ISSN 0974 - 9373

Vol. 14 No.3 (2010) **Journal of International Academy of Physical Sciences** pp. 295-303

Rotational MHD Flows in two annular regions of a clear Viscous Fluid and Fully Saturated Porous Medium*

Shikha
Mathematics Department, Bareilly College, Bareilly
Email: shikhasaxena.1984@yahoo.in

P. N. Tandon
Formerly with H. B. T. I. Kanpur & University Brunei Darussalam
Email: gopal1960@yahoo.co.uk

(Received February 20, 2010)

Abstract: The effects of constant applied magnetic field have been brought out and discussed in viscous fluid flows through a circular annular porous region bounded by an inner rigid permeable circular cylinder and an outer cylinder under different physical situations. The porous medium is extended to infinite extent and bounded by the outer cylinder. Computational results have been discussed for two cases: whether the inner cylinder is rotating with a constant velocity or at rest and for similar conditions for bounded outer cylinder. A clear viscous fluid flows through the inner cylinder whereas the porous medium is fully saturated with same fluid. The most general result has been observed in all cases that applied magnetic field retards the flow. Wall shearing stress has been calculated on the outer cylinder which is rotating at constant angular velocity while the inner cylinder kept stationary. For a fix value of slip parameter σ at the interface, the wall shearing stress on the outer cylinder decreases with increasing value of the magnetic parameter Mo and for a fixed value of the parameter Mo, the wall shearing stress decreases with increasing value of slip parameter at the interface.

Key Words: MHD Flows, Applied magnetic field, Rotational velocity, Effects of magnetic field, Slip velocity, Brinkman's equation, Porous media in contact with clear fluid

1. Introduction

The flow of a viscous fluid over and through a porous medium occurs in nature as well as in many engineering and medical problems. The flow of blood through lungs and capillaries are also examples of flow through porous media. The porous medium is in fact a non-homogeneous continua. One can approach a hypothetical homogeneous fluid under the action of averaged external forces[1,2]. The subject of Magnetohydrodynamic has also

*Presented at CONIAPS XI, University of Allahabad, Feb. 20-22, 2010.

attracted the attention of many research workers in view of its applications to Astrophysics, Geophysics and engineering [3-5]. As an external force, it retards the blood flow[6]. Rotational MHD Flows through porous materials with externally applied magnetic fields have recently been studied by various authors[7-10] Srivastava et.al[11] have developed a model for flow created by a rotation of circular cylinder with a constant angular velocity.

In this paper, we have discussed the magnetohydrodynamic flows through an fully saturated porous annulus $(\lambda_1 a \le r \le \lambda_2 a)$ bounded from inside by another annulus $(a \le r \le \lambda_1 a)$. The inner cylinder $(r = a)$ is rigid impervious whereas the outer one (interface) is rigid permeable and a clear viscous fluid flows through this annulus and same fluid flows through the porous medium also .following governing equations are developed by using Brinkmen's equations for porous media[12]. The rigid outer cylinder $(r = \lambda_2 a)$ is bounded or extended to infinity. Flows are assumed to be created by the rotation of inner or outer cylinders. The computational results for various cases are obtained for different physical situations subject to applied magnetic fields in radial direction.

2. Formulations of the Problem

Referring to the figures -1a and 1b, the flow is generated by rotating inner or outer rigid cylinders with a constant angular velocity Ω_0. Fluid flowing in zone-2 where the fluid flows through the pores of the porous material and both the regions are subject to a constant applied magnetic field in transverse direction. The only non-vanishing circumferential velocity is denoted by v*. In zone-1, the flow is governed by the Navier-stokes equation , which gives the following differential equation for v_1 in non-dimensional form :

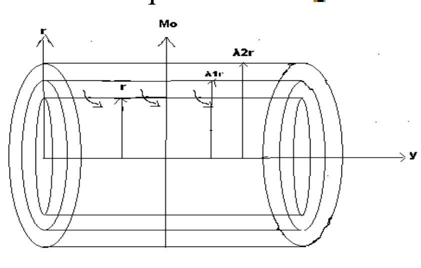


Fig-1(a)

