ISSN 0974 - 9373

Vol. 20 No. 1 (2016) **Journal of International Academy of Physical Sciences** pp. 1-16

Linear Stability of Equilibrium Points in the Restricted Three Body Problem with Perturbations*

Ram Kishor

Department of Mathematics, Central University of Rajasthan
Bandarsindari, Kishangarh, Ajmer-305817
Rajasthan, India
Email: kishor_math@curaj.ac.in, kishor.ram888@gmail.com

(Received December 24, 2015)

Abstract: We have considered the well known restricted three body problem under the influence of perturbations in the form of radiation pressure and lack of sphericity of the primaries, respectively. In the present article, author is interested to analyzed linear stability in case of three main resonances and hence, effect of perturbations on the stability regions. In order to achieve the goal, first, we have determined triangular equilibrium point then examined its linear stability and found that points are stable for the mass ratio $0 < \bar{\mu}_c < 0.0396478,$ in the presence of perturbations. Perturbed mass ratio for three main resonance cases is obtained and noticed that it is increasing function of radiation pressure but it decreases with respect to oblateness. It is also, observed that stability region expands with radiation pressure, in the presence and absence of oblateness but it contracts with oblateness. Again, effects of perturbations are analyzed and found that they affect the motion of restricted mass significantly, in space. Results are helpful to study more generalized problem in the presence of some other type of perturbations such as P-R drag and solar wind drag etc.

Keywords: RTBP, Linear Stability, Resonance, Radiation Pressure, Oblateness.

2000 Mathematics Subject Classification: 70F15, 70F07.

1. Introduction

Recent developments in the field of non-linear dynamics have revived interest in the dynamics of space objects, especially problems of three bodies. Restricted three body problem (RTBP) concerns the motion of a body with negligible mass under the gravitational influence of two massive bodies, called primaries, which orbit in circular Keplerian motion about their common center of mass on account of their mutual attractions. In classical RTBP, there are five equilibrium points out of them, three i.e. $L_{1,2,3}$

*Presented at CONIAPSXVIII, University of Allahabad during December 22-24, 2015.

are unstable, called collinear equilibrium points whereas, remaining two i.e. $L_{4,5}$ are stable for all mass ratio within the range of linear stability region $0 < \mu < 0.0385209$ Szebehely[1], called triangular equilibrium points.

The classical RTBP has no more existence when at least one of the interacting bodies is an intense emitter of radiation. There are many stellar dynamics problems where it is altogether inadequate to consider only gravitational force. For example, when a star acts upon a particle in a cloud of dust or gas, then in spite of gravitational attraction of star, the repulsive force of radiation pressure (Schuerman[2]), although drag forces, which have been neglected here, also work therein. Therefore, model is modified by superimposing a light repulsion field whose source is same as of gravitational field hence; the word ‘photogravitational’ is used. Several studies (Hamilton and Burns[3]; Singh and Ishwar[4]; Ishwar and Kushvah[5]; Singh[6]; Kushvah et al[7]; Kishor and Kushvah[8] of the restricted problem have included radiation pressure force. In RTBP, it is assumed that masses concerned are spherically symmetrical in homogeneous layers, but there are several celestial bodies (Saturn, Jupitor, Earth etc.) which are sufficiently oblate. Moreover, minor planets (e.g. Ceros) and a number of meteoroids have irregular shapes (Millis et al[9]; Norton and Chitwood[10]). Therefore, we have considered photo gravitational RTBP with an oblate body. The study of stability property of a dynamical system is a necessary step which brings not only the system to tackle many realistic problems of the world but also helps to understand the motion of test particle for a long time of evolution. Author is interested to examine the linear stability of the triangular equilibrium point under the influence of perturbations for three main cases of resonance and then to analyze effects of radiation pressure and oblateness on the stability region. Marchal[11] has discussed the linear stability in resonance cases whereas, Markellos et al[12] have examined the same in the presence of oblateness and observed that stability region decreases with respect to the oblateness. Kushvah[13] have performed the linear stability test in resonance cases in the generalized problem of RTBP in the presence of radiation and obtained perturbed mass ratio for the system whereas, Kishor and Kushvah[8] have discussed same for the Chermnykh-like problem in presence of a disc with power-law profile. In the present article, in spite of linear stability in the resonance case, we have analyzed the stability region under the influence of perturbations. In order to examine linear stability in photogravitational RTBP with an oblate body, in case of resonance, first, we have performed the test for for general case with the algorithms described in Moulton[14]; Murray and Dermott[15] and then proceeded for resonance cases.

The whole work is organized as follows: problem is formulated in Sec. (2) whereas; triangular equilibrium points and its linear stability are studied in Sec. (3) and (4) respectively. Linear stability in resonance cases is described in Sec. (5). Finally, the results are concluded in Sec. (6). Algebraic as well as numerical computation has been performed with the help of Mathematica® Wolfram[16] software package. For numerical computations, we have used $\mu = 9.0032789 \times 10^{-6}$, mass parameter of the Sun-Earth system in addition to the other parametric values.

