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Comparison of Spectra Using Matrix Representations for Sierpinski Graphs

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Abstract: There are a wide variety of graph matrix representations, among these are the adjacency matrix, incidence matrix, circuit matrix, Laplacian matrix and Signless Laplacian matrix. Spectra of Sierpinski graph can also be derived by studying eigenvalues. The choice of matrix representation clearly has a large effect on the suitability of spectrum in a number of pattern recognition tasks. The objective of this research are to find possible results using graph matrix representation.
Keywords: Signless Laplacian matrix, Spectra, Sierpinski Eulerian graph, Adjacency Matrix.

1. Introduction

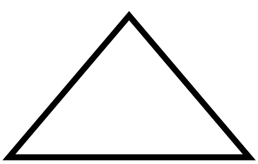
Graph theory is a branch of mathematics started by Euler as early as 1736. It took a hundred years before the second important contribution of Kirchhoff had been made for the analysis of electrical networks. There are many physical systems whose performance of a structure depends on the characteristics of its members. On the other hand, if the location of a member is changed, the properties of the structure will again be different. Therefore, the connectivity (topology) of the structure influences the performance of the entire structure. Hence, it is important to represent a system so that its topology can clearly be understood. The graph model of a system provides a powerful means for this purpose. In this section, basic concepts and definitions of graph theory are presented. Since some of the readers may be unfamiliar with the theory of graphs, simple examples are included to make it easier to understand the main concepts. Some of the uses of the theory of graphs in the context of civil engineering are as follows. A graph can be a model of a structure, a hydraulic network, a traffic network, a transportation system, a construction system or a resource allocation system.

These are only some of such models, and the applications of graph theory are much extensive. In this book, the theory of graphs is used as the model of a skeletal structure, and it is employed also as a means for transforming the connectivity properties of finite element meshes to those of graphs. This section will also enable the readers to develop their own ideas and methods in the light of the principles of graph theory. Some basics terminology and then discuss some important concepts in graph theory with many applications of graphs. Types of graph are pseudo graph, multiple graph, simple graph. A graph G consists of sets of vertices V and a set of edges E such that each edges is associated with an unordered pair of vertices then the graph is known as undirected graph and if each edges of graph is associated with an ordered pair of vertices then the graph is called directed graph or digraph. Although graphs are frequently stored in a computer as list of vertices and edges, they are pictured as diagrams in the plane in a natural way. Vertex set of graph is represented as a set of points in a plane and edge is represented by a line segment or an arc (not necessarily straight).

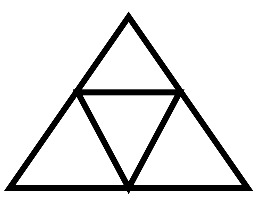
Sierpinski's Triangle is one of the most famous examples of a fractal although we should note that Benoit Mandelbrot first used the term fractal in 1975, almost sixty years after Sierpinski created his famous triangle. Sierpinski Gasket graphs are strongly related to the well known fractal called the Sierpinski Gasket. Sierpinski Gasket graphs appear in different areas of graph theory, topology, probability, pscychology. Sierpinski Gasket graphs S_n are the graphs naturally defined by the finite number of iterations.

To construct a Sierpinski triangle, first draw an equilateral triangle. Determine the midpoints of each side of the triangle. Connect the midpoints with straight lines to divide the original triangle into four smaller congruent equilateral triangles. Remove the middle triangle and repeat the same procedure for each remaining outer three triangles. Continue to repeat this entire process.

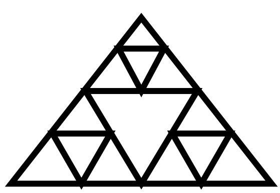
We denote by S_n the Sierpinski triangle obtained at the nth stage of the iterative process.



Step 1: S_1



Step 2: S_2



Step 3: S_3

The generalised Sierpinski graph, as per the above definition of the Sierpinski graphs $S(n,k)$. The vertex set of $S(n,k)$ consists of all $n-$tuples of the integers (for every $n \geq 1$ and $k \geq 1$) i.e. $V(S(n,k)) = \{1,2,3.........,k\}^n$. Two different vertices $u = (u_1, u_2, ..., u_n)$ and $v = (v_1, v_2, ..., v_n)$ are adjacent if and only if there exists an $h \in \{1,2,...., n\}$ such that

(i) $u_t = v_t$, for $t = 1,2,....., h-1$;

(ii) $u_h \neq v_h$; and

(iii) $u_t = v_h$ and $v_t = u_h$ for $t = h+1,......, n.$

In 1736 Euler noticed that the river Pregel flows through the city of Konigsberg dividing the city into four land regions of which, two are banks and two are islands and the four land regions were connected by 7 bridges. From here the existence of graph came out and now a day's it becomes an interesting part of study. Since earlier, Euler came out with the solution in terms of graph theory. He acknowledged that it was not possible to walk through the seven bridges exactly one time. He abstracted the case of Konigsberg by eliminating all unnecessary features. He drew a graphical picture consisting of "nodes" that represented the landmasses and the line-segments representing the bridges that connected those land masses. Euler not only proved that it is not possible, but also explained why it is not and what should be the characteristic of the graphs, so that its edge could be traversed exactly once. He came out with the then new concept of degree of nodes.

