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Overstable Magneto-Thermal Convection in a Viscoelastic Ferromagnetic Fluid Saturating a Porous Medium

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Abstract: The thermal stability of an incompressible, electrically conducting non-Newtonian ferromagnetic fluid saturated horizontal porous layer heated from below subjected to a magnetic field is investigated. The rheology of the ferromagnetic fluid is described by Walter's (model B') for calculating the shear stresses from the velocity gradients. The Darcy law for the non-Newtonian ferromagnetic fluid of the Walter's (model B') type is used to model the momentum equations. The boundaries are considered to be stress-free. The employed model incorporates the effects of buoyancy magnetization, kinematic viscoelasticity, medium porosity and medium permeability. The linear theory and normal mode technique are used to reduce the coupled non-linear partial differential equations to linear differential equations and the eigenvalue problem is solved analytically by using trial functions satisfying the boundary conditions in the Galerkin Weighted Residuals method. The criteria for both stationary and oscillatory modes are also derived analytically. Numerical results are computed using the software MATHEMATICA version 5.2 and presented graphically. It is observed that the magnetic field and buoyancy magnetization stabilize, whereas the medium permeability and medium porosity destabilize the physical system for both the cases of stationary and oscillatory motions. It is also found that oscillatory modes are not allowed in the absence of magnetic field and viscoelasticity implying thereby that principle of exchange of stabilities is valid.

Keywords Linear theory, Normal mode technique, Galerkin Weighted Residual method, Buoyancy magnetization, Exact solutions, Walter's (model B').

1. Introduction

Ferromagnetic fluid are magnetic fluids formed by a stable colloidal suspension of magnetic nanoparticles suspended in a carrier fluid usually organic solvent such as kerosene, heptane or water. These fluids deal with the mechanics of fluids motions influenced by strong forces of magnetic polarization. In the absence of an externally applied field ferrofluids does not retain its magnetization(Albrecht[1])and thus are often other classification as "Super paramagnets" rather than ferromagnets. In the recent years, the investigators on Ferrofluids has attracted researchers due to their applications in various fields like medical sciences, instrumentation, vacuum technology, oscillation damping, cooling of loudspeakers and high- speed silent printers etc.Convection instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical ferromagnetic fluid has been considered by Finlayson[2]. He explained the concept of thermomechanical interaction in ferromagnetic fluids. Thermoconvective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by Lalas and Carmi[3], whereas Shliomis[4] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability.

The $B'e$ nard convection in ferromagnetic fluid has been considered by many researchers(Siddheswar [5,6], Venkatasubramaniam[7] Aggarwal[8], Sunil et al.[9]). The medium has been considered to be non- porous in all the above studies. There has been a lot of interest, in recent years, in the study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of the convection flow. The original work on the stability of flow of a fluid through a porous medium using Darcy law was studied by Lapwood[10] and then by Wooding[11].Many researchers (Lee et al.[12], Borglin et al[13] Qin and Chadam[14] etc.) have investigated the onset of convection in ferromagnetic fluid saturating porous media due to its wide range of applications in oil reservoir modelling, petroleum industry, geothermal energy system, biological propulsion, solar collectors etc. The magnetic field, on the onset of convection has vast applications in physics and engineering. Practical cases of magnetic field are magneto- hydrodynamic generators, magnetic field sensor, magnetic storage media, cooling electronic devices, thermal insulators etc. Stileset at.[15] investigated the thermoconvective instability of a ferrofluid in a strong magnetic field and they found reduction in critical temperature gradient. Odenbach and Thurm[16] studied experimentally the magnetoviscous effects in ferrofluid and they investigated that the upper fraction containing only a negligible amount of large particles shows weak magneto viscous effect; whereas the lower fraction react noticeable on increasing magnetic field strength. They further found using rheumatically investigation that the application of an oscillatory mode gives direct access information about viscoelastic information.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigation of such fluids is desirable. There are many viscoelastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. Others classes of viscoelastic fluids are Rivlin- Ericksen and Walters' (model B') fluids, Walters[17] reported that mixture of polymethyl methacrylate and pyridine at 25 ^{o}C containing 30.5 g of polymer per liter with density 0.98g per litre behaves very nearly as the Walters' (model B') fluid. This types of viscoelastic fluid forms the basis for the manufacture of many important polymers and useful products in the manufacture of parts of space–crafts, aeroplanes, agriculture, communication and engineering appliances and in biomedical appliances.

Motivated by the various application mentioned above, an attempt has been made to study the thermal stability of a layer of ferromagnetic viscoelastic fluid heated from below saturating a porous medium in the presence of a horizontal magnetic field for stress free bounding surfaces. The rheology of the ferrofluids is described by the Walters' (model B'), which is an extension of the work on ferroconvective instability of fluidsby Vaidyanathan and Sekar[18]. In which they have shown the stability non-existence of oscillatory modes. The Galerkin-type weighted residuals method is used. We examine analytically the stability of the various non-dimensional parameters. It is observed that oscillatory modes occur due to the presence of a magnetic field, magnetization, medium permeability and medium porosity.