2. Equations of Motion

It is supposed that forces governing the motion of restricted body (the mass which does not affects gravitationally to the motion of finite bodies) are gravitational attractions of both the primaries (the Sun and the Earth). Radiation pressure force of the first primary is also taking into account and its opposing nature to the gravitational attraction results that the mass reduction factor $q_1 = (1 - \frac{F_p}{F_g})$ (Schuerman[2]), where F_p and F_g be the radiation pressure and gravitational attraction forces, respectively. The oblateness of the Earth comes into picture in the form of oblateness coefficients $A_2 = \frac{R_e^2 - R_p^2}{5R^2}$ (McCuskey[17]), where R_e and R_p be the equatorial and polar radii of the oblate body, respectively and R is the separation of both the primaries. The units of mass, distance and time, are normalized in such a way that Gaussian constant $k^2 = 1$ which results that the mean motion of the system is $n = \sqrt{q_1 + \frac{3}{2}A_2}$. Now, let us suppose that $(-\mu, 0)$ and $(1-\mu, 0)$ be the co-ordinates of the first and second primaries, respectively and $P(x, y, 0)$ be the coordinate of restricted body, relative to the synodic frame of reference $OXYZ$ where $\mu = \frac{M_e}{M_S + M_e}$ be the mass parameter be in the Sun-Earth system with M_S and M_e are masses of the Sun and the Earth respectively.

The equations of motion of the restricted body in xy-plane are written as (Kushvah et al[7]):

(2.1) $$\ddot{x} - 2n\dot{y} = \Omega_x,$$

(2.2) $$\ddot{y} - 2n\dot{x} = \Omega_y,$$

(2.3) $$\Omega_x = n^2 x - \frac{q_1(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x+\mu-1)}{r_2^3} - \frac{3\mu A_2(x+\mu-1)}{2r_2^5},$$

(2.4) $$\Omega_y = n^2 y - \frac{(1-\mu)q_1 y}{r_1^3} - \frac{\mu y}{r_2^3} - \frac{3\mu A_2 y}{2r_2^5},$$

with $r = \sqrt{x^2 + y^2}$, $r_1 = \sqrt{(x+\mu)^2 + y^2}$ and $r_2 = \sqrt{(x+\mu-1)^2 + y^2}$ be the distances of the restricted mass from both the primaries and from common center of mass of the system, respectively. The last term on the right hand side in equations (2.3) and (2.4) is due to oblateness.

3. Triangular Equilibrium Points $L_{4,5}$

The point, at which the motion of a moving particle ceases, is known as equilibrium point. The triangular equilibrium point (as it forms triangle with the first and second primary, hence name) can be obtained from equations (2.1) and (2.2) in addition to the vanishing conditions of the velocity as well as acceleration of the restricted body. In other words, we have evaluated the triangular equilibrium point of the problem by solving the equations

(3.1) $$\Omega_x = 0 \text{ and } \Omega_y = 0,$$

simultaneously for space variables. Analytically, to solve above equations for real x and y, is a cumbersome task. Therefore, for convenience it is assumed that perturbed distances $r_1 = q_1^{1/3}(1+\delta_1)$ and $r_2 = 1+\delta_2$, where $\delta_1, \delta_2 << 1$. Hence,

(3.2) $$\begin{cases} r_1^2 = (x+\mu)^2 + y^2 = q_1^{2/3}(1+\delta_1)^2 \\ r_2^2 = (x+\mu-1)^2 + y^2 = (1+\delta_2)^2, \end{cases}$$

which provides

$$\begin{cases} x = \dfrac{q_1^{\frac{2}{3}}}{2} - \mu + (q_1^{\frac{2}{3}}\delta_1 - \delta_2) = x_e, \text{(say)} \\ y = \pm q_1^{\frac{1}{3}}\left[1 - \dfrac{q_1^{\frac{2}{3}}}{4} + (2 - q_1^{\frac{2}{3}})\delta_1 + \delta_2\right]^{\frac{1}{2}} = y_e, \text{(say)} \end{cases} \tag{3.3}$$

under linear approximation of δ_1 and δ_2, where

$$\delta_1 = \frac{\left(1 - q_1 - \frac{3}{2}A_2\right)}{3}, \qquad \delta_2 = \frac{(1 - q_1)}{3\left(1 + \frac{5}{2}A_2\right)}. \tag{3.4}$$

In the expression of y_e, '+' sign corresponds to L_4 point and '–' for L_5. We study the dynamics of L_4 point whereas, motion about L_5 is similar to that of L_4. Above, δ's are obtained with the help of equations (3.1-3.4) in addition with a suitable approximation in simplification of the expression.