The Degree of Node can be defined as the number of edges touching a given node. Euler proposed that any given graph can be traversed with each edge traversed exactly once if and only if it had, zero or exactly two nodes with odd degrees. The graph following this condition is called, Eulerian circuit or path. Exactly two nodes are, (and must be) starting and end of your trip. If it has even nodes than we can easily come and leave the node without repeating the edge twice or more. Using this theorem, we can create and solve number of problems.

Trail that visits every edge of the graph once and only once is called Eulerian trail. Starting and ending vertices are different from the one on which it began. A graph of this kind is said to be traversable. An Eulerian circuit is an Eulerian trail that is a circuit i.e., it begins and ends on the same vertex. A graph is called Eulerian when it contains an Eulerian circuit. A digraph in which the in-degree equals the out-degree at each vertex.

The existence of an Euler path in a graph is directly related to the degrees of the graph's vertices. Euler formulated the following theorems of which the first two set a sufficient and necessary condition for the existence of an Euler circuit or path in a graph respectively.

Theorem 1.1: *An undirected graph has at least one Euler path if and only if it is connected and has two or zero vertices of odd degree.*

Theorem 1.2: *An undirected graph has an Euler circuit if and only if it is connected and has zero vertices of odd degree.*

Figure shown below, graphs indicating the distinct cases examined by the preceding theorems. Graph (a) has an Euler circuit, graph (b) has an Euler path but not an Euler circuit and graph (c) has neither a circuit nor path.

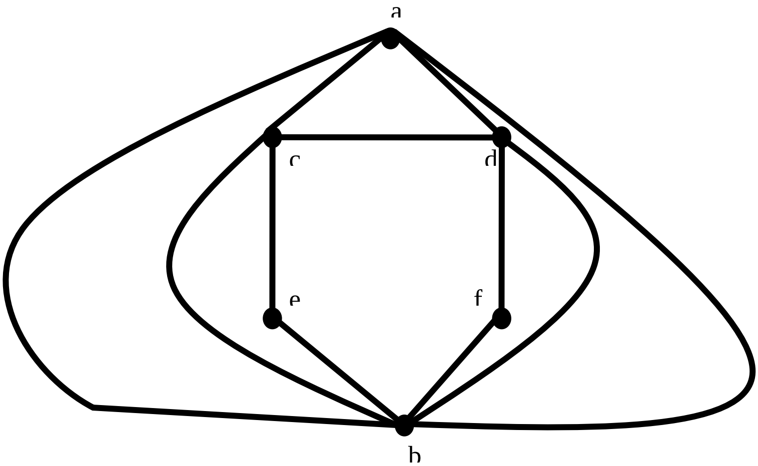


Figure 1.(a)

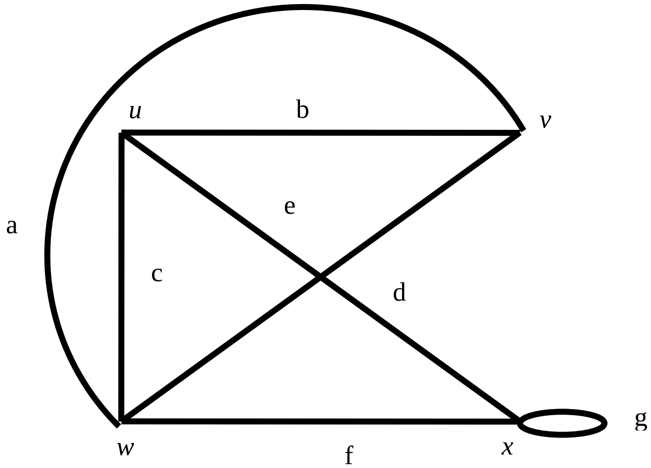


Figure 1. (b)

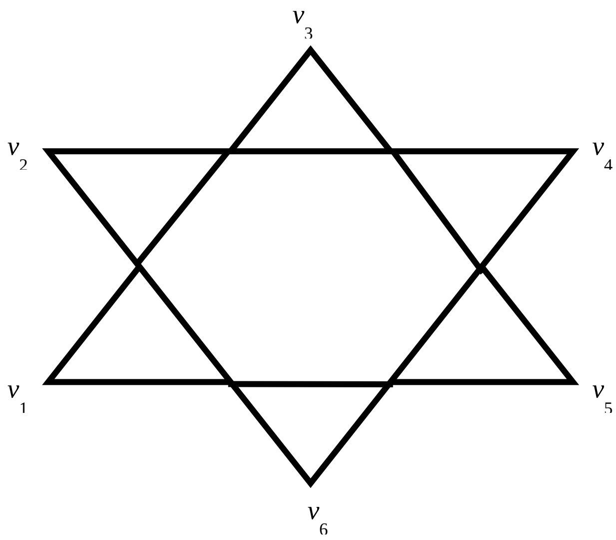


Figure 1.(c)