2. Nomenclature and Greek Symbols

$a =$ dimensionless wave number, [-], $d =$ depth of layer,[m],
$e =$ charge of an electron,[C], $E =$ Ratio of heat capacity,
$F =$ dimensionless viscoelastic number,[-], $\vec{g} =$ acceleration due to gravity,[ms^{-2}], $\vec{h} =$ perturbation in magnetic field H,[G],
$\vec{H}_f =$ magnetic field vector, [G], $K =$ Stokes' drag coefficient, [kgs^{-1}],
$k =$ wave number, [m^{-1}], $k_1 =$ medium permeability,[m^2],
M_f = Magnetization, N' = electron number density,[m^{-3}],
N = growth rate, [s^{-1}], P_1 = dimensionless medium permeability,[-],
p_f = fluid pressure,[Pa], p_1 =Prandtl number, [-],
p_2 =magnetic Prandtl number,[-], $Q =$ dimensionless Chandrasekhar number, [-],
$\overrightarrow{q_f}$ =filter velocity, [ms^{-1}], R_f = dimensionless thermal Rayleighnumber,[-],
T = temperature,[K], $\alpha_f =$ coefficient of thermal expansion,[K^{-1}],
$\beta_f =$ uniform temperature gradient, [K m^{-1}], $\eta_f =$ electrical resistivity, [m^2 s^{-1}], $\varepsilon =$ medium porosity , [-], $\theta =$ perturbation in temperature,[K],
$\kappa_f =$ thermal diffusivity, [m^2 s^{-1}], $\mu =$ dynamic viscosity,[kg m^{-1} s^{-1}],
$\mu_e =$ magnetic permeability, [H m^{-1}], $\nu =$ kinematic viscosity, [m^2 s^{-1}],
$\nu' =$ kinematic viscoelasticity, [m^2 s^{-1}], $\rho_f =$. density, [kg m^{-1}]

3. Theoretical Model

An infinite, incompressible, electrically non-conducting thin horizontal layer of Walters'(model B')viscoelastic ferromagnetic fluid, bounded by the planes z = 0 and z = d saturating in a porous medium is heated from below so that a uniform temperature gradient $\beta_f \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained.

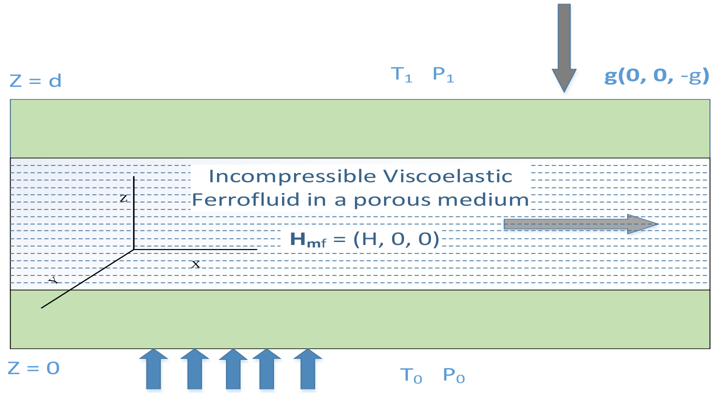

Figure 1. Geometrical configuration

The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1. The temperature T at the bottom and top surfaces are T_0 and T_1 respectively. The both bounding surfaces are assumed to be free. The thermo physical properties of viscoelastic ferrofluilds (viscosity, viscoelasticity, density, thermal conductivity and specific heat) are as constant for the analytical formulation. A uniform horizontal magnetic field $\overrightarrow{H_f}(H,0,0)$ perpendicular to z-axis and gravity force $\vec{g}(0,\ 0,\ -g)$ pervade the physical system. The mathematical equations of continuity, momentum and energy for viscoelastic ferromagnetic fluids through porous medium, using Boussinesq approximation relevant to the problimare

$$\nabla.\overrightarrow{q_f}=0, \tag{3.1}$$

$$\frac{1}{\varepsilon}\left[\frac{\partial\overrightarrow{q_f}}{\partial t^*}+\frac{1}{\varepsilon}\left(\overrightarrow{q_f}.\nabla\right)\overrightarrow{q_f}\right]=-\frac{1}{\rho_0}\nabla p_f+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right) \tag{3.2}$$

$$+\frac{\overrightarrow{M_f}.\nabla\overrightarrow{H_f}}{\rho_0}-\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{q_f}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\overrightarrow{H_f}\right)\times\overrightarrow{H_f},$$

$$E\frac{\partial T}{\partial t^*}+\left(\overrightarrow{q_f}.\nabla\right)T=k_f\nabla^2 T, \tag{3.3}$$

The magnetic permeability μ_e, the kinematic viscosity v and the thermal diffusivity k_f are all assumed to be constants and $E = \varepsilon + (1-\varepsilon)\left(\dfrac{\rho_s c_s}{\rho_i c_i}\right)$ where ρ_s, C_s and ρ_i, C_i are density and specific heat of solid (porous matrix) material and fluid.

The density equation of state is given as

$$\rho_f = \rho_0\left[1-\alpha_f\left(T-T_0\right)\right], \tag{3.4}$$

The magnetic induction and Gauss divergence equations are given as

$$\varepsilon\frac{\partial \overrightarrow{H_f}}{\partial t^*} = \nabla\times\left(\overrightarrow{q_f}\times\overrightarrow{H_f}\right) + \varepsilon\eta_f\nabla^2\overrightarrow{H_f}, \nabla.\overrightarrow{H_f} = 0. \tag{3.5}$$

The equation of state specifying $\overrightarrow{M_f}$ by two thermodynamic variables only ($\overrightarrow{H_f}$ and T), is necessary to complete the system. In the present case, it is assumed that the magnetization is independent of the magnetic field intensity so that $\overrightarrow{M_f} = M_f(T)$ only. As a first approximation, it is assumed that

$$\overrightarrow{M_f} = M_0\left[1-\gamma_f\left(T-T_0\right)\right] \tag{3.6}$$

where γ_f is the pyromagnetic coefficient and is defined by

$$\gamma_f = -\left(\frac{\partial M}{\partial T}\right)_{T_a}.$$

The primary flow representing the basic state is assumed to be quiescent (no setting of suspended magnetic nanoparticles) . Initially, no motions are present in the ferrofluid flow and time – independent solutions of equations (3.1)-(3.5) are taken. Therefore , the basic state satisfying equations (3.1)-(3.5) is given by

$$\overrightarrow{q_f} = q_b(0,0,0) = 0, p_f = p_b(z),$$

$$T = T_b(z) = T_0 - \beta z, \rho_f = \rho_0\left(1+\alpha_f\beta z\right) = \rho_b(z), \overrightarrow{M_f} = M_b(z).$$