Table 1: Co-ordinates of $L_{4,5}$: $(x_4, \pm y_4)$ for different values of parameters q_1, A_2 at mass parameter $\mu = 9.32789 \times 10^{-6}$ of the Sun-Earth system.

q_1	A_2	x_4	$\pm y_4$
1.0	0.0000	0.499991	0.866025
	0.0002	0.499891	0.865968
	0.0004	0.499791	0.865910
	0.0006	0.499691	0.865852
0.9	0.0000	0.463815	0.881440
	0.0002	0.463738	0.881378
	0.0004	0.463661	0.881317
	0.0006	0.463585	0.881256
0.8	0.0000	0.421663	0.888772
	0.0002	0.421610	0.888708
	0.0004	0.421557	0.888645
	0.0006	0.421504	0.888581

The coordinates (3.4) of the triangular equilibrium points are agree with the classical results for the classical values of parameters. Numerically, we have computed co-ordinates of $L_{4,5}$ for the Sun-Earth system at different values of mass reduction factor and oblateness (Table-1). From, Table (1), it is observed that positions of the triangular equilibrium points deviated from the classical results in the presence of radiation pressure as well as oblateness.

4. Linear Stability

We have examined linear stability of triangular equilibrium point by linearising the equations of motion of the restricted body in the vicinity of $L_4 : (x_e, y_e)$ point (Moulton[14]; Murray and Dermott[15]). Let us take a small displacement and a small velocity so that coordinates of restricted body and components of its velocity are $x = x_e + X, y = y_e + Y, \dot{x} = \dot{X}$ and $\dot{y} = \dot{X}$, where $X, Y, \dot{X}$, and $\dot{Y}$ are initially very small quantity. Substituting these into equations (2.1) and (2.2), and then expanding right hand side of resulting equations about (x_e, y_e) by Taylor's series up to first order terms in X and Y, we have new differential equations in the neighborhood of equilibrium point such as

$$(4.1) \qquad \ddot{X} - 2n\dot{Y} = X\Omega^0_{xx} + Y\Omega^0_{xy},$$

$$(4.2) \qquad \ddot{Y} + 2n\dot{X} = X\Omega^0_{yx} + Y\Omega^0_{yy},$$

where $\Omega^0_{xx}, \Omega^0_{xy}, \Omega^0_{yx}$ and Ω^0_{yy} are second order partial derivatives with respect to x and y respectively, of the effective potential Ω of the system and which are obtained with the help of equations (3) and (4). Superfix'0' denotes the corresponding value at equilibrium point. In order to solve equations (9) and (10), let

$$(4.3) \qquad X = Pe^{\lambda t} \quad \text{and} \quad Y = Qe^{\lambda t},$$

where P, Q are constants and λ is parameter. On substituting these into equations (4.1) and (4.2) and dividing out by common factors, a linear system of equations is obtained as follows:

$$(4.4) \qquad (\lambda^2 - \Omega^0_{xx})P + (-2n\lambda - \Omega^0_{xy})Q = 0,$$

$$(4.5) \qquad (2n\lambda - \Omega^0_{yx})P + (\lambda^2 - \Omega^0_{yy})Q = 0.$$

Since, for nontrivial solutions

$$\begin{vmatrix} \lambda^2 - \Omega^0_{xx} & -2n\lambda - \Omega^0_{xy} \\ 2n\lambda - \Omega^0_{yx} & \lambda^2 - \Omega^0_{yy} \end{vmatrix} = 0.$$

On simplifying the above determinant, we get a bi-quadratic equation in λ known as characteristic equation:

(4.6) $$\lambda^4 + A\lambda^2 + B = 0,$$

(4.7) $$A = 4n^2 - \left(\Omega^0_{xx} + \Omega^0_{yy}\right) \text{ and } B = \Omega^0_{xx}\Omega^0_{yy} - (\Omega^0{}_{xy})^2.$$

Substituting the values of Ω^0_{xx}, Ω^0_{xy}, Ω^0_{yx} and Ω^0_{yy} in equation (4.7), it is found that,

(4.8) $$A = \left[n^2 - \frac{3\mu A_2}{r^5_{2,0}}\right] \quad \text{and} \quad B = 9\mu(1-\mu)y_0^2\left[\frac{q_1}{r^5_{1,0}r^5_{2,0}}\left(1 + \frac{5A_2}{2r^2_{2,0}}\right)\right]$$

where, subscript '0' indicates value at triangular equilibrium point. Let $\lambda_1, \lambda_2, \lambda_3$ and λ_4 be the roots of bi-quadratic (4.6). Then, general solution of the system of linear differential equations with constant coefficients (4.1) and (4.2), can be expressed in terms of exponential such as