(a) A graph containing an Euler circuit

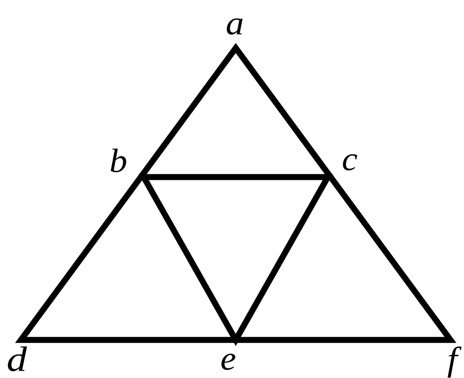
$(c-d-f-b-e-c-a-d-b-a-b-c)$

(b) Containing an Euler path $(b-a-c-d-g-f-e)$ but not Euler circuit.

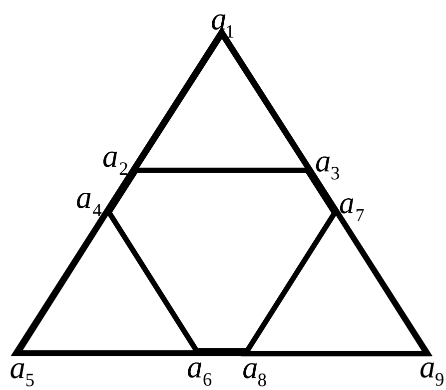
(c) A non-Eulerian graph, it does not contain Eulerian circuit since it is not connected.

Proposition 1.1: *Sierpinski's Gasket has an Euler circuit if and only if it is has two or zero vertices of odd degree.*

For the case of no odd vertices, the path can begin at any vertex and will end there; for the case of two odd vertices, the path must begin at one odd vertex and end at the other. Any finite connected graph with two odd vertices is traversable. A traversable trail may begin at either odd vertex and will end at the other odd vertex.



Eulerian



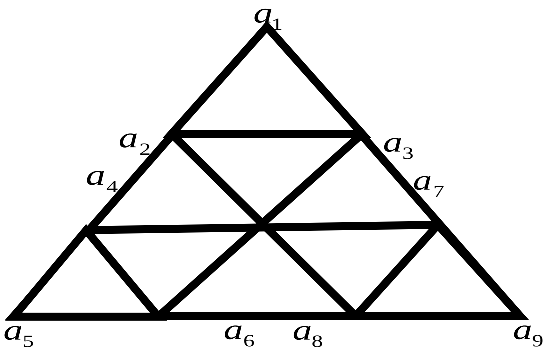
not Eulerian

Proposition 1.2: *Sierpinski's Gasket is Eulerian if and only if its vertices are all of even degree.*

Proof: **Case 1. (Eulerian as shown in figure):** Suppose *G* be a Sierpinski Graph is Eulerian, then G has an Eulerian trail which begins and ends at "*a*". If traverse along the trail then each and every time traverse a vertex having two edges. It is necessary condition that starting and ending nodes are same and each and every vertices must contain even degree (*deg(v)*) of vertices.

Case 2. (not Eulerian as shown in figure): Suppose *G* be a Sierpinski Graph is not Eulerian, then G has not Eulerian trail which begins at "a_1" but does not ends at "a_1". If traverse along the trail then each and every time traverse a vertex having two odd vertices or even vertices but above figure does not satisfy the Eulerian condition. Since each vertex in the middle of the trail is associated with three edges (G can not have just one odd vertex).

Let a_2, a_3, a_4, a_6, a_7 and a_8 be odd vertices in the connected graph G(not Eulerian). If we connect the vertices in pair $(a_2, a_8), (a_3, a_6)$ and (a_4, a_7) then the not Eulerian graph becomes the Sierpinski Eulerian. Hence all the vertices become even after connecting the odd vertices.



Sierpinski Eulerian

2. Eigenvalues of a Graph

Let *A* be the adjacency matrix of the graph *Γ* of order *N*. Let *I* be the identity matrix of order *N*, and let *λ* be a scalar. Then the determinant $|A-\lambda I|$ which is an ordinary polynomial in λ of *N*-th degree with scalar coefficients, is called the characteristic polynomial of *Γ*. The roots of the equation $|A-\lambda I| = 0$ are called the eigenvalues of the graph *Γ* (also of the matrix *A*). The set of eigenvalues is called the spectrum of the graph *Γ*. The multiplicity of an eigenvalue *λ* is called the algebraic multiplicity of *λ*. The equation $Au = \lambda u$ is called an eigenvalue equation. A nonzero solution u of the equation is called an eigenvector or eigenfunction for the eigenvalue *λ*. The vector space constructed from the set of eigenvectors corresponding to a particular eigenvalue *λ* is called the eigenspace of *λ*. The dimension of the eigenspace of an eigenvalue *λ* is the geometric multiplicity of *λ*. For a symmetric matrix, the geometric and algebraic multiplicities of an eigenvalue are equal.