To study the stability of the system, infinitesimal perturbations onto basic state, which are of the forms

$$(3.7)\qquad \begin{cases} \overrightarrow{q_f} = 0 + \overrightarrow{q_f^*}, T = T_b + T^*, \\ p_f = p_b + p^*, \rho_f = \rho_b + \rho^*, \\ \overrightarrow{H_f} = H_b + \overrightarrow{h^*}, \overrightarrow{M_f} = M_b + M^*, \end{cases}$$

where $\rho^*, p^*, M^*, T^*, h^*(h_x, h_y, h_z)$ and $q_f^*(u,v,w)$ denote the perturbations in density ρ_f, pressure p_f, magnetization M_f, temperature T, magnetic field $H_f(H,0,0)$ and filter velocity q_f (zero initially), respectively which are superimposed into the basic state. The change in magnetization M^* and density ρ^* caused by the perturbations T^* and γ_f in temperature and concentration, is given by

$$(3.8)\qquad M^* = -\gamma_f M_0 T^*, \rho^* = -\rho_0 \alpha_f T^*.$$

Using the equation (3.7) in the equations (3.1)-(3.6), as we obtain

$$(3.9)\qquad \frac{1}{\varepsilon}\frac{\partial \overrightarrow{q_f^*}}{\partial t^*} = -\frac{1}{\rho_0}\nabla p^* - g\alpha_f T^* - \frac{\gamma_f M_0 . \nabla \overrightarrow{H_f}}{\rho_0} T^*$$

$$-\frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)\overrightarrow{q_f^*} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla \times \overrightarrow{h^*}\right) \times \overrightarrow{H_f},$$

$$(3.10)\qquad \nabla . \overrightarrow{q_f^*} = 0,$$

$$(3.11)\qquad E\frac{\partial T^*}{\partial t^*} = \beta w + k_f \nabla^2 T,$$

$$(3.12)\qquad \varepsilon\frac{\partial \overrightarrow{h^*}}{\partial t^*} = \left(\overrightarrow{H_f} . \nabla\right)\overrightarrow{q_f^*} + \varepsilon\eta_f \nabla^2 \overrightarrow{h^*},$$

$$(3.13)\qquad \nabla . \overrightarrow{h^*} = 0 .$$

Cartesian form of above equation (3.9), (3.10), (3.12) and (3.13) becomes

(3.14) $$\frac{1}{\varepsilon}\frac{\partial u}{\partial t^*} = -\frac{1}{\rho_0}\frac{\partial}{\partial x}p^* - \frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^* - \frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)u,$$

(3.15) $$\frac{1}{\varepsilon}\frac{\partial v}{\partial t^*} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}p^* - \frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^* - \frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)v$$
$$+\frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\left(\frac{\partial}{\partial x}h_y - \frac{\partial}{\partial y}h_x\right),$$

(3.16) $$\frac{1}{\varepsilon}\frac{\partial w}{\partial t^*} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}p^* + g\alpha_f T^* - \frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^*$$
$$-\frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)w + \frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\left(\frac{\partial}{\partial x}h_z - \frac{\partial}{\partial z}h_x\right),$$

(3.17) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

(3.18) $$\varepsilon\frac{\partial h_x}{\partial t^*} = \left(\overrightarrow{H_f}.\frac{\partial}{\partial x}\right)u + \varepsilon\eta_f \nabla^2 h_x,$$

(3.19) $$\varepsilon\frac{\partial h_y}{\partial t^*} = \left(\overrightarrow{H_f}.\frac{\partial}{\partial x}\right)v + \varepsilon\eta_f \nabla^2 h_y,$$

(3.20) $$\varepsilon\frac{\partial h_z}{\partial t^*} = \left(\overrightarrow{H_f}.\frac{\partial}{\partial x}\right)w + \varepsilon\eta_f \nabla^2 h_z,$$

(3.21) $$\frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + \frac{\partial h_z}{\partial z} = 0.$$

Operating equation (3.9) by $\frac{\partial}{\partial x}$ equation (3.10) by $\frac{\partial}{\partial y}$ and then adding, we get

(3.21) $$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1}{k_1}\left(\nu - \frac{\partial}{\partial t^*}\nu'\right)\right]\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{1}{\rho_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)p^*$$

$$-\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)\frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^*+\frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\left(\frac{\partial^2}{\partial x \partial y}h_y-\frac{\partial^2}{\partial y^2}h_x\right),$$

which upon using equation (3.17) and then operating by $\frac{\partial}{\partial z}$ yields

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\right]\left(\frac{\partial^2 w}{\partial z^2}\right)=-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)p^*$$
$$+\frac{\partial}{\partial z}\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)\frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^*-\frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x \partial y}h_y-\frac{\partial^2}{\partial y^2}h_x\right), \tag{3.23}$$

Operating equation (3.16) by $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$, we have

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\right]\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w=-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)p^*$$
$$+g\alpha_f\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)T^*-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}T^*$$
$$+\frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial}{\partial x}h_z-\frac{\partial}{\partial z}h_x\right), \tag{3.24}$$

Adding equation (9.5) and (9.6), we obtain

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\right]\nabla^2 w=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$
$$\times\left(g\alpha_f-\frac{\gamma_f M_0 \nabla.\overrightarrow{H_f}}{\rho_0}\right)T^*+\frac{\mu_e \overrightarrow{H_f}}{4\pi\rho_0}\left(\nabla^2\frac{\partial h_z}{\partial x}\right), \tag{3.25}$$

Operating equation (3.14) and (3.15) by $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and algebraic simplification, one obtain

$$\left[\frac{1}{\varepsilon}\frac{\partial}{\partial t^*}+\frac{1}{k_1}\left(\nu-\frac{\partial}{\partial t^*}\nu'\right)\right]\zeta=\frac{\mu_e H_f}{4\pi\rho_0}\left(\frac{\partial \xi}{\partial x}\right), \tag{3.26}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity, $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density.