(4.9) $$X(t) = \sum_{j=1}^{4} P_j e^{\lambda_j t}; \quad Y(t) = \sum_{j=1}^{4} Q_j e^{\lambda_j t},$$

where constants Q_j are related to arbitrary constants $P_j, j = 1,2,3,4$ respectively, by the mean of linear equations (4.4) and (4.5). From, solution (4.9), it is clear that if $\lambda_j, j = 1,2,3,4$ are pure imaginary then X and Y are expressible in the form of periodic function and hence the solution (4.9) are said to be stable. However, if roots are of same magnitude i.e. multiple pure imaginary, then due to presence of secular term in the solution, it will be unstable. On the other hand, if $\lambda_j, j = 1,2,3,4$ are real then solutions are said to be unstable. In case of complex roots, if real part of at least one root is positive, then due to presence of exponential term in the solution, it unstable whereas, if all the real parts of the complex roots have negative sign then solution is said to be asymptotically stable (Boccaletti and Pucacco[18]). Now, from the characteristic equation (4.6), we have

$$(4.10)\qquad \lambda_{1,2} = \pm\left[\frac{(-A+\sqrt{A^2-4B})}{2}\right]^{\frac{1}{2}},\ \lambda_{3,4} = \pm\left[\frac{(-A-\sqrt{A^2-4B})}{2}\right]^{\frac{1}{2}}.$$

These four roots depend in a simple manner on the parameters μ, A_2 and q_1. In order to insure the stability of $L_{4,5}$ points, four roots $\lambda_j, j=1,2,3,4$ of the characteristic equation (4.6) must be pure imaginary. In other words, sign of the quantity (A^2-4B) in the equation (4.10) determine the scenario of stability property. Therefore, three cases arise which are given as

(i) $A^2-4B=0$ or $\mu=\bar{\mu}_c$

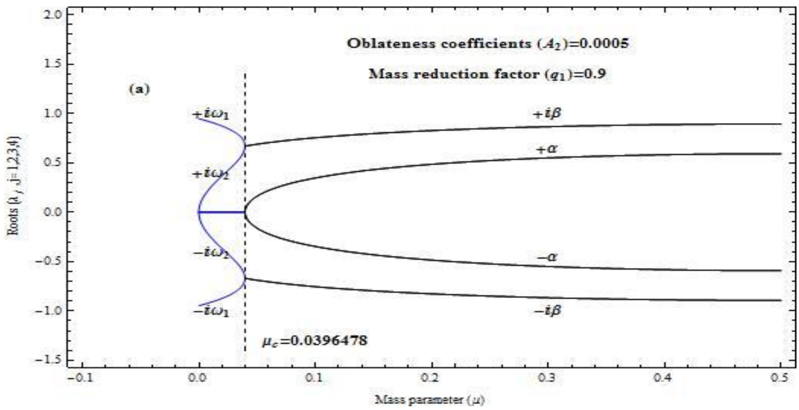
(ii) $A^2-4B<0$ or $\bar{\mu}_c<\mu\le\frac{1}{2}$

(iii) $A^2-4B>0$ or $0<\mu<\bar{\mu}_c$.

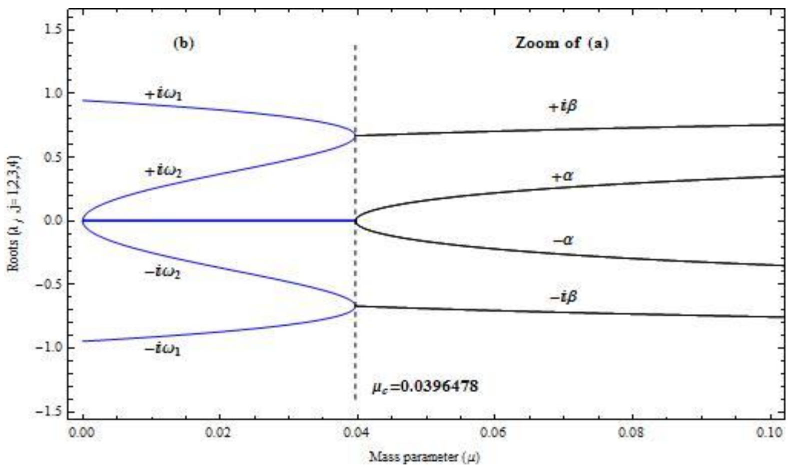
Case (i) corresponds to the critical value of the mass ratio $\bar{\mu}_c$. That is

$$(4.11)\quad A^2-4B=0=\left[n^2-\frac{3\mu A_2}{r_{2,0}^5}\right]^2-36\mu(1-\mu)y_0^2\left[\frac{q_1}{r_{1,0}^5 r_{2,0}^5}\left(1+\frac{5A_2}{2r_{2,0}^2}\right)\right]$$

provides the value of $\bar{\mu}_c=0.0396478$ at $q_1=0.9$ and $A_2=0.0005$ under the Taylor's series expansion about $\mu=0$ up to order second. At the classical values of parameters i.e. $q_1=1$ and $A_2=0$, equation (4.11) provides Routh's value of critical mass ratio $\bar{\mu}_c=\mu_c=0.0385209$. Moreover, when $A^2-4B=0$, equation (4.10) gives $\lambda_{1,3}=i\sqrt{\frac{A}{2}}$ and $\lambda_{2,4}=-i\sqrt{\frac{A}{2}}$. That is the characteristic equation (4.6) have multiple pure imaginary roots. Since, the multiple roots give secular terms in the solution of the equations of motion of restricted body and hence, triangular equilibrium points are unstable in this case.