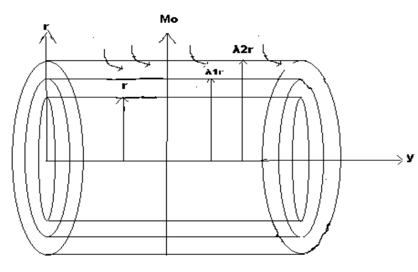


Fig-1(b)

$$y^2 \frac{d^2 v_1}{dy^2} + y \frac{dv_1}{dy} - (1 + M_0^2 y^2) v_1 = 0 \tag{2.1}$$

where $v_1 = \frac{v_1^*}{a\Omega}$, $y = \frac{r}{a}$ and $M_0^2 = \sigma'^2 \mu_p^2 H_0^2 y^2$

The solution of this equation is

(2.2) $$v_1 = AI_1(M_0 y) + Bk_1(M_0 y)$$

In zone-2, the flow is governed by the Brinkman equation[12], which gives the following differential equation in non-dimensional form for v_2 :

(2.3) $$y^2 \frac{d^2 v_2}{dy^2} + y \frac{dv_2}{dy} - [Q^2 y^2 + 1] v_2 = 0$$

where $v_2 = \frac{v_2^*}{a\Omega}$, $\varpi = \frac{a}{\sqrt{k}}$ and $Q^2 = \sigma^2 + M_0^2$

The solution of the equation is

(2.4) $$v_2 = CI_1(Qy) + Dk_1(Qy)$$

The matching conditions at the interface $y = \lambda_1$ suggested by Ochoa-Tapia and Whitaker[13] in the present notation, this can be written as:

(2.5a) $$v_1(\lambda_1) = v_2(\lambda_1) \qquad at\ y = \lambda_1$$

(25b) $$\frac{\partial v_1}{\partial x} - \frac{\partial v_2}{\partial x} = \alpha\sigma v_1 \qquad at\ y = \lambda_1$$

where α is a dimensionless constant depending on the surface of the porous material. The boundary conditions of the problem are

(2.6a) $$v_1 = 1 \qquad at\ y = 1$$

(2.6b) $$v_2 = 0 \qquad at\ y = \lambda_2$$

The solutions of the equations satisfying the boundary and matching conditions at the interface is given by

(2.7) $$v_1 = A\left[\frac{I_1(M_0 y)K_1(M_0) - I_1(M_0)K_1(M_0 y)}{K_1(M_0)}\right] + \frac{K_1(M_0 y)}{K_1(M_0)}$$

and

(2.8) $$v_2 = DK_1(QY),$$

where

(2.9a) $$A = \frac{Qk_0(Q\lambda_1)k_1(M_0\lambda_1) - k_1(Q\lambda_1)[M_0 k_0(M_0\lambda_1) + \alpha\sigma k_1(M_0\lambda_1)]}{\begin{array}{c}[I_1(M_0)k_1(M_0\lambda_1) - I_1(M_0\lambda_1)k_1(M_0)] * \\ [Qk_0(Q\lambda_1) - \alpha\sigma k_1(Q\lambda_1)] * \\ -M_0 k_1(Q\lambda_1)[I_0(M_0\lambda_1)k_1(M_0) + I_1(M_0)k_0(M_0\lambda_1)]\end{array}}$$

and

(2.9b) $$D = \frac{-M_0[I_1(M_0\lambda_1)k_0(M_0\lambda_1)+I_0(M_0\lambda_1)k_1(M_0\lambda_1)]}{\begin{array}{c}[I_1(M_0)k_1(M_0\lambda_1)-I_1(M_0\lambda_1)k_1(M_0)]* \\ [Qk_0(Q\lambda_1)-\alpha\sigma k_1(Q\lambda_1)]-M_0k_1(Q\lambda_1)* \\ [I_0(M_0\lambda_1)k_1(M_0)+I_1(M_0)k_0(M_0\lambda_1)]\end{array}}$$

Now we present the results for different cases

2.1. Bounded Porous Medium

(2.1) case-1 Inner cylinder is rotating with constant angular velocity and the outermost is stationary

When inner cylinder is rotating and the outer most bounding impervious cylinder $r = \lambda_2$ is stationary. The boundary conditions of the problem are

(2.1.1) $v_1 = 1$ at $y = \lambda_1$,

(2..1.2) $v_2 = 0$ $at\ \ y = \lambda_2$,

The solutions of the equations satisfying the boundary conditions are

(2.1.3) $$v_1 = \frac{A[I_1(M_0y)k_1(M_0) - I_1(M_0)k_1(M_0y)] + k_1(M_0y)}{k_1(M_0)},$$

and

(2.1.4) $$v_2 = F\left[\frac{k_1(QY)I_1(Q\lambda_2) - k_1(Q\lambda_2)I_1(Qy)}{I_1(Q\lambda_2)}\right].$$

where A and F are constants determined using the matching conditions at the interface and boundary conditions as given above