Operating equation (3.18) and (3.19) by- $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and using equation (3.21), we get

(3.27) $$\left(\frac{\partial}{\partial t^*} - \eta_f \nabla^2\right)\xi = \frac{H_f}{\varepsilon}\left(\frac{\partial \zeta}{\partial x}\right)$$

z- component of equation (3.20) is

(3.28) $$\left(\frac{\partial}{\partial t^*} - \eta_f \nabla^2\right)h_z = \frac{H_f}{\varepsilon}\left(\frac{\partial w}{\partial x}\right)$$

4. Normal Mode Analysis

Analyzing the disturbances into normal modes, the perturbation quantities are assumed to be of the form

(4.1) $$\left[w, T^*, \xi, \zeta, h_z\right] = \left[W(z), \Theta(z), X(z), Z(z), K(z)\right] \times \exp\left(ik_x x + ik_y y + n^* t\right),$$

where k_x and k_y are wave numbers along x-and y-directions, respectively, $k\left[=\left(k_x^2 + k_y^2\right)^{\frac{1}{2}}\right]$ is the resultant wave number of the disturbance and n^* is the growth rate (in general, a complex constant). For functions with this dependence on x, y and t, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -k^2\right)$ and $\left(\nabla^2 = \frac{\partial^2}{\partial z^2} - k^2\right)$.

Using the expression (4.1) in the equations (3.26), (3.27), (3.27), (3.28) and (3.13) in which we already eliminating horizontal component of filter velocity, magnetic field and pressure p^* in non-dimensional form as

(4.2) $$\left(\frac{\sigma}{\varepsilon} + \frac{1}{P_1} - \sigma F\right)\left(D^2 - a^2\right)W = -\frac{\alpha_f a^2 d^2}{\nu}\left(g - \frac{\gamma_f M_0 \nabla H}{\rho_0 \alpha_f}\right)\Theta + \frac{ik_x \mu_e H d^2}{4\pi\rho_0 \nu}\left(D^2 - a^2\right)K,$$

(4.3) $$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)Z=\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}X,$$

(4.4) $$\left(D^2-a^2-p_2\sigma\right)K=-\frac{ik_x Hd^2}{\varepsilon\eta}W,$$

(4.5) $$\left(D^2-a^2-p_2\sigma\right)X=-\frac{ik_x Hd^2}{\varepsilon\eta}Z,$$

(4.6) $$\left(D^2-a^2-Ep_1\sigma\right)\Theta=-\frac{\beta d^2}{\kappa}W.$$

where $a=kd, k_x=k\cos\theta, \sigma=\frac{nd^2}{\nu}, D=\frac{d}{dz}$ and the non-dimensional parameters are $p_1=\frac{\nu}{\kappa}$ is the thermal Prandtl number, $p_2=\frac{\nu}{\eta}$ is the magnetic Prandtl number, $P_1=\frac{k_1}{d^2}$ is the dimensionless medium permeability and $F=\frac{\nu'}{k_1}$ is non – dimension viscoelastic parameter.

Operating equation (4.2) by $\left(D^2-a^2-Ep_1\sigma\right)$ and using equation (4.6), one gets

(4.7) $$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)-R_f a^2\right]W$$

$$-\frac{ik_x\mu_e Hd^2}{4\pi\rho_0\nu}\left(D^2-a^2\right)\left(D^2-a^2-Ep_1\sigma\right)K=0,$$

Eliminating Z from equation (4.3) and (4.5), we get

(4.8) $$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}-\sigma F\right)\left(D^2-a^2-p_2\sigma\right)-\frac{k_x^2 d^2 Q}{\varepsilon}\right]X=0.$$

The appropriate boundary conditions for both bounding surfaces stress- free; using expression (4.1) in non- dimensional form transform to

(4.9) $$W=D^2W=0, X=DX=0, \Theta=0, D^2Z=0, K=0 \text{ at } z=0 \text{ and } z=1.$$

5. Method of Solution

The Galerkin weighted residuals method is used to obtain an approximate solution to the W, X and K of equations with the corresponding boundary conditions (4.9). In this method, the weighted functions are the same as the trial functions. Accordingly, the base functions W , X and K are taking in the following way

(5.1) $$W=\sum_{q=1}^{N} A_q W_q,\ X=\sum_{q=1}^{N} B_q X_q \text{ and } K=\sum_{q=1}^{N} C_q K_q ,$$

where $W_q=\left(z^q-2z^{q+2}+z^{q+3}\right),\ X_q=\left(z^{q+1}-2z^{q+2}+z^{q+3}\right), K_q=\left(z^q-z^{q+1}\right).$