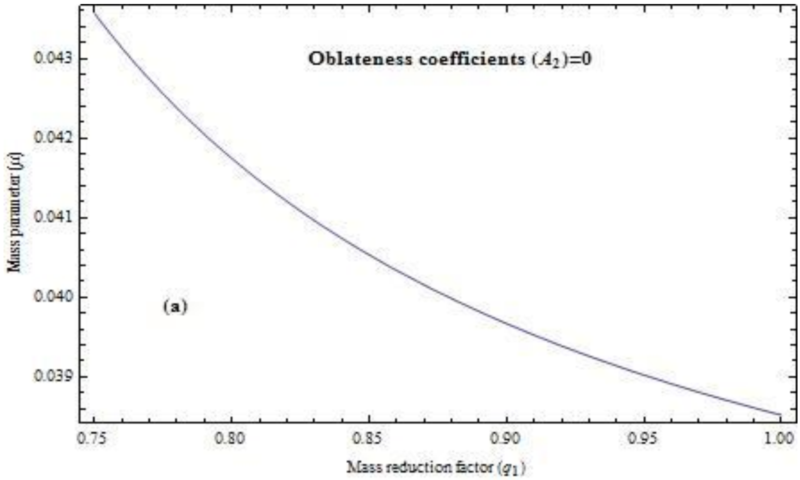
(a)



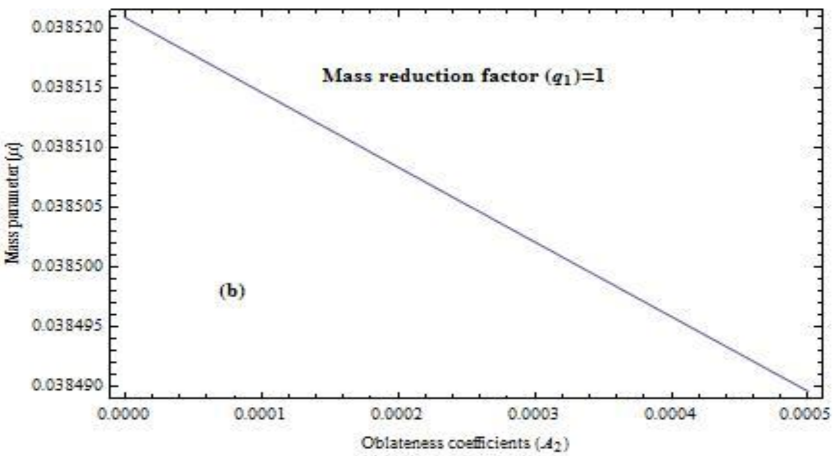
(b)

Figure 1: Numerical values of the real and imaginary components of the root λ_j, $j = 1, 2, 3, 4$ of the characteristic equation for triangular equilibrium points as the function of $\mu \in (0, 0.5)$.

In case (ii) i.e. when $\bar{\mu}_c < \mu \leq \frac{1}{2}$, roots are of the form $\pm\alpha \pm i\beta$ and so there will always be a positive real part hence, perturbed motion is unstable.. However, in case (iii) i.e. when $0 < \mu < \bar{\mu}_c$, roots are of the form $\pm i\omega_1$, $\pm i\omega_2$ and so, perturbed motion is stable. Figure1 (a, b) shows, how the nature of roots varies with the mass parameter. The nature of curves shown in Figure1 (a, b) can be understand by considering the analytical solutions to the characteristic equation (4.6). Figure 2 (a, b) show the variations of critical mass ratio in the absence of oblateness as well as radiation pressure force. It is observed that value of critical mass ratio increases with radiation pressure as well as oblatenessof the primaries, respectively



(a)



(b)

Figure 2: Variation of critical value of mass ratio μ_c in the absence of (a) oblateness, (b) radiation pressure.

5. Resonance Cases and Perturbed Mass Ratio μ_κ

In this section, we have described resonances in the frequencies of stable solution. The three main cases of the resonance (Marchal[11]; Markellos et al[12]; Kishor and Kushvah[8]) of the problem are obtained as follows:

(5.1) $$\omega_1 - \kappa\omega_2 = 0, \quad \kappa = 1,2,3.$$

Now, with the help of equation (18), we get

(5.2) $$\kappa^2 = \left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{-A - \sqrt{A^2 - 4B}}{-A + \sqrt{A^2 - 4B}},$$

and then using equation (15), we have

(5.3) $$9\mu(1-\mu)y_0^2\left(\frac{\kappa^2+1}{\kappa}\right)^2\left[\frac{q_1}{r_{1,0}^5 r_{2,0}^5}\left(1+\frac{5A_2}{2r_{2,0}^2}\right)\right]-\left[n^2-\frac{3\mu A_2}{r_{2,0}^5}\right]^2 = 0.$$

