q_1=.2$	$q_1=.3$
SET 1	(.01,1)	(5, 25.07)	(4, 22.72)	(4, 26.09)	(4, 24.40)
SET 2		(5, 30.53)	(4, 25.99)	(4, 30.06)	(4, 27.46)
SET 3		(4, 34.26)	(4, 36.07)	(4, 40.45)	(4, 40.05)
SET 4		(5, 47.61)	(5, 42.76)	(5, 49.64)	(5, 46.44)
SET 1	(.05,1.1)	(5, 25.07)	(4, 22.72)	(4, 31.47)	(4, 29.44)
SET 2		(5, 36.48)	(4, 30.95)	(4, 36.13)	(4, 32.86)
SET 3		(4, 41.50)	(4, 43.81)	(4, 49.40)	(4, 48.83)
SET 4		(5, 56.96)	(5, 51.12)	(5, 59.81)	(5, 55.79)

Figs 1 (a-b) exhibits the combined effect of arrival rate λ along with N^* on $T_c(N_1)$ for different sets of cost elements. It can be easily observed that $T_c(N_1)$ first decreases and then increases sharply on increasing λ for all sets of cost elements. By using the fuzzy toolbox, the ANFIS network results for $E[NS_1]$ are displayed in figs 2(a)-2(b). For this purpose, the ANFIS networks are trained for 10 epochs where λ treats as input variable which takes the different linguistic values like as low, average, high, very high, etc.. The shape of the corresponding membership function for fig 2(b) is displayed in fig 2(a). We plot the graph for $E[NS_1]$ for the different values of N in figs 2(b) where the analytical (ANFIS) results are represented by

continuous (discrete) lines. We observe from fig. 2(b) that $E[NS_1]$ increases on increasing λ .

Finally, we conclude that ANFIS provides an easy and fast solution which is at par with analytical results and is useful to analysts and decision makers to manage the queueing systems for which exact results using classical queue theoretic approaches are difficult to obtain.

8. Conclusion

In this paper, the state dependent unreliable $M^X/H_2/1$ queue and vacation under N-policy has been the topic of our investigation. The incorporation of more realistic assumptions viz. unreliable server, vacation and threshold N policy make our study more versatile and feasible to deal with congestion situations encountered in computers and communication networks, manufacturing and production systems, and many others. The results are helpful to system designers to minimize the expected total operating cost.

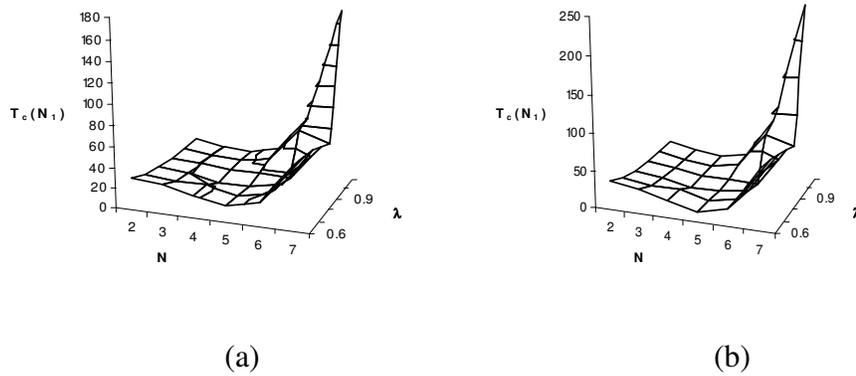


Fig. 1: Effect of N and λ on the expected total cost for (a) Set 1 (b) Set 2

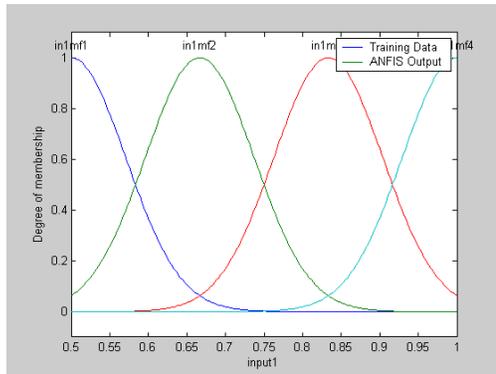


Fig. 2 (a): Membership functions for input Parameters λ

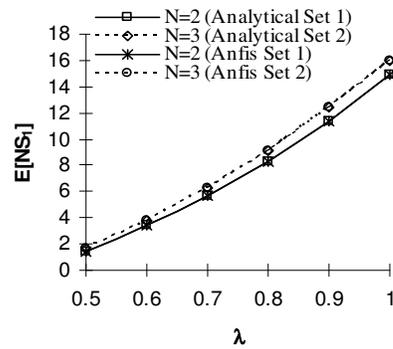


Fig. 2(b): $E[NS_1]$ by varying λ for different values of N

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