The trial solutions satisfying the dimensionless boundary conditions A_q, B_q and C_q are unknown coefficients, and q = *1,2,3,4,.....N*. Using the expressions for W , X and K from (5.1) in equations (4.4), (4.7) and (4.8) for a first approximation i.e., N = *1* and multiplying the first and third equation by K_q and second by W_q and then integrating in the limits from zero to unity, we obtain following a set of linear homogeneous equations. This set of equations admit non-trivial solution only if its determinant is equal to zero. Therefore, it equating the determinant is equal to zero gives rise to the characteristic equation of the system in terms of the Rayleigh number for ferromagnetic fluid $R_f\left(=\left[g-\frac{\gamma_f M_0\nabla.H}{\rho_0\alpha_f}\right]\frac{\alpha_f\beta d^4}{\nu\kappa}\right)$ as follows

(5.2) $$R_f=\left[-28L_1\left(-\varepsilon+P_1\left(-1+F\varepsilon\right)\sigma\right)L_2+51a^2P_1QL_3\cos^2\theta\right]\left(868a^2P_1\varepsilon L_1\right)^{-1},$$

where $L_3=17a^4+168(10+Ep_1\sigma)+a^2(336+17Ep_1\sigma),$

$L_1=(10+a^2+p_2\sigma), L_2=3024+31a^4+306Ep_1\sigma+a^2(612+31Ep_1\sigma),$

It is observed from expression (5.2) that R_f is function of the non-dimensional parameters, $Q\left(=\frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta_f}\right)$ is the Chandrasekhar number accounting for magnetic field, p_1, p_2, P_1 and F .

6. Mathematical Analysis

(a) Stationary Convection: When the instability sets in as stationary convection, the marginal state is characterized by putting $\sigma = 0$ in eq.(5.2),which yields the Rayleigh number for the stationary convection R_f^s as

(6.1) $$R_f^s = \frac{\begin{bmatrix} 28\left(30240 + 9144a^2 + 922a^4 + 31a^6\right)\varepsilon \\ +51a^2\left(1680 + 336a^2 + 17a^4\right)P_1 Q\cos^2\theta \end{bmatrix}}{868a^2 P_1 \varepsilon\left(10 + a^2\right)}.$$

The critical cell size at the onset of instability is obtained from the condition $\left(dR_f^s / da^2\right)_{a^2 = a_c^2} = 0$, which gives a very complicated equation in a_c^2. Therefore, the critical values of the wave number a_c^2 and the corresponding critical Rayleigh number R_{fc}^s are obtained numerically using Mathematica 5.2. It is noteworthy from expression (6.1) that the thermal Rayleigh number is independent for stationary convection viscoelastic parameter due to vanishing of σ. Thus the viscoelastic ferrofluid behaves like a regular (Newtonian) Ferrofluid. In order to investigate the effects of magnetic field, medium permeability and medium porosity, we examine analytically the behavior of $\frac{dR_f^s}{dQ}, \frac{dR_f^s}{dP_1}$ and $\frac{dR_f^s}{d\varepsilon}$.

Eq. (6.1) gives

(6.2) $$\frac{dR_f^s}{dP_1} = \frac{-(3024 + 612a^2 + 31a^4)}{31a^2 P_1^2},$$

(6.3) $$\frac{dR_f^s}{dQ} = \frac{51(1680 + 336a^2 + 17a^4)cos^2}{868\varepsilon(10 + a^2)},$$

(6.4) $$\frac{dR_f^s}{d\varepsilon} = -\frac{51(1680 + 336a^2 + 17a^4)Qcos^2}{868\varepsilon^2(10 + a^2)}.$$

It is observed from eqs. (6.2) and (6.4) that the medium permeability and medium porosity has always a destabilizing effect for all values of the wave numbers. Eq.(6.3) depicts that magnetic field has a stabilizing effect on the stationary modes for all wave numbers.

In order to investigate the analytical effect of magnetization, R_f^s is replaced by $\frac{R_f^s}{\pi^4}$ in equation (6.1), which yields

(6.5) $$R_f^s = \frac{\left[\begin{array}{l} 28\left(30240+9144a^2+922a^4+31a^6\right)\varepsilon \\ +51a^2\left(1680+336a^2+17a^4\right)P_1 Q\cos^2\theta \end{array}\right]}{868a^2 P_1\varepsilon\left(10+a^2\right)} \times \frac{\pi^4}{1-\dfrac{\gamma_f M_0 \nabla H_f}{\rho_0 \alpha_f g}}$$

To see the effect of magnetization, we examine analytically the behavior of $\frac{dR_f^s}{dM_0}$. Equation (6.2) gives

(6.6) $$\frac{dR_f^s}{dM_0} = \frac{\nabla H_f \pi^4 \rho_0 \alpha_f \gamma_f g \left[\begin{array}{l} 28\left(30240+9144a^2+922a^4+31a^6\right)\varepsilon \\ +51a^2\left(1680+336a^2+17a^4\right)P_1 Q\cos^2\theta \end{array}\right]}{868a^2 P_1\varepsilon\left(10+a^2\right)\left(\gamma_f M_0 \nabla H_f - \rho_0 \alpha_f g\right)^2}$$

which shows that magnetization parameter has always stabilizing effect on the system.

(b) Oscillatory motion: Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of viscoelasticity parameters F. Equating the imaginary parts of equation (5.2) by putting $\sigma = i\sigma_i$,we obtain

(6.7) $$\sigma_i[(-28a_3(10+a^2)^2 + 51Qa^2P_1b_2cos^2\theta) - \sigma_i^{\,2}(28a_3p_2^{\,2})] = 0,$$

It is evident from equation (6.7) that σ_i may be either zero or non-zero, meaning thereby that the modes may be either non-oscillatory or oscillatory. In the absence of magnetic prandtl parameter, p_2 and viscoelastic parameter, F , equation (6.7) reduces to

(6.8) $$\sigma_i[(-28a_3(10+a^2)^2 + 51Qa^2P_1b_2cos^2\theta)] = 0,$$

which is identical with the earlier result by Vaidhyanathan et al.[19]. The term inside the square bracket of equation (6.8) is positive definite for all a. Hence $\sigma_i = 0$, which implies that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium in the absence of magnetic prandtl parameter and viscoelastic parameter. Thus from equation (6.1), it is observed that the oscillatory modes are introduced due to the presence of magnetic prandtl parameter and viscoelastic parameter, which were non- existent in their absence.