3. Laplacian Matrix

We consider graphs which has no loops or parallel edges, unless stated otherwise. Thus a graph $G = (V(G), E(G))$, consist of a finite set of vertices, $V(G),$ and a set of edges, $E(G),$ each of whose elements is a pair of distinct vertices. Given a graph, one associates a variety of matrices with the graph. Some of the important ones will be defined now. Let G be a graph with $V(G) = \{1,2,....., n\}, E(G) = \{e_1, e_2,...., e_n\}$. The adjacency matrix $A(G)$ of G

is an $n \times n$ matrix with its rows and columns indexed by $V(G)$ and with the $(i, j)-$entry equal to 1 if vertices i, j are adjacent (i.e., joined by an edge) 0(zero) otherwise. Thus $A(G)$ is a symmetric matrix with its $i-$th row (or column) sum equal to $d_i(G)$, which by definition is the degree of the vertex i, $i = 1,2,.....,n.$ Let $D(G)$ denote the $n \times n$ diagonal matrix, $i-$th diagonal entry is $d_i(G), i = 1,2,.....,n.$

The Laplacian matrix of G, denoted by $L(G)$, is simply the matrix $D(G) - A(G).$

There is another way to view the Laplacian matrix. Suppose each edge of G is assigned an orientation, which is arbitrary but fixed. The (vertex-edge) incidence matrix of G, denoted by $Q(G)$, is the $n \times m$ matrix defined as follows. The rows and the columns of $Q(G)$ are indexed by $V(G), E(G)$ respectively. The $(i, j)-$entry of $Q(G)$ is 0(zero) if vertex i edge e_j are not incident and otherwise it is 1 or -1 according as e_j originates or terminates at i respectively.

A simple verification reveals that the Laplacian matrix $L(G)$ equals $Q(G)Q(G)^T$, (where T denotes transpose), suggests that the Laplacian might depend on the orientation, although it is evident from the definition that the Laplacian is independent of the orientation.

4. Signless Laplacian Matrix

The adjacency matrix $A(G)$ of G is an $n \times n$ matrix with its rows and columns indexed by $V(G)$ and with the $(i, j)-$entry equal to 1 if vertices i, j are adjacent (i.e., joined by an edge) 0(zero) otherwise. Thus $A(G)$ is a symmetric matrix with its $i-$th row (or column) sum equal to $d_i(G)$, which by definition is the degree of the vertex i, $i = 1,2,.....,n.$ Let $D(G)$ denote the $n \times n$ diagonal matrix, $i-$th diagonal entry is $d_i(G), i = 1,2,.....,n.$

The Signless Laplacian matrix of G, denoted by $L(G)$, is simply the matrix $D(G) + A(G).$

Theorem 4.1: *Let G be a graph on n vertices with vertex degrees* $d_1, d_2, \ldots\ldots, d_n$ *and largest Q-eigenvalue* q_1. *Then* $2\min d_i \le q_1 \le 2\max d_i$. *For a connected graph G, equality holds in either of these in equalities if and only if G is regular.*

Theorem 4.2: *Let G be a graph on n vertices with vertex degrees* $d_1, d_2, \ldots\ldots, d_n$ *and largest Q-eigenvalue* q_1. *Then*

$$\min(d_i + d_j) \le q_1 \le \max(d_i + d_j),$$

where (i,j) runs over all pairs of adjacent vertices of G. For a connected graph G, equality holds in either of these inequalities if and only if G is regular or semi-regular bipartite.

Proof: The line graph L(G) of G has largest eigenvalue $q_1 - 2$. Consider an edge u of G which joins vertices i and j. The vertex u of L(G) has degree $d_i + d_j - 2$. Hence, $\min(d_i + d_j - 2) \le q_1 - 2 \le \max(d_i + d_j - 2)$, which proves the theorem.

Lemma 4.1*:* *Let* $p(x)$ *be a given polynomial. If* λ *is an eigenvalue of* A, *while* x *is an associated eigenvector, then* $p(\lambda)$ *is an eigenvalue of the matrix* $p(A)$ *and* x *is an eigenvector of* $p(A)$ *associated with* $p(\lambda)$. *The characteristic polynomial of A is defined by*

$$\chi_A(t) = \det(tI - A)$$

Notes: The roots of the characteristic polynomial χ_A are exactly the eigenvalues of A. By the Fundamental Theorem of Algebra we know that every polynomial with degree n has exactly n complex roots (counted with multiplicities).

Lemma 4.2: *Let A be a* $n \times n -$ *matrix with eigenvalues* $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$. *Then* $\mathrm{tr}(A) = \sum_{i=1}^{n} \lambda_i$.

Lemma 4.3: *Let A be a symmetric real matrix. Suppose v and w are eigenvectors of A associated with the eigenvalues* λ *and* μ *respectively. If* $\lambda \neq \mu$ *then v* $\perp$ *w, i.e. v and w are orthogonal.*

Proposition 4.1: *The least eigenvalue of the signless Laplacian of a connected graph is equal to 0 if and only if the graph is bipartite. In this case 0 is a simple eigenvalue.*

Proof: Let $x^T = (x_1, \ldots\ldots, x_n)$.For a non-zero vector x we have Qx=0 if and only if $R^T x = 0$. The later holds if and only if $x_i = -x_j$ for every edge, i.e. if and only if G is bipartite. Since the graph is connected, x is determined up to a scalar multiple by the value of its coordinate corresponding to any fixed vertex i.