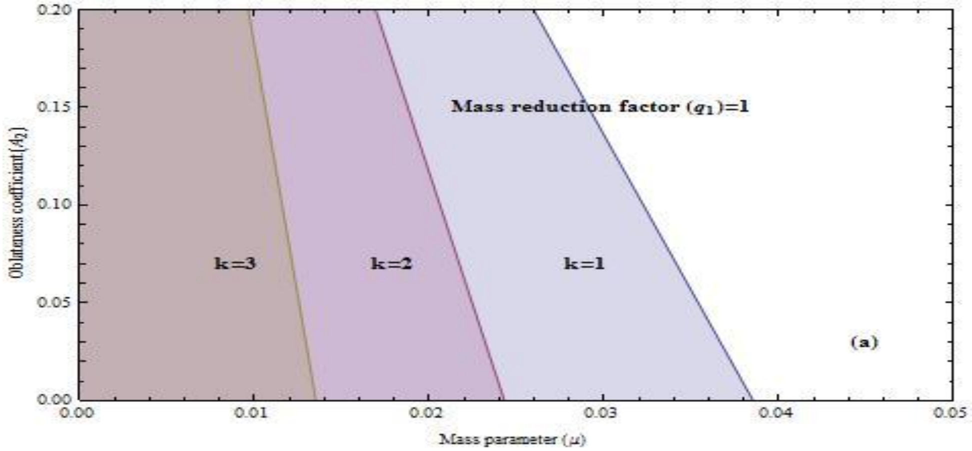
Since, $q_1 \in (0,1]; A_2 << 1$, so let us suppose that $q_1 = 1-\epsilon,\ \epsilon \ll 1$. Now, expanding above expression using Taylor's formula up to order second, about $\mu = 0$ under a suitable approximation, we have obtained a quadratic equation in μ which provides the critical mass ratio μ_κ for these three main resonance cases as follows:

(5.4) $$\mu_1 = 0.0385209 + 0.0089175\epsilon - 0.0627796A_2 + 0.1575610\epsilon A_2$$

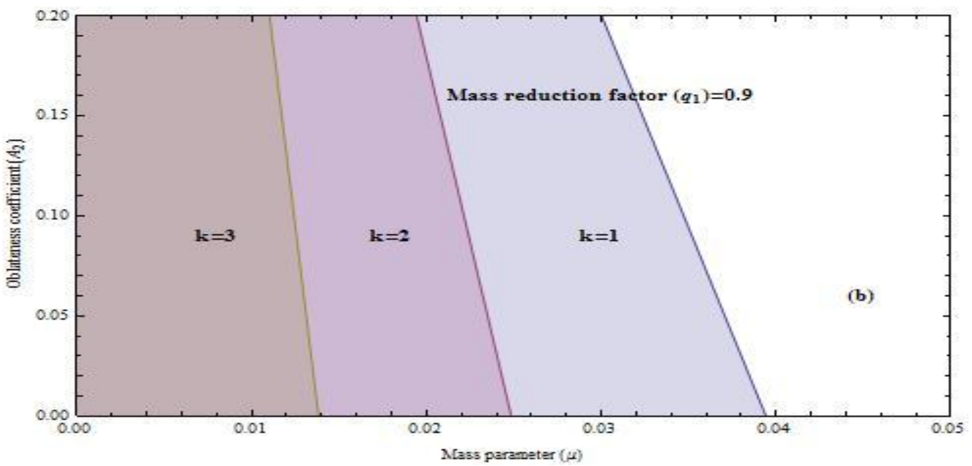
(5.5) $$\mu_2 = 0.0242939 + 0.0055365\epsilon - 0.0368506A_2 + 0.0977076\epsilon A_2$$

(5.6) $$\mu_3 = 0.0135160 + 0.0030453\epsilon - 0.0193830A_2 + 0.0536684\epsilon A_2$$

It can be see that mass ratio depends significantly on the radiation pressure and oblateness of the primaries, respectively. These expressions of the perturbed mass ratio agree with that of Deprit and Deprit-Bartholome[19] for classical values of parameters i.e. at $\epsilon = A_2 = 0$ moreover, it is similar to that of Markellos et al[12] for $\epsilon = 0,$ and $0 \le A_2 \le 0.1$ up to first order in A_2, Kushvah[20] and Kishor and Kushvah[8] etc. The values of perturbed mass ratio μ for several values of parameters q_1 and A_2, are computed up to seven decimal places (Table-2). It is notice that perturbed mass ratio is a decreasing function of oblateness A_2 and κ, whereas it is an increasing function of radiation pressure. It provides the information about the linear stability region i.e. it shows the upper bound of the stability regions on the μ-axis at the different values of q_1 and A_2 with $\kappa = 1,2,3$. Figures (3-4) represent the linear stability regions corresponding to main resonance curves in the $\mu - A_2$ and $\mu - q_1$ spaces, respectively. From Figure3 (a, b), it is observed that stability region decreases with the increment in the value of A_2 for all three main resonance cases, in the absence of radiation pressure but in the presence of it, rate of decrement become slow.