(c)The case of overstability: Since for overstability, we wish to determine the thermal Rayleigh number for the onset of instability is determined via a state of pure oscillations, it suffices to find conditions for which equation (6.7) will admit of solutions with σ_i real. The pure oscillatory motion of increasing amplitude are obtained by putting $\sigma = i\sigma_i \neq 0$ in eq. (5.2), and after some algebraic simplification, we have

(6.9) $$R_f = \Delta_r + i\sigma_i\Delta_i \ ,$$

where

(6.10) $$\Delta_r = \frac{-28(a_1(10+a^2)^2 + \sigma_i^2(a_1 p_2^2 - a_2(10+a^2)^2) - \sigma_i^4 a_2 p_2^2) + 51Qa^2P_1(b_1+\sigma_i^2 b_3)\cos^2\theta}{868a^2P_1\varepsilon((10+a^2)^2+\sigma_i^2 p_2^2)},$$

(6.11) $$\Delta_i = [(-28a_3(10+a^2)^2 + 51Qa^2P_1b_2\cos^2\theta) - \sigma_i^2(28a_3 p_2^2)] = 0.$$

Since $\sigma_i \neq 0$ for oscillatory modes, therefore equation (6.9) implies that $\Delta_i = 0$ which on simplification yields a dispersion relation as

(6.12) $$28a_3 p_2^2\sigma_i^2 + [28a_3(10+a^2)^2 - 51Qa^2P_1b_2cos^2\theta] = 0,$$

Also equation (6.12) with $R_f^{OSC} = \Delta_r$ on simplification gives the thermal Rayleigh number for oscillatory modes as

(6.13) $$R_f^{OSC} = \frac{-28(a_1(10+a^2)^2 + \sigma_i^2(a_1 p_2^2 - a_2(10+a^2)^2) - \sigma_i^4 a_2 p_2^2) + 51Qa^2P_1(b_1+\sigma_i^2 b_3)\cos^2\theta}{868a^2P_1\varepsilon((10+a^2)^2+\sigma_i^2 p_2^2)}.$$

where $a_1 = -\varepsilon(3024+31a^4+612a^2), a_2 = P_1(-1+F\varepsilon)(306Ep_1+31a^2Ep_1),$

$$a_3 = P_1(-1+F\varepsilon)(3024+31a^4+612a^2)-\varepsilon(306Ep_1+31a^2Ep_1),$$
$$b_1 = (1680+17a^4+336a^2)(10+a^2), b_3 = p_2(168Ep_1+17a^2Ep_1)$$
$$b_2 = (10+a^2)(168Ep_1+17a^2Ep_1)-p_2(1680+17a^4+336a^2).$$

Since σ_i is real for overstability, the two values of σ_i must be positive. The oscillatory neutral solutions of equation (6.13) are obtained by firstly determining the roots of eq. (6.12). For overstability σ_i is real, at most one root must be positive of eq. (6.12) for which the critical thermal Rayleigh number for oscillatory modes is obtained for various values of non-dimensional wave numbers.

7. Results and Discussions

Expressions of thermal Rayleigh number for both stationary and oscillatory motions are given by equation (6.1) and (6.13), respectively. The variation of thermal Rayleigh number with respect to non- dimensional wave number has been plotted by using eq.(25) for stationary case and eq. (6.13) for overstability case with $Q=50, P_1=1.4,\ Q=50, P_1=1.4,\ p_1=7,$ $p_2=1, \varepsilon=0.5, F=2.5, \theta=45^o, \alpha_f=10,\ \gamma_f=0.5, \rho_o=10, \nabla H=10,$ where in the experimental values and the fixed permissible values of the dimensionless parameters are used to investigate the effects of various parameters o the system numerically. These computations have been carried out by using the software Mathematica -5.2.

Figures 2 and 3 represent the variation of thermal Rayleigh number R_f versus wavenumber a for various values of medium permeability parameter P_1=1.2, 1.4, 1.6. For stationary convection and oscillatory motion respectively the Rayleigh number decreases with the increase in the parameter P_1 showing thereby the destabilizing effect of the medium permeability parameter on the system.

Figures 4 and 5 display the variation of thermal Rayleigh number R_f versus wavenumber a for the stationary convection and the case of overstability respectively, for various values of the magnetic field parameter Q=30, 50, 70. It is clear from the Figures 4 and 5 that the Rayleigh number increases with the increase in the parameter Q implying thereby the stabilizing effect of the magnetic field parameter on the system.

The variation of Rayleigh number R_f for the stationary convection and the case of overstability corresponding wavenumber a for different values of medium porosity parameter ε=0.5, 0.6, 0.7(stationary) and ε =0.50, 0.52

(overstability) is displaced in the figures 6 and 7. From the figures, it is found that the Rayleigh numbers for both the cases decreases with the increases in the parameter ε, showing thereby the destabilizing effect of the medium porosity parameter on the system.

Fig. 8 and 9 depicted that the variation of R_f verses a for oscillatory and non- oscillatory found that stabilizing effect of the magnetization on the system. Figure 10 illustrates the effect of thermal Rayleigh number R_f (oscillatory) with wavenumber a for different values of viscoelastic parameter F = 2.5, 2.6.The thermal Rayleigh number decrease with increase in the viscoelastic parameter F depicting the destabilizing effect of the viscoelastic parameter on the system. It is clear from the figures 2-10 that overstability is the dominant mode of stability (seen in table 1 and 2).

