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Impact of Ecological Factors on the Spread of Carrier Dependent Infectious Diseases: A Model

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Abstract: In this paper, a four dimensional non-linear model is proposed and analyzed to study the impact of ecological factors on the spread of carrier dependent infectious diseases such as Cholera, Typhoid fever, TB, etc. caused by various types of carriers including house flies etc. It is assumed that the density of carrier population follows a general logistic model and its intrinsic growth rate and carrying capacity increase as the cumulative biomass density of ecological factors increases. It is further assumed that the cumulative density of ecological factors is also governed by a general logistic model, the growth rate of which increases bilinearly with the human population density. The proposed model is analyzed by using the stability theory of differential equations and computer simulation. The analysis shows that as the cumulative density of ecological factors increases (decreases), the spread of carrier dependent infectious disease increases (decreases). The computer simulation of the proposed model confirms this analytical result.

Keywords: Carriers, Ecological Factors, Stability, Lyapunov's function.

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1. Introduction

In general, the spread of infectious diseases in human population depends upon many factors such as the number of infectives and susceptibles; population migration; modes of transmission (carriers, vectors etc.); socio-economic factors; environmental and ecological conditions in the habitat. The carrier populations play a very important role in the spread of the infectious diseases such as cholera, typhoid fever, TB, dysentery, etc. For example, carriers such as flies, cockroaches, ticks, mites etc., grow and survive in the bushes in residential areas, parks, grasses, open drainage, garbage stores, ponds, etc. They also grow in the mounds formed by the roots of the bushes as well as on branches and leaves of plants and their populations increase further as the bushes and plants become denser and denser. The carriers carry bacteria of infectious diseases on their body parts discharged from infectives to susceptibles contaminating their food and drinking water and thus spread infectious diseases in human population indirectly[1].

The modelling and analysis of various infectious diseases have been conducted by many researchers in the past[2-6]. However, it is pointed out that very little attention has been paid to the study of the spread of such diseases by considering the effect of carrier population transporting infectious agents, the density of which increases due to natural or human population density related ecological factors[7-9]. Singh et. al.[10, 11] presented some mathematical models for the carrier dependent infectious disease by considering environmental and ecological effects with constant immigration of susceptible population. They concluded that the spread of the infectious disease increases, when the growth of carrier caused by conducive environmental and ecological factors due to population density related factors, increases.

In view of the above, in this paper, therefore, the spread of carrier dependent infectious diseases by considering explicitly the ecological factors, is modelled and analyzed. It is assumed that the density of carrier population is governed by a general logistic model, the growth rate and the carrying capacity of which increases as the cumulative biomass density of ecological factors increases. It is further, assumed that cumulative biomass density of ecological factors is also governed by a general logistic model, the growth rate of which increases bilinearly as the human population density increases. Our main aim is to show that the ecological factors play important roles in the spread of carrier dependent infectious diseases. The model is analyzed by the stability theory of differential equations. The numerical simulation of the model is also performed to see the influence of certain key parameters on the spread of the carrier dependent infectious diseases.

2. An SIS Model

Let $X(t)$ and $Y(t)$ denote densities of susceptible and infective classes respectively of total human population density $N(t) = X(t) + Y(t)$. Let $C(t)$ be the carrier population density and $B(t)$ is the cumulative biomass density of ecological factors. By assuming simple mass action interaction, an SIS model for the growth of a carrier dependent infectious disease, can be proposed as follows:

$$(2.1)\qquad \begin{cases} \dfrac{dX}{dt} = A - \beta XY - \lambda XC + \nu Y - dX \\ \dfrac{dY}{dt} = \beta XY + \lambda XC - (\nu + \alpha + d)Y \\ \dfrac{dC}{dt} = sC - \dfrac{s_0 C^2}{L} + s_1 BC + s_2 BC^2 \\ \dfrac{dB}{dt} = rB - \dfrac{r_0 B^2}{K} + r_1 N + r_2 BN, \end{cases}$$

where $X + Y = N$ and $X(0) > 0, Y(0) \geq 0, C(0) \geq 0, B(0) \geq 0.$

In the model (2.1) A is the constant immigration rate of human population from outside the region, d is the natural death rate constant, β and λ are the transmission coefficients due to infectives and carrier population respectively, α is the disease related death rate constant, ν is the recovery rate constant, s is the intrinsic growth rate of carrier population, s_0 is the coefficient related to intraspecific competition coefficient (i.e. S_0/L), L is the measure of the carrier population density and sL/s_0 is its carrying capacity as compared to the usual logistic model. Further, s_1 is the growth rate coefficient of carrier population and s_2 is the growth rate coefficient of the carrying capacity caused by the growth of cumulative density of ecological factors. Similarly, r is the natural growth rate coefficient cumulative biomass density of ecological factors, r_0 is the coefficient related to intraspecific coefficient s_0/L, L is the measure of cumulative biomass density and rK/r_0 is the carrying capacity as compared to the usual logistic model. Also r_1 is the growth rate coefficient of $B(t)$ due to human population density related factors, r_2 is the growth rate coefficient of $B(t)$ caused by the bilinear interaction of human population density.