(2.1. 6) $$A = \frac{\begin{array}{c}[M_0k_0(M_0\lambda_1) + \alpha\sigma k_1(M_0\lambda_1)] \\ [I_1(Q\lambda_1)k_1(Q\lambda_2) - I_1(Q\lambda_2)k_1(Q\lambda_1) + \\ Qk_1(M_0\lambda_1)[I_0(Q\lambda_1)k_1(Q\lambda_2) + I_1(Q\lambda_2)k_0(Q\lambda_1)]\end{array}}{\begin{array}{c}Q[I_0(Q\lambda_1)k_1(Q\lambda_2) + I_1(Q\lambda_2)k_0(Q\lambda_1)]* \\ [I_1(M_0)k_1(M_0\lambda_1) - I_1(M_0\lambda_1)k_1(M_0)]* \\ +[I_1(Q\lambda_1)k_1(Q\lambda_2) - I_1(Q\lambda_2)k_1(Q\lambda_1)]* \\ [M_0\{I_0(M_0\lambda_1)k_1(M_0) + I_1(M_0)k_0(M_0\lambda_1)\} - \\ \alpha\sigma\{I_1(M_0\lambda_1)k_1(M_0) + I_1(M_0)k_1(M_0\lambda_1)\}]\end{array}}$$

and

(2.1.6) $$F = \frac{M_0I_1(Q\lambda_2)[I_0(M_0\lambda_1)k_1(M_0\lambda_1) + I_1(M_0\lambda_1)k_0(M_0\lambda_1)]}{\begin{array}{c}Q[I_0(Q\lambda_1)k_1(Q\lambda_2) + I_1(Q\lambda_2)k_0(Q\lambda_1)]* \\ [I_1(M_0)k_1(M_0\lambda_1) - I_1(M_0\lambda_1)k_1(M_0)]* \\ +[I_1(Q\lambda_1)k_1(Q\lambda_2) - I_1(Q\lambda_2)k_1(Q\lambda_1)]* \\ [M_0\{I_0(M_0\lambda_1)k_1(M_0) + I_1(M_0)k_0(M_0\lambda_1)\} - \\ \alpha\sigma\{I_1(M_0\lambda_1)k_1(M_0) + I_1(M_0)k_1(M_0\lambda_1)\}]\end{array}}.$$

The results for this case are presented in figures 2 to 5.

2.2. Case-2 When the porous medium is extended to infinity

These results may be obtained from above by taking $\lambda_2 \longrightarrow \propto$ from above equations in case 1. Results are presented in figures 5 to 7.

(2.3.1) Case -3: Inner Cylinder is stationary and outer one rotating with constant angular velocity

We consider the case when an impervious solid infinite circular cylinder $r=\lambda_1$ is surrounded by a porous medium $\lambda_1 \le r \le \lambda_2$ fills the space which is zone-2. The viscous fluid fills the space $1 \le r \le \lambda_1$ which is zone 1. We assume that the outer cylinder $r=\lambda_2$ is rotating and the inner one is stationary. A constant applied magnetic field in transverse direction is applied to both the zones. In this case, the boundary conditions of the problem are

(2.3.1) $v_2 = 1$ at $y=\lambda_2$,

((2.3.2) $v_1 = 0$ at $y=\lambda_1$,

The matching conditions at the interface remain the same and the slip coefficient α can be positive or negative. The velocities v_1 and v_2 satisfying the boundary conditions are given by