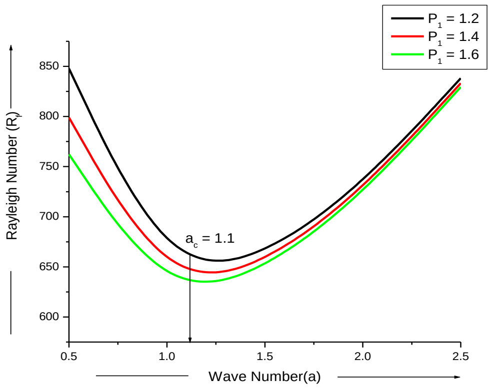

Figure 2. The variation of thermal Rayleigh number R_f (stationary) versus wavenumber a for three different values of the medium permeability parameter P_1 =1.2, 1.4, 1.6.

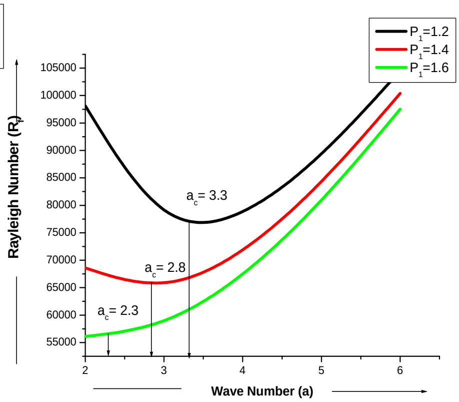

Figure 3: The variation of thermal Rayleigh number R_f (overstability) versus wave number a for three different values of the medium permeability parameter P_1 =1.2, 1.4, 1.6.

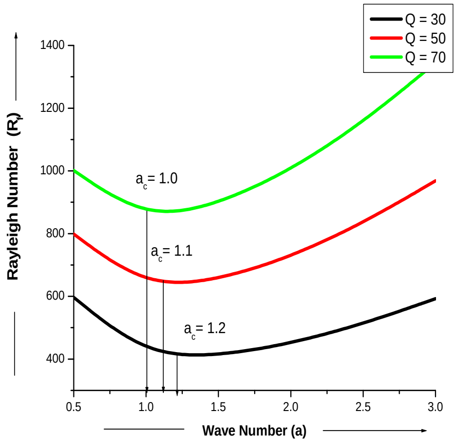

Figure 4. The variation of thermal Rayleigh number R_f (stationary) versus wavenumber a for three different values of the magnetic field parameter $Q = 30, 50, 70$.

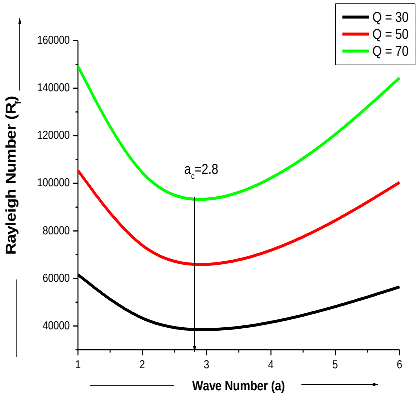

Figure 5. The variation of thermal Rayleigh number R_f (overstability) versus wavenumber a for three different values of the magnetic field parameter $Q = 30, 50, 70$.

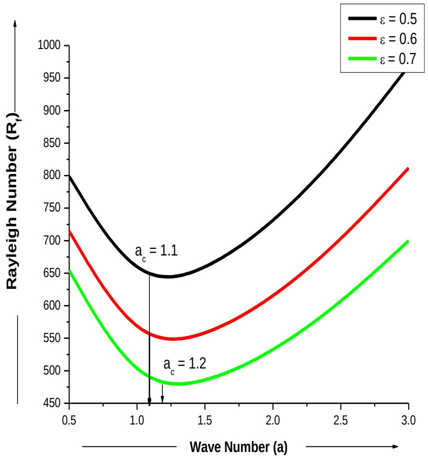

Figure 6: The variation of thermal Rayleigh number R_f (stationary) versus wavenumber a for three different values of the medium porosity parameter $\varepsilon = 0.5, 0.6, 0.7$.

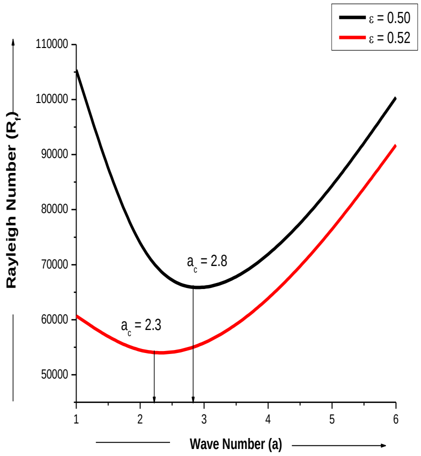

Figure 7: The variation of thermal Rayleigh number R_f (overstability) versus wavenumber a for two different values of the medium porosity parameter $\varepsilon = 0.50, 0.52$.

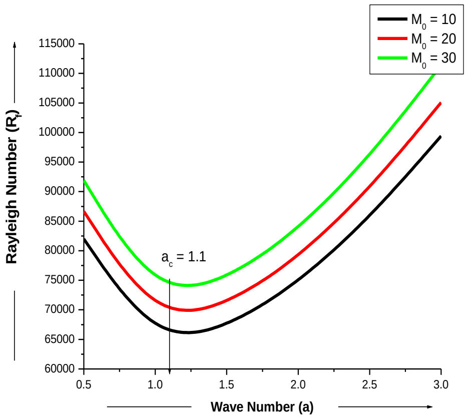

Figure 8. The variation of thermal Rayleigh number R_f (stationary) versus wavenumber a for three different values of the magnetization parameter $M_0 = 10, 20, 30$.