3. Equilibrium Analysis

Since $X + Y = N$, thus model (2.1) can be transformed as follows:

$$\text{(3.1)} \qquad \begin{cases} \dfrac{dY}{dt} = \beta(N-Y)Y + \lambda(N-Y)C - (\nu+\alpha+d)Y \\ \dfrac{dN}{dt} = A - d\,N - \alpha Y \\ \dfrac{dC}{dt} = sC - \dfrac{s_0 C^2}{L} + s_1 BC + s_2 BC^2 \\ \dfrac{dB}{dt} = rB - \dfrac{r_0 B^2}{K} + r_1 N + r_2 BN \end{cases}$$

It would suffice to analyze the model (3.1) instead of (2.1) and for this we need the following lemma which establishes region of attraction for the system (3.1).

Lemma 3.1: *The set*

$$\Omega = \left\{ (Y,N,C,B) : 0 \le Y \le N \le \frac{A}{d}, \frac{A}{d+\alpha} \le N \le \frac{A}{d}, 0 \le C \le C_m, 0 \le B \le B_m \right\},$$

attracts all solutions initiating in the positive orthant, where

$$\text{(3.2)} \qquad C_m = \frac{s + s_1 B_m}{\frac{s_0}{L} - s_2 B_m},\ B_m = \frac{K}{2r_0}\left[\left(r + r_2\frac{A}{d}\right) + \sqrt{\left(r + r_2\frac{A}{d}\right)^2 + \frac{4r_0 r_1 \frac{A}{d}}{K}}\right]$$

provided

$$\text{(3.3)} \qquad s_0 > s_2 B_m L.$$

Proof: Here we give only a brief outline of the proof, the detail proof can be seen in Freedman and So[12]. From the second equation of model (3.1), we have

$$\frac{dN}{dt} = A - dN - \alpha Y \le A - dN$$

and

$$\frac{dN}{dt} = A - dN - \alpha Y \ge A - (d+\alpha)N.$$

These equations imply that

(3.4) $$\frac{A}{\alpha+d} \le N \le \frac{A}{d}.$$

From the equation for carrier population density in (3.1), we have

$$\frac{dC}{dt} \le sC - \frac{s_0 C^2}{L} + s_1 B_m C + s_2 B_m C^2 = (s + s_1 B_m)C - (\frac{s_0}{L} - s_3 B_m)C^2$$

which gives

(3.5) $$0 \le C \le C_m = \frac{s + s_1 B_m}{\frac{s_0}{L} - s_2 B_m},$$

which is positive provided $s_0 > s_2 B_m L.$

Similarly from the last equation of model (3.1), we have

$$\frac{dB}{dt} \le rB - \frac{r_0 B^2}{K} + r_1 \frac{A}{d} + r_2 B \frac{A}{d} = -\frac{r_0 B^2}{K} + \left(r + r_2 \frac{A}{d}\right) B + r_1 \frac{A}{d}$$

which gives

(3.6) $$0 \le B \le B_m = \frac{K}{2r_0}\left[\left(r + r_2 \frac{A}{d}\right) + \sqrt{\left(r + r_2 \frac{A}{d}\right)^2 + \frac{4 r_0 r_1 \frac{A}{d}}{K}}\right].$$

Theorem 3.1: *The system (3.1) has following three equilibria:*

(i) $E_0\left(0, \frac{A}{d}, 0, B_m\right)$, *the disease free and carrier free equilibrium, where* B_m *is given by (3.6).*

(ii) $E_1\left(\bar{Y}, \bar{N}, 0, \bar{B}\right)$, *the carrier free equilibrium, which exists if* $R_0 = \frac{\beta A}{d\left(\nu + \alpha + d\right)} > 1$, *where* R_0 *is the basic reproductive number. The components of* E_1 *are given as follows:*

Here $\bar{Y} = \frac{\beta A - d_1(\nu + \alpha + d)}{\beta(\alpha + d)}$, $\bar{N} = \frac{\beta A + \alpha(\nu + \alpha + d)}{\beta(\alpha + d)}$,

$$and\ \bar{B} = \frac{K}{2r_0}\left[\left(r + r_2\bar{N}\right) + \sqrt{\left(r + r_2\bar{N}\right)^2 + \frac{4r_0 r_1 \bar{N}}{K}}\right].$$

(iii) $E^*\left(Y^*, N^*, C^*, I^*\right)$*: The endemic equilibrium.*

The existence of E_0 *or* E_1 *is obvious. The equilibrium point* E^* *is given as the solutions of system of following equations, which are obtained from (3.1) by taking left hand sides to zero:*

(3.7) $$\beta Y^2 + Y\left[(\nu + \alpha + d) - \beta N + \lambda C\right] - \lambda NC = 0,$$

(3.8) $$Y = \frac{A - dN}{\alpha},$$

(3.9) $$s - \frac{s_0 C}{L} + s_1 B + s_2 BC = 0,$$

(3.10) $$rB - \frac{r_0 B^2}{K} + r_1 N + r_2 BN = 0\ .$$

Now, eliminating Y *between equations (3.7) and (3.8), we get*

(3.11) $$F(N) = \left(\frac{\beta}{\alpha^2}\right)(A - dN)^2 + \left(\frac{A - dN}{\alpha}\right)\left[\left(\nu + \alpha + d\right) - \beta N + \lambda C\right] - \lambda NC = 0,$$

where C *is given as a function* N *by using (3.9) and (3.10). From equation (3.11) we note the following:*

(3.12) $$F\left(\frac{A}{\alpha + d}\right) = \frac{A}{\alpha + d}(\nu + \alpha + d) > 0,$$

(3.13) $$F\left(\frac{A}{d}\right) = -\frac{\lambda A}{d}C < 0.$$

Thus, there exists a root N^* *of* $F(N) = 0$ *in the interval* $\frac{A}{d + \alpha} \le N \le \frac{A}{d}$.