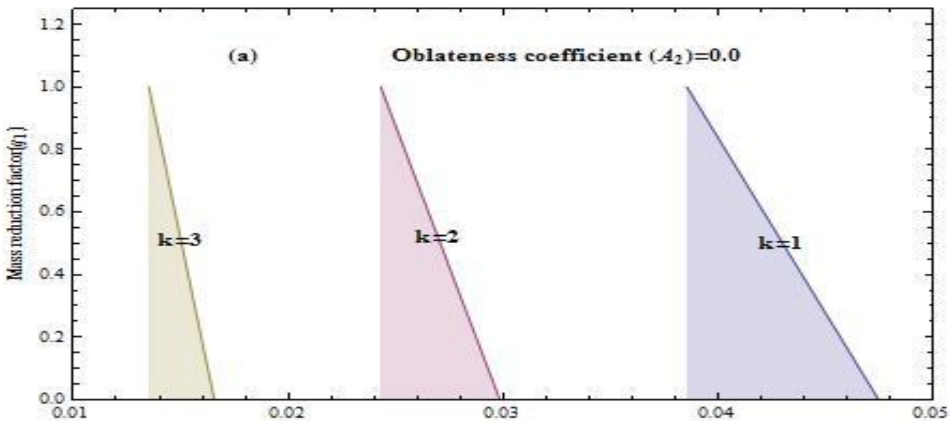


(a)

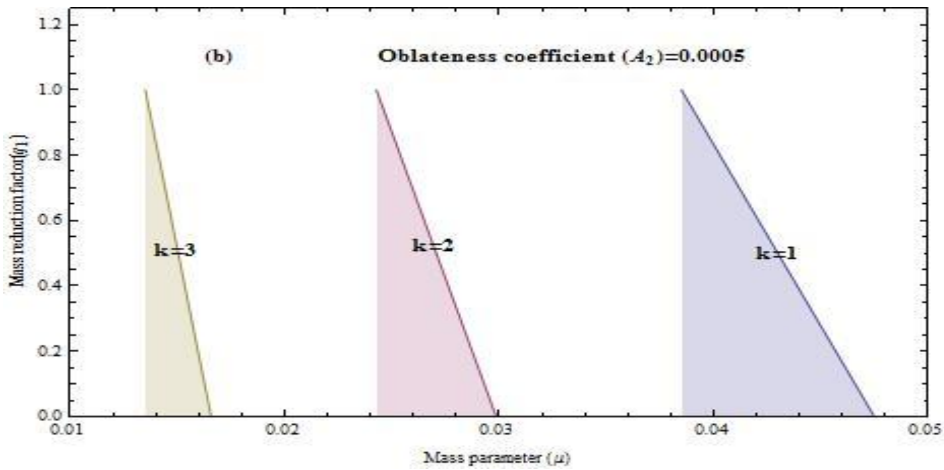


(b)

Figure 3: Linear stability region of L_4 in the $\mu - A_2$ space and the resonance curves $\omega_1 - \kappa\omega_2 = 0,\ \kappa = 1, 2, 3.$ (a) without radiation pressure (b) with radiation pressure.



(a)



(b)

Figure 4: Linear stability region of L_4 in the $\mu - q_1$ space and the resonance curves $\omega_1 - \kappa\omega_2 = 0,\ \kappa = 1,2,3.$ (a) without oblateness (b) with oblateness.

Figure 4 (a, b), shows that stability region increases with radiation pressure in the absence as well as presence of oblateness. On the basis of numeric as well as graphical results, it is observed that the perturbation factors have a significant effect on the motion of a spacecraft or satellite in space. It is noticed that the nature of the motion is unaffected but the stability region of the motion varies with the variations of radiation pressure and oblateness. The influence of oblateness of smaller primary is very less but considerable where as radiation pressure force affects significantly.

Table 2: Mass ratio $\mu_\kappa(q_1, A_2)$, $\kappa = 1, 2, 3, 4, 5$ at different values of q_1 and A_2 .

q_1	κ	$\mu_\kappa(q_1, 0.0)$	$\mu_\kappa(q_1,\ 0.0002)$	$\mu_\kappa(q_1,\ 0.0004)$	$\mu_\kappa(q_1,\ 0.0006)$
1.0	1	0.0385209	0.0385083	0.0384958	0.0384832
	2	0.0242939	0.0242865	0.0242792	0.0242718
	3	0.0135160	0.0135121	0.0135083	0.0135044
0.9	1	0.0394126	0.0394032	0.0393938	0.0393844
	2	0.0248475	0.0248421	0.0248367	0.0248313
	3	0.0138205	0.0138177	0.0138149	0.0138121
0.8	1	0.0403044	0.0402981	0.0402919	0.0402856
	2	0.0254012	0.0253977	0.0253943	0.0253908
	3	0.0141251	0.0141233	0.0141216	0.0141199