Theorem 4.3: *(Spectral Theorem) Let A be a n×n symmetric real matrix. Then there are n pairwise orthogonal (real) eigenvectors v_i of A associated with real eigenvalues of A.*

Consider $\lambda_1(A) \leq \ldots \leq \lambda_n(A)$ are eigenvalues of a symmetric matrix A. Some of these eigenvalues can be equal; we say that those eigenvalues have multiplicity greater than 1. Thus we denote the spectrum of A also in the form $\overline{\lambda}_1^{[m_1]}, \ldots\ldots, \overline{\lambda}_2^{[m_2]}$, where $\overline{\lambda}_i$ is an eigenvalue with multiplicity m_i.

Theorem 4.4: *(Rayleigh-Ritz) Let A be an n×n real symmetric matrix, and let $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$ be the eigenvalues of A. Then*

$$\lambda_n = \max_{x \neq 0} \frac{x^T Ax}{x^T x} = \max_{x^T x = 1} x^T Ax,$$

$$\lambda_1 = \min_{x \neq 0} \frac{x^T Ax}{x^T x} = \min_{x^T x = 1} x^T Ax.$$

Definiton 4.1: *(Adjacency eigenvalues) The eigenvalues of A(G) are called the adjacency eigenvalues of G. The set of all the adjacency eigenvalues are called the (adjacency) spectrum of the graph G.*

Lemma[1] 4.4: *Let G be a graph on n vertices.*

i) The maximum eigenvalue of G lies between the average and the maximum degree of G, i.e. $\overline{d} \leq \lambda_n \leq \Delta$.

ii) The range of all the eigenvalues of a graph is $-\Delta \leq \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \leq \Delta$.

Proof: i) The Rayleigh quotient for some special vector is greater than $\overline{d}$. This suffices to get the first inequality, because the maximum of the Rayleigh quotient is λ_n. The other inequality in (i) follows from the second point. Set x = (1, 1,..., 1)T. The Rayleigh quotient for this vector equals:

$$R(A;x) = \frac{x^T Ax}{x^T x} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} x_i A_{ij} x_j}{\sum_{i=1}^{n} x_i^2} = \frac{\sum_{i=1}^{n} \sum_{j:j\sim i} 1}{n} = \frac{\sum_{i=1}^{n} d_i}{n} = \overline{d}$$

ii) We have to show that the absolute value of every eigenvalue is less than or equal to the maximum degree. Let u be an eigenvector corresponding to the eigenvalue λ, and let u_j denote the entry with the largest absolute value. We have

$$|\lambda||u_j| = |\lambda u_j| = |(Au)_j| = \left|\sum_{i\sim j} u_i\right| \le \sum_{i\sim j}|u_i| \le d_j|u_j| \le \Delta|u_j|.$$

Thus we have $|\lambda| \le \Delta$ as required.

Definiton 4.2: *(Laplacian eigenvalues) The eigenvalues of L(G) are called the Laplacian eigenvalues. The set of all the Laplacian eigenvalues are called the (Laplacian) spectrum of the graph G.*

Lemma[2] 4.5: *Let G be a graph on n vertices with Laplacian eigenvalues $\lambda_1 = 0 \le \lambda_2 \le \ldots \le \lambda_n$ and maximum degree Δ. Then $0 \le \lambda_i \le 2\Delta$ and $\lambda_n \ge \Delta$.*

Proof: All eigenvalues are nonnegative by positive semidefinite matrices. Let u be an eigenvector corresponding to the eigenvalue λ, and let u_j denote the entry with the largest absolute value. We have

$$|\lambda||u_j| = |\lambda u_j| = \left|d_j u_j - \sum_{i\sim j} u_i\right| \le d_j|u_j| + \sum_{i\sim j}|u_i| \le 2d_j|u_j| \le 2\Delta|u_j|.$$

Thus, we have $|\lambda| \le 2\Delta$ as required. Let j be the vertex with maximal degree, i.e. $d_j = \Delta$. We define the characteristic vector x:

$$x_i = \begin{cases} 1, & \text{if } i = j; \\ 0, & \text{otherwise.} \end{cases}$$

Now, the desired inequality follows:

$$\lambda_n = \max_{\tilde{x}\neq 0} \frac{\tilde{x}^T \tilde{x}}{\tilde{x}^T \tilde{x}} \ge \frac{x^T L x}{x^T x} = \frac{\sum_{\{u,v\}\in E}(x_u - x_v)^2}{1} = \Delta$$

5. Result and Discussion

The Adjacency, Laplacian and Signless laplacian eigenvalues of the representation matrices A, L and $\overline{L}$ of Siepinski graph and Sierpinski Eulerian graph are shown in Fig.. The eigenvalue spectra become more comparable via the proposed notations $\lambda 1, \lambda 2$ of Sierpinski graph and Sierpinski Eulerian respectively.