Further, this root will be unique if $F'(N) < 0,\ \frac{A}{d + \alpha} \le N \le \frac{A}{d}$.

To show this, we differentiate (3.11) to get

$$F'(N) = -\frac{2\beta d}{\alpha^2}\left(A - dN\right) - \frac{d}{\alpha}\left[\left(\nu + \alpha + d\right) - \beta N + \lambda C\right] \tag{3.14}$$

$$-\frac{\beta}{\alpha}\left(A - dN\right) - \lambda C - \frac{\lambda}{\alpha}\left[N\left(\alpha + d\right) - A\right]\frac{dC}{dN}.$$

Using (3.11) in (3.14), we get on simplification

$$F'(N) = -\frac{\beta d}{\alpha^2}\left(A - dN\right) - \frac{d}{\left(A - dN\right)}\lambda NC - \frac{\beta}{\alpha}\left(A - dN\right) \tag{3.15}$$

$$-\lambda C - \frac{\lambda}{\alpha}\left[N\left(\alpha + d\right) - A\right]\frac{dC}{dN},$$

which is negative as

$$\frac{dC}{dN} = \left[\frac{s_1 + s_2 C}{\left(\frac{s_0}{L} - s_2 B\right)}\right]\frac{dB}{dN} \quad and \quad \frac{dB}{dN} = \frac{\left(r_1 + r_2 B\right)B}{\left(r_1 N + \frac{r_0 B^2}{K}\right)}$$

are positive under the condition (3.3).

After, knowing the value of N^*, *the value of* Y^*, C^* *and* B^* *can be uniquely determined from (3.8), (3.9), (3.10). Hence* E^* *exists in* Ω.

Remark: These conditions imply that the number of infectives increases not only on as the carrier population increases but also as the biomass density of ecological factors increases.

From (3.8) we note that $\frac{dY}{d\nu} = -\frac{d}{\alpha}\frac{dN}{d\nu}$ for any parameter ν. Now differentiating (3.11) w.r.t. N and using it again, we can easily show that $\frac{dN}{d\nu} < 0$ in the region $\frac{A}{\alpha + d} \le N \le \frac{A}{d}$. This gives $\frac{dY}{d\nu} > 0$ and we have, $\frac{dY}{ds} > 0$, $\frac{dY}{ds_1} > 0$, $\frac{dY}{ds_2} > 0$, $\frac{dY}{dr} > 0$, $\frac{dY}{dr_1} > 0$, $\frac{dY}{dr_2} > 0$.

4. Stability Analysis

We shall now study the stability behaviour of above equilibria. The local stability result of equilibria E_0, E_1 and E^* are given in the following theorem:

Theorem 4.1: *The equilibria E_0 and E_1 are locally unstable and the equilibrium E^* is locally asymptotically stable provided the following conditions are satisfied*

(4.1) $$\alpha\lambda^2 C^{*2} < d\beta^2 Y^{*2},$$

(4.2) $$2\alpha\lambda^2 B^{*2}(N^* - Y^*)^2\left(s_1 + s_2 C^*\right)^2\left(r_1 + r_2 B^*\right)^2 < \beta^2 Y^{*2} d\left(\frac{s_0}{L} - s_2 B^*\right)^2\left(\frac{r_0 B^{*2}}{K} + r_1 N^*\right)^2.$$

Proof: The local stability behaviour of each of the two equilibria E_0 or E_1 is studied by computing corresponding variational matrices for system (3.1) and for the nontrivial equilibrium point E^* it is studied by using Lyapunov's theory.

The variational matrix M_i corresponding to equilibrium points is given by:

$$M_i = \begin{bmatrix} \beta N - 2\beta Y - \lambda C - (\nu + \alpha + d) & \beta Y + \lambda C & \lambda(N - Y) & 0 \\ -\alpha & -d & 0 & 0 \\ 0 & 0 & \begin{array}{c} s - \dfrac{2s_0}{L}C \\ + s_1 B + 2 s_2 C B \end{array} & s_1 C + s_2 C^2 \\ 0 & r_1 + r_2 B & 0 & r - \dfrac{2r_0}{K}B + r_2 N \end{bmatrix}$$

Local Stability Behaviourof $E_0\left(0, \frac{A}{d}, 0, B_m\right)$:

The variational matrix corresponding to equilibrium point E_0 is given by:

$$M_0 = \begin{bmatrix} \beta\dfrac{A}{d} - (\nu + \alpha + d) & 0 & \lambda\dfrac{A}{d} & 0 \\ -\alpha & -d & 0 & 0 \\ 0 & 0 & s + s_1 B_m & 0 \\ 0 & r_1 + r_2 B_m & 0 & r - \dfrac{2r_0}{K}B_m + r_2\dfrac{A}{d} \end{bmatrix}$$

Here one of the eigen value $s+s_1B_m$ is positive and so E_0, if exists, will be unstable.

Local Stability Behaviourof $E_1\left(\bar{Y},\bar{N},0,\bar{B}\right)$:

In this case the variational matrix is given as follows:

$$M_1=\begin{bmatrix} \beta\bar{N}-2\beta\bar{Y}-(\nu+\alpha+d) & \beta\bar{Y} & \lambda(\bar{N}-\bar{Y}) & 0 \\ -\alpha & -d & 0 & 0 \\ 0 & 0 & s+s_1\bar{B} & 0 \\ 0 & r_1+r_2\bar{B} & 0 & r-\dfrac{2r_0}{K}\bar{B}+r_2\bar{N} \end{bmatrix}$$

This variational matrix has a positive eigen value $s+s_1\bar{B}$ and so E_1, if exists, will be unstable.