(2.3.3)
$$v_1 = G\left[\frac{I_1(Qy)k_1(Q) - I_1(Q)k_1(Qy)}{k_1(Q)}\right]$$

(2.3.4)
$$v_2 = \frac{\lambda_2 I_1(M_0 y) - S[I_1(M_0 y)k_1(M_0\lambda_2) - I_1(M_0\lambda_2)k_1(M_0 y)]}{I_1(M_0\lambda_2)}$$

where

(2.3.5)
$$G = \frac{M_0\lambda_2 k_1(Q)[I_1(M_0\lambda_1)k_0(M_0\lambda_1) + I_0(M_0\lambda_1)k_1(M_0\lambda_1)]}{\begin{array}{c} M_0[I_0(M_0\lambda_1)k_1(M_0\lambda_2) + I_1(M_0\lambda_2)k_0(M_0\lambda_1)] * \\ [I_1(Q\lambda_1)k_1(Q) - I_1(Q)k_1(Q\lambda_1)] + \\ [I_1(M_0\lambda_1)k_1(M_0\lambda_2) - I_1(M_0\lambda_2)k_1(M_0\lambda_1)] * \\ [\alpha\sigma\{I_1(Q\lambda_1)k_1(Q) - I_1(Q)k_1(Q\lambda_1)\} - Q\{I_0(Q\lambda_1)k_1(Q) + k_0(Q\lambda_1)I_1(Q)\}] \end{array}}$$

and

(2.3.6)
$$S = \frac{\begin{array}{c} \lambda_2[M_0 I_0(M_0\lambda_1) + \alpha\sigma I_1(M_0\lambda_1)] * \\ [I_1(Q\lambda_1)k_1(Q) - I_1(Q)k_1(Q\lambda_1)] - Q\lambda_2 I_1(M_0\lambda_1) * \\ [I_0(Q\lambda_1)k_1(Q) + I_1(Q)k_0(Q\lambda_1)] \end{array}}{\begin{array}{c} M_0[I_0(M_0\lambda_1)k_1(M_0\lambda_2) + I_1(M_0\lambda_2)k_0(M_0\lambda_1)] * \\ [I_1(Q\lambda_1)k_1(Q) - I_1(Q)k_1(Q\lambda_1)] + \\ [I_1(M_0\lambda_1)k_1(M_0\lambda_2) - I_1(M_0\lambda_2)k_1(M_0\lambda_1)] * \\ [\alpha\sigma\{I_1(Q\lambda_1)k_1(Q) - I_1(Q)k_1(Q\lambda_1)\} - Q\{I_0(Q\lambda_1)k_1(Q) + k_0(Q\lambda_1)I_1(Q)\}] \end{array}}$$

In this case, the results are presented through figures 8 to 10.

2. Results and Discussions

In figures 2 -5, the graphs of the velocities against the distance from the inner cylinder $(r = 1)$ have been plotted for various values of magnetic field Mo and slip parameter when inner cylinder rotates and outer is stationary.

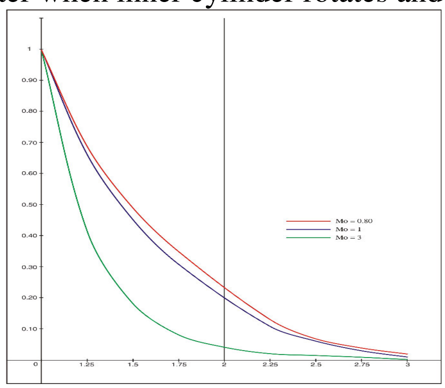
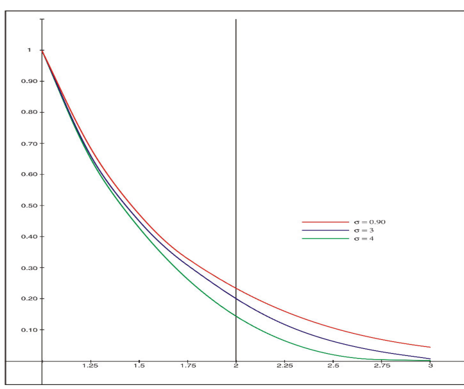
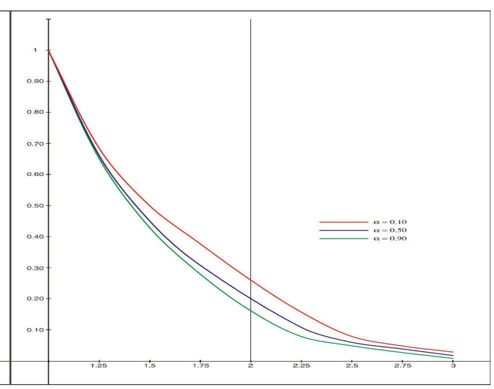


Fig-2 variation of velocities with magnetic field

From figure -2, it is seen that in both the regions, the velocities decrease with increases of the magnetic field Mo.