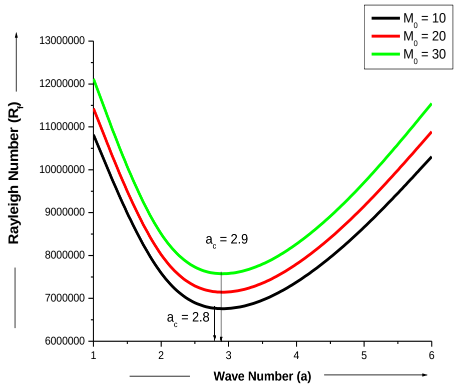

Figure 9. The variation of thermal Rayleigh number R_f (overstability) versus wavenumber a for three different values of the magnetization parameter $M_0 = 10, 20, 30$.

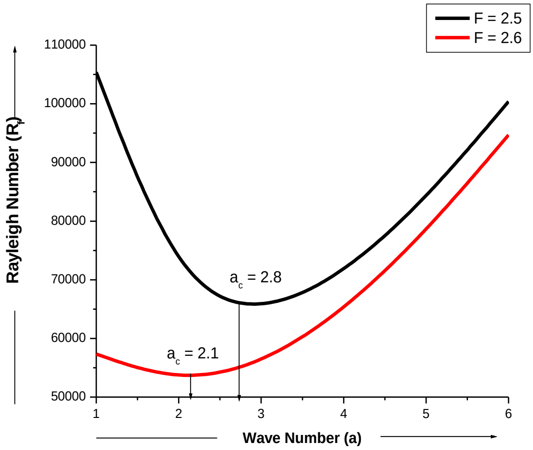

Figure 10. The variation of thermal Rayleigh number R_f (overstability) versus wavenumber a for different values of the viscoelastic number parameter $F = 2.5, 2.6$.

8. Conclusions

In the present paper, the combined effect of medium permeability, horizontal magnetic field, medium porosity, viscoelastic number and magnetization has been considered on the thermal stability of a ferromagnetic fluid. The effect of various parameters such as medium permeability, horizontal magnetic field, medium porosity, viscoelastic number and magnetization has been investigated analytically as well as graphically. The main results from the analysis of the paper are as follows.

(i) It is found that magnetic field and magnetization have a stabilizing effect whereas medium permeability and medium porosity have a destabilizing effect on the stationary modes. The reasons for stabilizing effect of magnetic field are accounted by Chandrasekhar[20].This is also valid for second- order fluids as well.

(ii) It is found that for overstability convection magnetic field and magnetization have a stabilizing effect whereas medium permeability and medium porosity as well as viscoelastic number parameter have a destabilizing effect on the system.

(iii) In the absence of magnetic prandtl parameter(and hence magnetic field) and viscoelastic parameter oscillatory modes are not allowed and the principle of exchange of stabilities is valid for a porous medium.

Table 1. Rayleigh number with respect to wave number for stationary convection with $Q = 50, P_1 = 1.4, p_1 = 7, p_2 = 1, \varepsilon = 0.5, F = 0,\ \theta = 45^o, \alpha_f = 10,\ \gamma_f = 0.5, \rho_o = 10, \nabla H = 10.$

a	0.5	1	1.5	2	2.5	3
$P_1 = 1.2$	847.12	641.532	659.295	731.747	838.095	973.231
$P_1 = 1.4$	798.88	627.45	651.516	726.017	833.143	968.519
$P_1 = 1.6$	762.257	616.888	645.681	721.72	829.428	964.985
$Q = 30$	596.524	410.267	409.58	449.361	511.771	592.42
$Q = 70$	1001.24	844.632	893.452	1002.67	1154.51	1344.62
$\varepsilon = 0.6$	714.565	536.957	550.709	610.744	699.238	811.811
$\varepsilon = 0.7$	654.34	472.319	478.705	528.406	603.592	699.877
$M_0 = 10$	82002	64405.3	66875.6	74522.9	85518.9	99414.7
$M_0 = 20$	86661.2	68064.7	70675.3	78757.1	90377.9	105063
$M_0 = 30$	91881.7	72165	74932.9	83501.5	95822.3	111392

Table 2. Rayleigh number with respect to wave number for overstablility convection with $Q=50, P_1=1.4, p_1=7, p_2=1, \varepsilon=0.5, F=2.5$, $\theta=45^o, \alpha_f=10, \gamma_f=0.5, \rho_o=10$, $\nabla H=10$.

A	1	2	3	4	5	6
$P_1=1.2$	522967	98142.3	74881.2	77436.5	88604	104491
$P_1=1.4$	105379	68599.5	63994.3	70909.1	83717	100391
$P_1=1.6$	65587	56115.7	57771.1	66723.4	80405.1	97525.3
$Q=30$	61558.6	40269.7	37375.4	41038.3	47838.6	56421.6
$Q=70$	149199	96929.3	90613.1	100780	119596	144360
$\varepsilon=0.52$	60687	52880	54697	63176.3	75953.2	91755.5
$M_0=10$	1.08167E7	7.04147E6	6.56876E6	7.27854E6	8.59323E6	1.03047E7
$M_0=20$	1.14313E7	7.44156E6	6.94199E6	7.6921E6	9.08148E6	1.08902E7
$M_0=30$	1.21199E7	7.88984E6	7.36018E6	8.15548E6	9.62856E6	1.15462E7
$F=2.6$	57340.7	52477	55451.5	64680.5	78156.7	94690.5

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