Local Stability Behaviourof $E^*\left(Y^*,N^*,C^*,I^*\right)$:

We study the stability behaviour or E^* by Lyapunov's method by linearising (3.1) with the transformations

$$Y=Y^*+y,\ N=N^*+n,\ C=C^*+c,\ B=B^*+b$$

and using the following positive definite function,

(A.1) $$V=\frac{1}{2}y^2+\frac{k_1}{2}n^2+\frac{k_2}{2}c^2+\frac{k_3}{2}b^2$$

where k_1, k_2 and k_3 are positive constants to be chosen appropriately.

Differentiating (A.1) w.r.t. t and using (3.1), $\dfrac{dV}{dt}$ can be written as:

$$\frac{dV}{dt}=-\left(\beta Y^*+\lambda\frac{N^*C^*}{Y^*}\right)y^2-\left(k_1d\right)n^2-k_2\left(\frac{s_0}{L}C^*-s_2C^*B^*\right)c^2$$
$$-k_3\left(\frac{r_0B^*}{K}+\frac{r_1N^*}{B^*}\right)b^2+\left(\beta Y^*+\lambda C^*-k_1\alpha\right)yn+\lambda\left(N^*-Y^*\right)yc$$
$$+k_3\left(r_1+r_2B^*\right)nb+k_2C^*\left(s_1+s_2C^*\right)cb$$

$$
\begin{aligned}
&= \left(\beta Y^* - k_1\alpha\right) yn - \lambda \frac{N^* C^*}{Y^*} y^2 - \left[\left(\frac{\beta Y^*}{2}\right) y^2 - \left(\lambda . C^*\right) yn + \frac{k_1 d}{2} n^2\right] \\
&\quad - \left[\left(\frac{\beta Y^*}{2}\right) y^2 - \lambda\left(N^* - Y^*\right) yc + \frac{k_2}{2}\left(\frac{s_0}{L} - s_2 B^*\right) C^* c^2\right] \\
&\quad - \left[\frac{k_2}{2}\left(\frac{s_0}{L} - s_2 B^*\right) C^* c^2 - k_2 C^*\left(s_1 + s_2 C^*\right) cb + \frac{k_3}{2}\left(\frac{r_0 B^*}{K} + \frac{r_1 N^*}{B^*}\right) b^2\right] \\
&\quad - \left[\frac{k_3}{2}\left(\frac{r_0 B^*}{K} + \frac{r_1 N^*}{B^*}\right) . b^2 - k_3\left(r_1 + r_2 B^*\right) nb + \frac{k_1 d}{2} n^2\right].
\end{aligned}
$$

Choosing $k_1 = \dfrac{\beta Y^*}{\alpha}$, the conditions for $\dfrac{dV}{dt}$ to be negative definite are as follows:

(A.2) $$\alpha \lambda^2 C^{*2} < d \beta^2 Y^{*2},$$

(A.3) $$k_2 > \frac{\lambda^2 (N^* - Y^*)^2}{\beta Y^* \left(\frac{s_0}{L} - s_2 B^*\right) C^*},$$

(A.4) $$k_2 < \frac{\left(\frac{s_0}{L} - s_2 B^*\right)\left(\frac{r_0 B^*}{K} + \frac{r_1 N^*}{B^*}\right)}{C^*\left(s_1 + s_2 C^*\right)^2} k_3,$$

(A.5) $$k_3 < \frac{\beta Y^* d \left(\frac{r_0 B^*}{K} + \frac{r_1 N^*}{B^*}\right)}{\alpha\left(r_1 + r_2 B^*\right)^2}.$$

Now if we choose $k_3 = \dfrac{1}{2}\dfrac{\beta Y^* d \left(\frac{r_0 B^*}{K} + \frac{r_1 N^*}{B^*}\right)}{\alpha\left(r_1 + r_2 B^*\right)^2}$, then inequality (A.5) will satisfy automatically. Now we can choose k_2 satisfying inequality (A.3) and (A.4) provided the following inequality is satisfied,

(A.6) $$2\alpha\lambda^2 B^{*2}(N^*-Y^*)^2\left(s_1+s_2C^*\right)^2\left(r_1+r_2B^*\right)^2$$
$$<\beta^2 Y^{*2} d\left(\frac{s_0}{L}-s_2B^*\right)^2\left(\frac{r_0B^{*2}}{K}+r_1N^*\right)^2$$

Hence E^* is locally stable if (A.2) and (A.6) are satisfied.

The nonlinear stability result for E^* is stated in two following theorem:

Theorem 4. 2: *The equilibrium point* E^* *is nonlinearly asymptotically stable in* Ω *provided the following inequalities are satisfied:*

(4.3) $$\alpha\lambda^2 C_m{}^2 < d\beta^2 Y^{*2}\ ,$$

(4.4) $$2\alpha\lambda^2 B_m{}^2(N^*-Y^*)^2\left(s_1+s_2C_m\right)^2\left(r_1+r_2B^*\right)^2$$
$$< d\,\beta^2 Y^{*2}\left(\frac{s_0}{L}-s_2B^*\right)^2\left(\frac{r_0}{K}B^*B_m+r_1\frac{A}{d+\alpha}\right)^2 .$$

Proof: We prove the above theorem by using the following positive definite function:

(B.1) $$V=\left(Y-Y^*-Y^*\ln\frac{Y}{Y^*}\right)+\frac{m_1}{2}\left(N-N^*\right)^2$$
$$+m_2\left(C-C^*-C^*\ln\frac{C}{C^*}\right)+m_3\left(B-B^*-B^*\ln\frac{B}{B^*}\right)$$

where m_1, m_2 and m_3 are positive constants to be chosen appropriately.