Table 1. Spectrum of Sierpinski graph 2nd iteration on 3 vertices (i.e. $S(2,3)$) round off to 2 decimal placces

Sr. No.	Adjacency matrix	Laplacian matrix	Signless Laplacian Matrix
1	-2.00	0.00	1.00
2	-1.53	0.70	1.00
3	-1.53	0.70	1.00
4	-0.73	3.00	1.44
5	-0.35	3.00	2.38
6	-0.35	3.00	2.38
7	1.88	4.30	4.62
8	1.88	4.30	4.62
9	2.73	5.00	5.56

Table 2. Spectrum of Sierpinski graph 3rd iteration on 3 vertices (i.e. $S(3,3)$) round off to 2 decimal places

Sr. No.	Adjacency matrix	Laplacian matrix	Signless Laplacian Matrix
1	-2.00	0.00	1.00
2	-2.00	0.14	1.00
3	-2.00	0.14	1.00
4	-2.00	0.70	1.00
5	-1.87	0.70	1.00
6	-1.87	0.70	1.00
7	-1.47	1.10	1.09
8	-1.41	1.10	1.30
9	-1.41	1.38	1.30
10	-1.00	3.00	1.87
11	-1.00	3.00	1.87
12	-0.62	3.00	2.20
13	-0.46	3.00	2.38
14	-0.40	3.00	2.38
15	-0.40	3.00	2.38

16	0.00	3.62	3.00
17	0.00	3.90	3.00
18	0.00	3.90	3.00
19	1.62	4.30	4.62
20	1.76	4.30	4.63
21	1.76	4.30	4.62
22	2.00	4.86	4.80
23	2.17	4.86	5.12
24	2.17	5.00	5.12
25	2.75	5.00	5.70
26	2.75	5.00	5.70
27	2.93	5.00	5.91

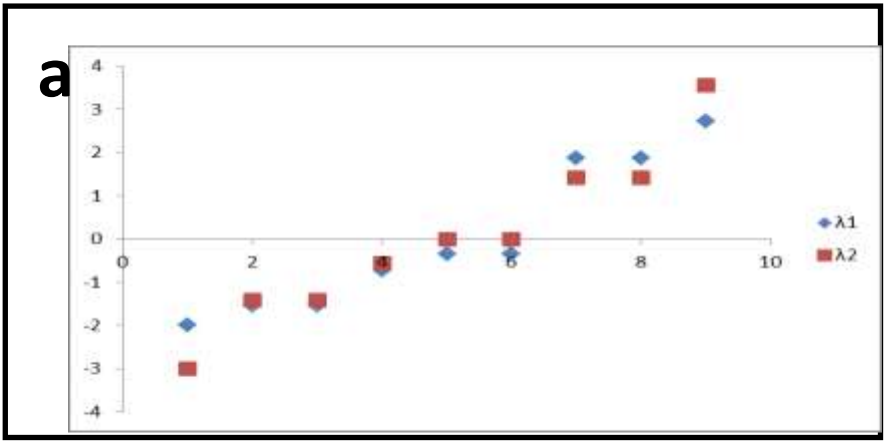
Table 3. Spectrum of Sierpinski Eulerian graph 2^{nd} iteration on 3 vertices (i.e. $S(2,3)$) round off to 2 decimal places

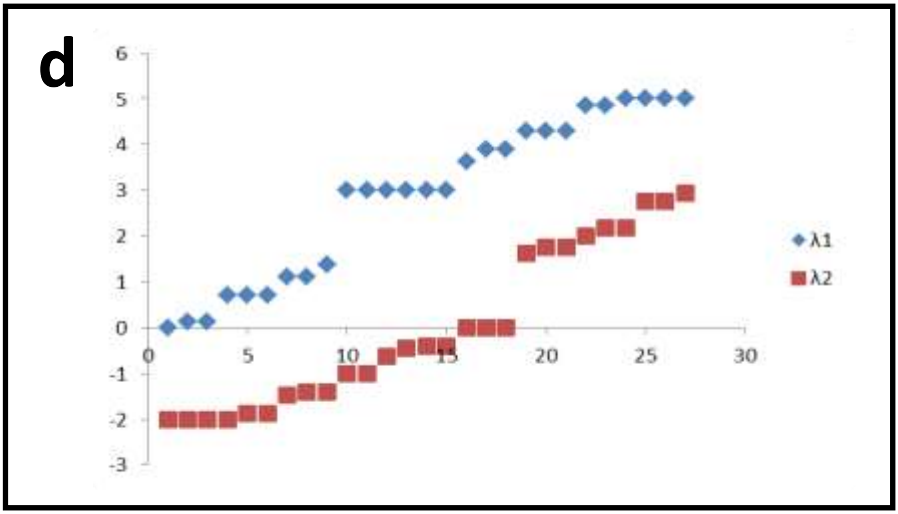
Sr. No.	Adjacency matrix	Laplacian matrix	Signless Laplacian Matrix
1	-3.00	0.00	1
2	-1.41	1.27	1.27
3	-1.41	1.27	1.27
4	-0.56	3.00	1.63
5	0.00	4.00	4.00
6	0.00	4.00	4.00
7	1.41	4.73	4.73
8	1.41	4.73	4.73
9	3.56	7	4.73