6. Conclusion

We have considered a photo gravitational RTBP with an oblate body and then analyzed the linear stability of triangular equilibrium points, in case of three main resonances. In order to examined linear stability, first, we have determined triangular equilibrium points under the influence of perturbations for the Sun-Earth system which are agree with classical results in the absence of perturbations. Linear stability of the equilibrium points are examined in the presence of radiation pressure and oblateness and it is found that these are stable under Routh's condition of critical mass ratio $\bar{\mu}_c = 0.0396478$. Again, perturbed mass ratio μ_κ, $\kappa = 1,2,3$ for the three main cases of resonance, are obtained and it is noticed that these increase with radiation pressure but decrease with respect to oblateness. Further, stability region in case of resonance is observed and found that it spans with radiation pressure in presence and absence of oblateness whereas, it contracts with oblateness. Thus, it is concluded that presence of radiation pressure and oblatenessaffects the motion of restricted body (spacecraft, asteroid, satellite etc.) significantly and these results are very helpful to observe the same in the Sun-Planet system. The present study and observations are also, applicable to analyze more generalized problems in addition with some other type of perturbations like P-R drag, solar wind drag etc.

Acknowledgement

Author is thankful to the IUCAA Library, Pune for important references as well as partial financial support. Thanks also go to host university for providing basic facility.

References

1. V. Szebehely, *Theory of orbits. The restricted problem of three bodies*, Academic Press, 1967.

2. D. W. Schuerman, The restricted three-body problem including radiation pressure, *The Astrophysical Journal*, **238** (1980) 337–342.

3. D. P. Hamilton and J. A. Burns, Orbital stability zones about asteroids: II - The destabilizing effects of eccentric orbits and of solar radiation, *Icarus,* **96** (1992) 43–64.

4. J. Singh and B. Ishwar, Stability of triangular points in the generalised photo gravitational restricted three body problem, *Bulletin of the Astronomical Society of India,* **27** (1999) 415.

5. B. Ishwar and B. S. Kushvah, Linear Stability of Triangular Equilibrium Points in the Generalized Photo gravitational Restricted Three Body Problem with Poynting-Robertson Drag, *Journal of Dynamical Systems & Geometric Theories*, **4** (2006) 79-86.

6. J. Singh, Combined Effects of Oblateness and Radiation on the Nonlinear Stability of L_4 in the Restricted Three-Body Problem, *The Astronomical Journal*, **137** (2009) 3286–3292.

7. B. S. Kushvah, R. Kishor and U. Dolas, Existence of equilibrium points and their linear stability in the generalized photogravitational Chermnykh-like problem with power-law profile, *Astrophysics and Space Sciences*, **337** (2012) 115–127.

8. R. Kishor and B. S. Kushvah, Linear stability and resonances in the generalized photogravitational Chermnykh-like problem with a disc, *MNRAS*, **436** (2013) 1741–1749.

9. R. L. Millis, L. H. Wasserman, O. G. Franz, R. A. Nye, R. C. Oliver, T. J. Kreidl, S. E. Jones, W. Hubbard, L. Lebofsky, R. Goff, R. Marcialis, M. Sykes, J. Frecker, D. Hunten, B. Zellner, H. Reitsema, G. Schneider, E. Dunham, J. Klavetter, K. Meech, T. Oswalt, J. Rafert, E. Strother, J. Smith, H. Povenmire, B. Jones, D. Kornbluh, L. Reed, K. Izor, M. F. A'Hearn, R. Schnurr, W. Osborn, D. Parker, W. T. Douglas, J. D. Beish, A. R. Klemola, M. Rios, A. Sanchez, J. Piironen, M. Mooney, R. S. Ireland and D. Leibow, The size, shape, density and albedo of Ceres from its occultation of BD+8 deg 471, *Icarus*, **72** (1987) 507–518.

10. O. R. Norton and L. A. Chitwood, *Field Guide to Meteors and Meteorites*, Springer, 2008.

11. C. Marchal, *The three-body problem*, Elsevier, 1990.

12. V. V. Markellos, K. E. Papadakis and E. A. Perdios, Non-Linear Stability Zones around Triangular Equilibria in the Plane Circular Restricted Three-Body Problem with Oblateness, *Astrophysics and Space Sciences*, **45** (1996) 157–164.

13. B. S. Kushvah, The effect of radiation pressure on the equilibrium points in the generalized photogravitational restricted three body problems. *Astrophysics and Space Sciences*, **315(b)** (2008) 231–241.

14. F. R. Moulton, *An introduction to celestial mechanics*, The MacMillan Company, 1914.

15. C. D. Murray and S. F. Dermott, *Solar System Dynamics*, Cambridge University Press, 2000.

16. S. Wolfram, *The Mathematica Book*, Wolfram Media, 2003.

17. S. W. McCuskey, *Introduction to celestial mechanics*, Addison-Wesley Publishing Company, 1963.

18. D. Boccaletti and G. Pucacco, *Theory of Orbits*, Springer –Verlag Berlin Heidelberg, 1996.

19. A. Deprit and A. Deprit-Bartholome, Stability of the triangular Lagrangian points, *The Astronomical Journal*, **72** (1967) 173–173.

20. B. S. Kushvah, Linear stability of equilibrium points in the generalized photogravitational Chermnykh's problem, *Astrophysics and Space Sciences,* **318(a)** (2008) 41–50.