Differentiating (B.1) w.r.t. t and using (3.1), we get

$$\frac{dV}{dt}=\left(\frac{Y-Y^*}{Y}\right)\frac{dY}{dt}+m_1\left(N-N^*\right)\frac{dN}{dt}+m_2\left(\frac{C-C^*}{C}\right)\frac{dC}{dt}+m_3\left(\frac{B-B^*}{B}\right)\frac{dB}{dt}$$
$$=-\left[\beta+\lambda\frac{NC}{YY^*}\right](Y-Y^*)^2-m_1d(N-N^*)^2-m_2\left[\frac{s_0}{L}-s_2B^*\right](C-C^*)^2$$
$$-m_3\left(\frac{r_0}{K}+\frac{r_1N}{BB^*}\right)(B-B^*)^2+\left[\beta-m_1\alpha+\lambda\frac{C}{Y^*}\right](Y-Y^*)\left(N-N^*\right)$$
$$+\lambda\left[\frac{N^*}{Y^*}-1\right](Y-Y^*)(C-C^*)+m_2\left[s_1+s_2C\right](C-C^*)(B-B^*)$$
$$+m_3\left[\frac{r_1}{B^*}+r_2\right](B-B^*)(N-N^*).$$

Taking $m_1 = \dfrac{\beta}{\alpha}$,

(B.2)
$$\begin{aligned}\frac{dV}{dt} &= -\lambda\frac{NC}{YY^*}(Y-Y^*)^2 - \frac{\beta}{2}(Y-Y^*)^2 + \lambda\frac{C}{Y^*}(Y-Y^*)\left(N-N^*\right)\\
&\quad -\frac{m_1 d}{2}(N-N^*)^2 - \frac{\beta}{2}(Y-Y^*)^2 + \lambda\left(\frac{N^*}{Y^*}-1\right)(Y-Y^*)(C-C^*)\\
&\quad -\frac{m_2}{2}\left(\frac{s_0}{L}-s_2B^*\right)(C-C^*)^2 - \frac{m_2}{2}\left(\frac{s_0}{L}-s_2B^*\right)(C-C^*)^2\\
&\quad +m_2\left(s_1+s_2C\right)(C-C^*)(B-B^*) - \frac{m_3}{2}\left(\frac{r_0}{K}+\frac{r_1N}{BB^*}\right)(B-B^*)^2\\
&\quad -\frac{m_1 d}{2}(N-N^*)^2 + m_3\left(\frac{r_1}{B^*}+r_2\right)(B-B^*)(N-N^*)\\
&\quad -\frac{m_3}{2}\left(\frac{r_0}{K}+\frac{r_1N}{BB^*}\right)(B-B^*)^2.\end{aligned}$$

Now $\dfrac{dV}{dt}$ will be negative definite if following conditions holds,

(B.3)
$$\alpha\lambda^2C^2 < d\beta^2Y^{*2},$$

(B.4)
$$m_2 > \frac{\lambda^2\left(\dfrac{N^*}{Y^*}-1\right)^2}{\beta\left(\dfrac{s_0}{L}-s_2B^*\right)},$$

(B.5)
$$m_2 < \frac{\left(\dfrac{s_0}{L}-s_2B^*\right)\left(\dfrac{r_0}{K}+\dfrac{r_1N}{BB^*}\right)}{(s_1+s_2C)^2}m_3,$$

(B.6)
$$m_3 < \frac{(\beta/\alpha)d\left(\dfrac{r_0}{K}+\dfrac{r_1N}{BB^*}\right)}{\left(\dfrac{r_1}{B^*}+r_2\right)^2}.$$

Now on maximizing left hand side and minimizing right hand side of (B.3), (B.5) and (B.6), we get

(B.7) $$\alpha\lambda^2 C_m{}^2 < d\beta^2 Y^{*2} ,$$

(B.8) $$m_2 > \frac{\lambda^2\left(\frac{N^*}{Y^*}-1\right)^2}{\beta\left(\frac{s_0}{L}-s_2 B^*\right)} ,$$

(B.9) $$m_2 < \frac{\left(\frac{s_0}{L}-s_2 B^*\right)\left(\frac{r_0}{K}+\frac{r_1}{B_m B^*}\frac{A}{\alpha+d}\right)}{(s_1+s_2 C_m)^2} m_3 ,$$

(B.10) $$m_3 < \frac{(\beta/\alpha)d\left(\frac{r_0}{K}+\frac{r_1}{B_m B^*}\frac{A}{\alpha+d}\right)}{\left(\frac{r_1}{B^*}+r_2\right)^2} .$$