Table 4. Spectrum of Sierpinski Eulerian graph 3^{rd} iteration on 3 vertices (i.e. $S(3,3)$) round off to 2 decimal places

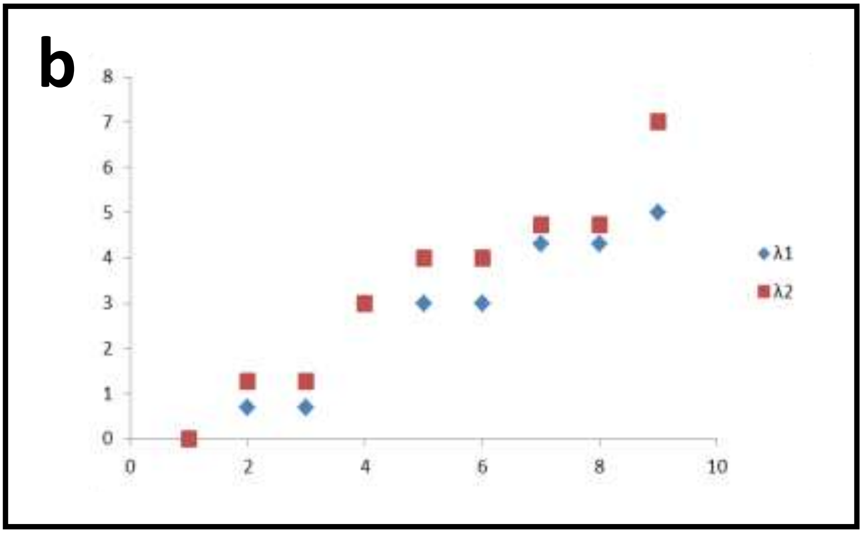
Sr. No.	Adjacency matrix	Laplacian matrix	Signless Laplacian Matrix
1	3.90	0	7.87
2	3.44	0.33	7.37
3	-2.84	1.29	5.68
4	3.49	6.83	7.42
5	1.93	0.30	4.82
6	-1.60	1.71	3.89

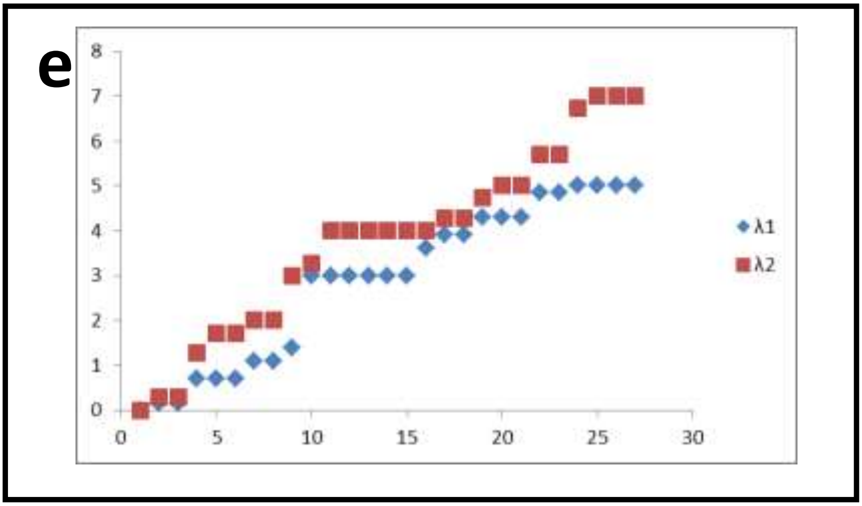
7	-1.18	5.58	2.43
8	-1.18	3.05	1.14
9	0.42	4.53	1.43
10	-0.18	4.53	1.36
11	1.29	4.14	1.26
12	-2.73	6.73	1.35
13	-1.78	1.72	2.40
14	1.29	5.72	4.83
15	0.73	3.27	4.73
16	2.00	4.28	6.00
17	2.00	7.00	6.00
18	-3.00	7.00	1.00
19	-3.00	2.00	1.00
20	-1.00	2.00	3.00
21	-1.00	4.00	3.00
22	-1.00	4.00	4.00
23	0.00	5.00	4.00
24	0.00	5.00	4.00
25	0.00	4.00	4.00
26	0.00	4.00	4.00
27	0.00	4.00	4.00