If we choose

$$m_3 = \frac{1}{2}\frac{(\beta/\alpha)d\left(\frac{r_0}{K}+\frac{r_1}{B_m B^*}\frac{A}{\alpha+d}\right)}{\left(\frac{r_1}{B^*}+r_2\right)^2},$$

the inequality (B.10) will satisfy automatically and we can choose m_2 satisfying inequality (B.8) and (B.9) provided

$$\frac{\lambda^2\left(\frac{N^*}{Y^*}-1\right)^2}{\beta\left(\frac{s_0}{L}-s_2 B^*\right)} < m_2 < \frac{1}{2}\frac{(\beta/\alpha)d\left(\frac{r_0}{K}+\frac{r_1}{B_m B^*}\frac{A}{\alpha+d}\right)}{\left(\frac{r_1}{B^*}+r_2\right)^2}\frac{\left(\frac{s_0}{L}-s_2 B^*\right)\left(\frac{r_0}{K}+\frac{r_1}{B_m B^*}\frac{A}{\alpha+d}\right)}{(s_1+s_2 C_m)^2}$$

or

(B.11) $$2\alpha\lambda^2 B_m{}^2 (N^*-Y^*)^2 \left(s_1+s_2 C_m\right)^2\left(r_1+r_2 B^*\right)^2 < d\,\beta^2 Y^{*2}\left(\frac{s_0}{L}-s_2 B^*\right)^2\left(\frac{r_0}{K}B^* B_m + r_1\frac{A}{d+\alpha}\right)^2 .$$

Hence E^* is nonlinearly asymptotically stable if (B.7) and (B.11) are satisfied.

5. Numerical Simulation

Here we shall discuss the numerical analysis of the existence and stability of the nontrivial equilibrium point by taking the following set of parameter values using MAPLE.

$$A = 500,\ d = 0.02,\ \alpha = 0.03,\ \beta = 0.000005,\ \lambda = 0.000001,\ \nu = 0.05,$$
$$s = 0.25,\ s_0 = 0.99,\ L = 25000,\ s_1 = 0.002,\ s_2 = 0.00000001,$$
$$r = 0.3,\ r_0 = 0.99,\ K = 1000,\ r_1 = 0.002,\ r_2 = 0.000025.$$

For these values of parameters the E^* can be found as follows:

$$Y^* = 6827,\ N^* = 14758,\ C^* = 51949,\ B^* = 717.$$

The eigen values corresponding to E^* can be found from the corresponding jacobian matrix as, -0.0537, -0.1114, -0.7527, -1.6843, which are all negative. Hence E^* is locally stable. Now numerical simulation is performed between B and Y by solving (3.1) for the different initial starts and displayed in the Figure1, which shows the nonlinear stability of the point $\left(B^*, Y^*\right)$ in $B - Y$ plane.

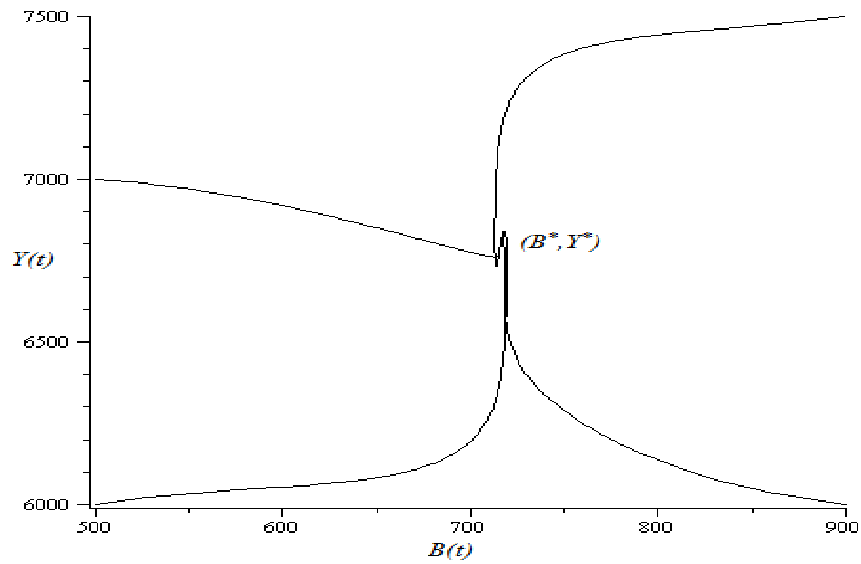

Figure 1: Phase plot between B and Y

The model (3.1) has also been solved by using MAPLE and the graphs of the variable Y with respect to t for various values of parameters have been plotted in Figures 2-10. The following conclusion may be drawn:

(i) From Figure 2, it is noted that $Y(t)$ increases as β increases.

(ii) From Figure 3, it is noted that $Y(t)$ increases as λ increases.

The above results are expected, as the infectives increases with the increase in interaction coefficients. Further,

(iii) From Figure 4, it is noted that $Y(t)$ increases as s increases.

(iv) From Figure 5, it is seen that $Y(t)$ increases as s_1 increases.

(v) From Figure 6, we note that $Y(t)$ increases as s_2 increases.

The above results are expected, as the carrier population increases with the parameters s, s_1, s_2. Also

(vi) From Figure 7, it is seen that $Y(t)$ increases as r increases.

(vii) From Figure 8, we note that $Y(t)$ increases as r_1 decreases.

(viii) From Figure 9, it is seen that $Y(t)$ increases as r_2 increases.

These results are also expected as increase in the cumulative biomass density of ecological factors causes increase in the density of carrier population, resulting in the increase of the density of infectives.

Further, from Figure 10 it is noted that $Y(t)$ increases as A increases. This implies that the disease becomes more endemic.

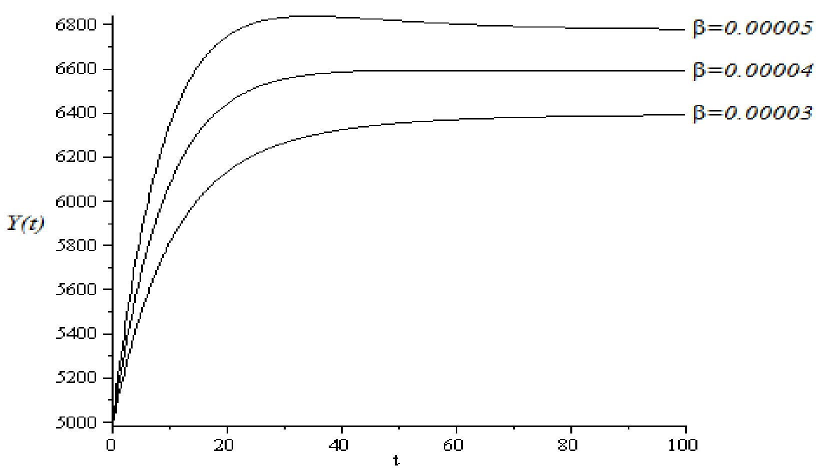

Figure 2: Plot between Y and t for different values of β

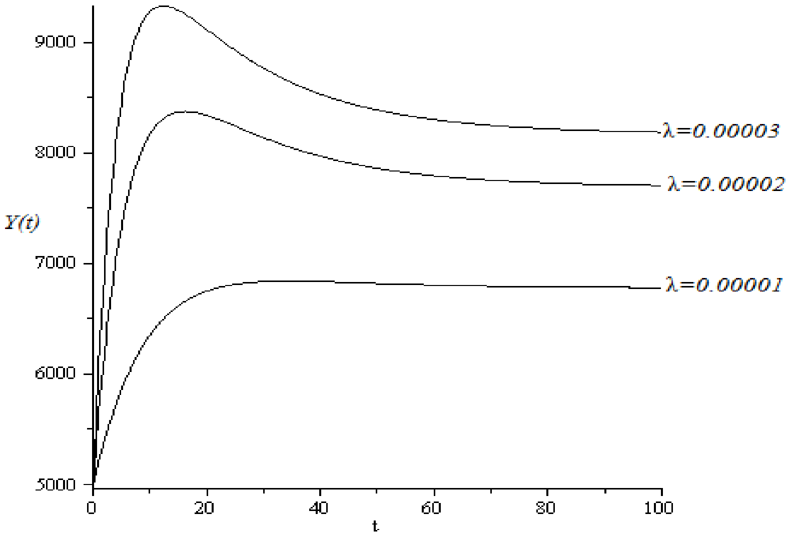

Figure 3: Plot between Y and t for different values of λ

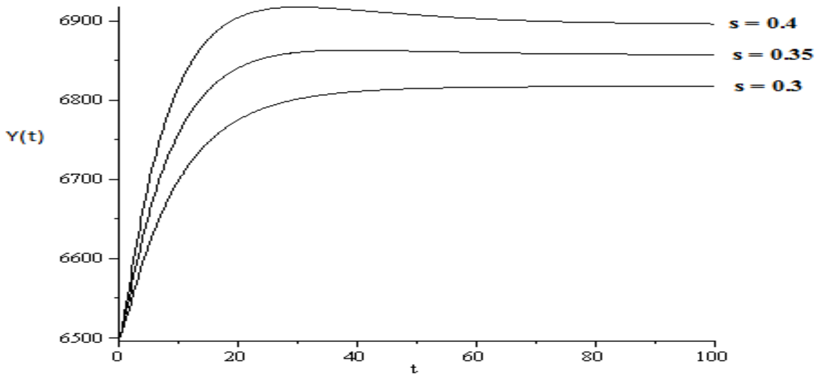

Figure 4: Plot between Y and t for different values of s

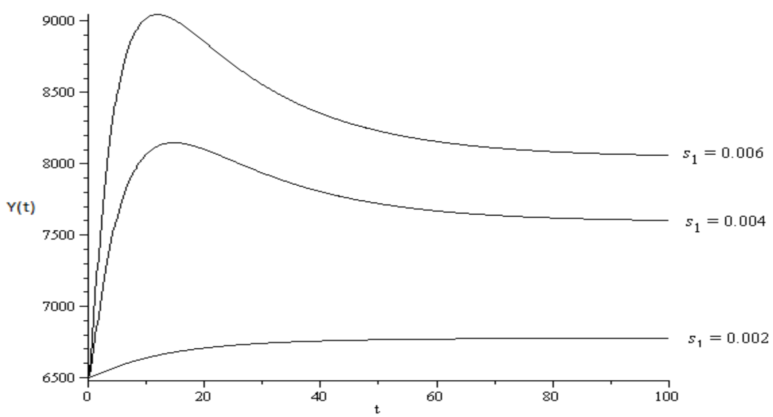

Figure 5: Plot between Y and t for different values of s_1

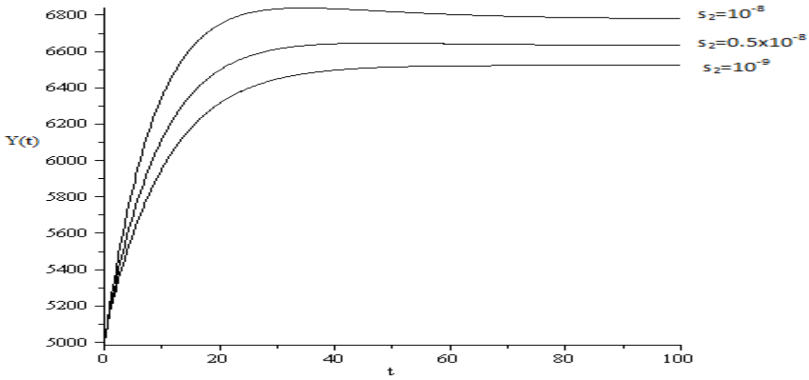

Figure 6: Plot between Y and t for different values of s_2

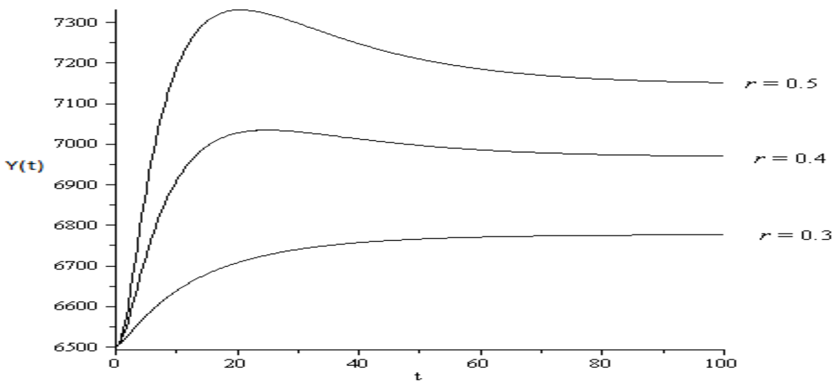

Figure 7: Plot between Y and t for different values of r

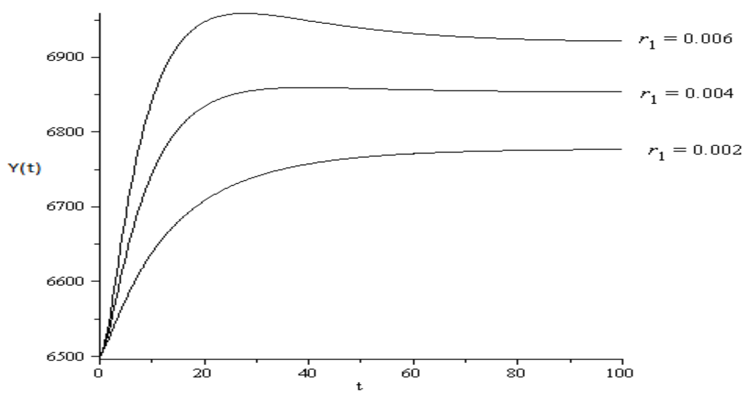

Figure 8: Plot between Y and t for different values of r_1

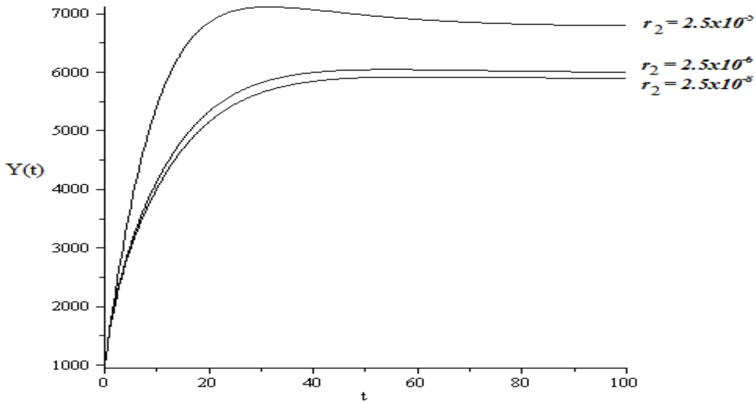

Figure 9: Plot between Y and t for different values of r_2

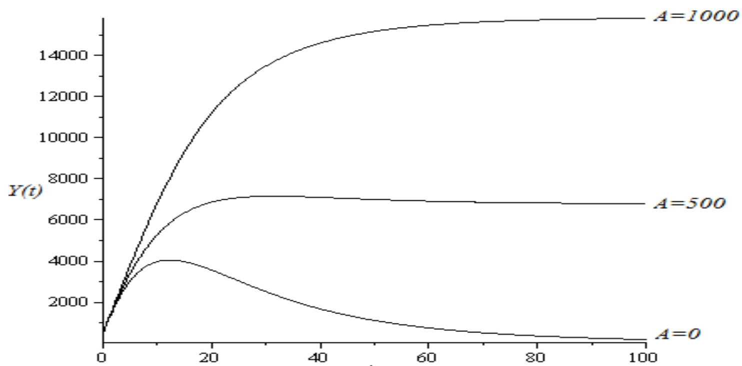

Figure 10: Plot between Y and t for different values of A

6. Conclusions

In this paper, a non linear mathematical model has been proposed and analyzed to study the impact of biomass density of ecological factors on the spread of carrier dependent infectious diseases. The equation governing the density of carrier population has been assumed to be a generalized logistic model with specific growth rate and carrying capacity which increase with the biomass density of ecological factors. It has been assumed further that the cumulative biomass density of such ecological factors is also governed by a generalized logistic model, which depends on human population density bilinearly. The model has been analyzed analytically and by computer simulation. The effects of parameters governing the ecological factors, conducive to the growth of carrier population, have been studied. It has been shown that the disease spread faster due to growth of biomass density of ecological factors and the growth of carrier population as well as due to their natural growth. It has been found that an infectious disease becomes more endemic due to immigration.

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