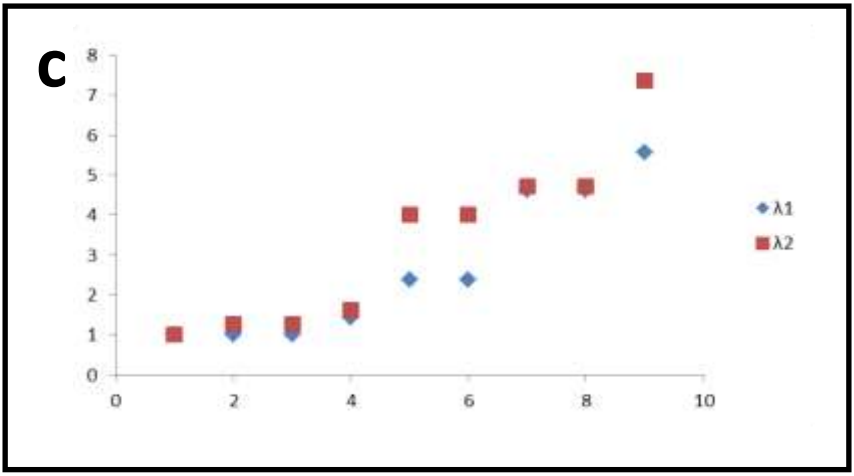


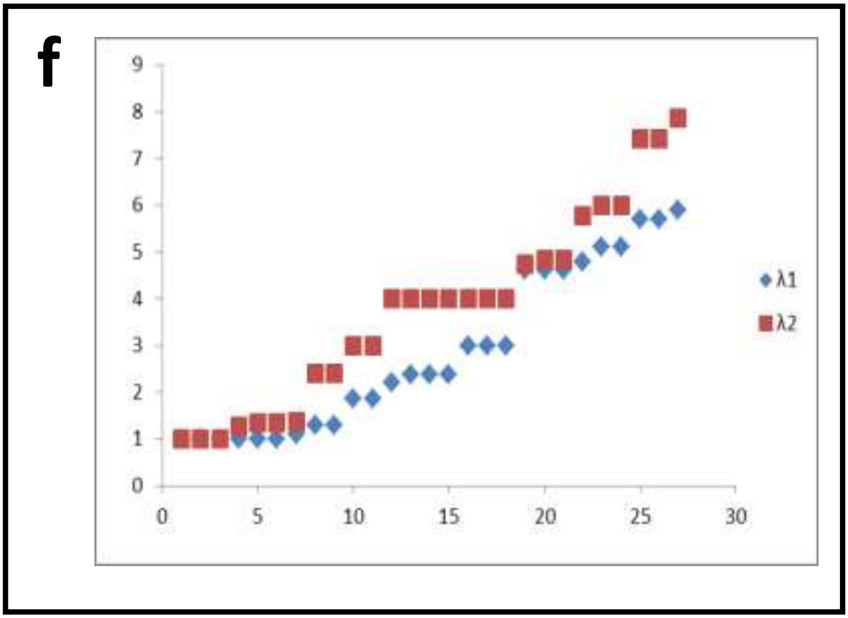
Results: for adjacency matrix of Sierpinski and Sierpinski Eulerian graph dataset: (a) eigenvalues λ1 of adjacency matrix of Sierpinski Graph and (d) eigenvalues λ2 of adjacency matrix of Sierpinski Eulerian graph.





Results: for Laplacian matrix of Sierpinski and Sierpinski Eulerian graph dataset. (b) eigenvalues λ1 of Laplacian matrix of Sierpinski Graph and (e) eigenvalues λ2 of Laplacian matrix of Sierpinski Eulerian graph.





Results for Signless Laplacian matrix of Sierpinski and Sierpinski Eulerian graph dataset:: (c) eigenvalues λ1 of Signless Laplacian matrix of Sierpinski Graph and (f) eigenvalues λ2 of Signless Laplacian matrix of Sierpinski Eulerian graph.

The specific manipulation of graphs i.e. addition of connected components which allows us to order the spectra of the graphs observed in the above graphs. As above we see that we can obtain Sierpinski Eulerian graph from Sierpinski graph by adding an edges between pair of odd degree vertices and that we can obtain Sierpinski Eulerian graph by adding the connected components. Having in view the above data, in applications the Signless Laplacian-spectrum and Laplacian-spectrum is used to encode graphs rather than Adjacency-spectrum, i.e. the Signless Laplacian -spectrum, Laplacian-spectrum has more representational power than the Adjacency-spectrum, in terms of resulting of above graphs. The above data show that it is even better to use signless Laplacian eigenvalues and Laplacian eigenvalues since they have stronger characterization properties. Recently, a spectral theory of graphs based on the signless Laplacian has been developed[3-5]. There are many results in the mathematical literature on spectral characterizations of particular classes of graphs. For example, complete graphs, paths and circuits are characterized by their A-spectra up to an isomorphism. There are also characterizations with some exceptional cospectral maps and graphs. However, these results hardly could be applied to graphs which appear in applications to computer science, science, mathematics and other aspects of NP problems. This is a strong basis for believing that almost all graphs are determined by their spectra when "*n*" tends towards the infinity, as conjectured[6,7].

6. Conclusion

We have compared the spectra of the three graph representation matrices: the adjacency matrix A, the Laplacian matrices and the Signless Laplacian matrices and found differences in the spectra corresponding to generalised Sierpinski graphs. As a result of this work, we hope to have increased awareness about the importance of the choice of representation matrix for graph signal processing applications and other fields of communications[1,8].

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