..

Fig-3Variation of velocities with slip parameter   Fig-4Variation of velocity profile with α

The velocities for both the cases are shown in figures 3 and 4 respectively for various values of σ and α. These figures show that the velocities decrease in both regions with the increase of the slip parameter σ as well as with the increase of the α.

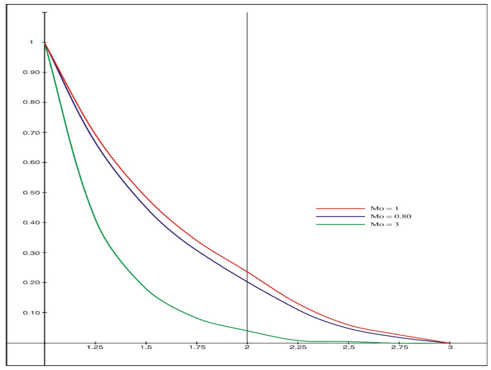


Fig-5Variation of velocities with magnetic field

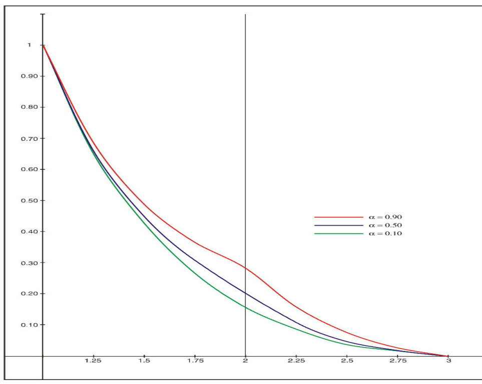


Fig-6Variation of velocity profile with α

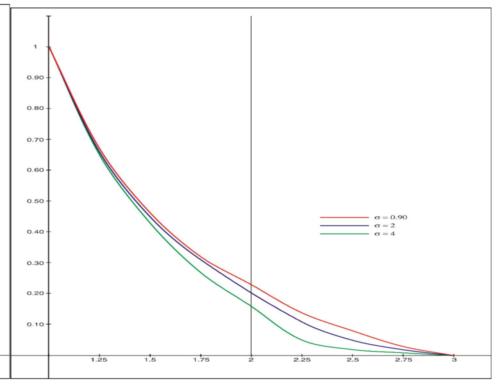


Fig-7Variation of velocities with slip parameter

In figures 5-7, the results are presented to the case when porous region is extended to infinity $((\lambda_2 \rightarrow \propto)$. The effects of applied magnetic field are presented in figure-5. The effects of slip parameter σ and α respectively on velocity profile of fluid in both regions decrease sharply and in porous region to zero for a particular value of Mo.

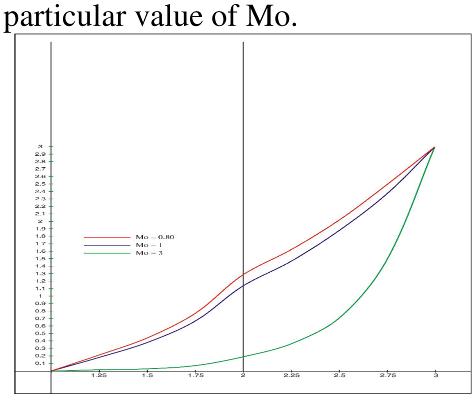


Fig-8Variation of velocities with magnetic field

Figures 8 - 10 show the graphs of the velocities against the distance from the inner cylinder for the case when the outer cylinder rotates and the inner one $(r = 1)$ is stationary. In this case also, the velocity of the fluid, in both the regions, decreases with the increase of the magnetic field parameter Mo. The velocity is maximum in the porous medium due to the prescribed boundary condition.

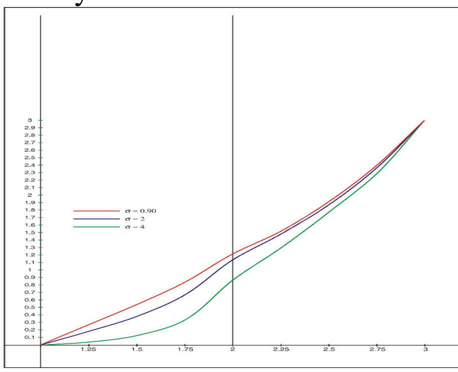


Fig-9Variation of velocities with slip parameter

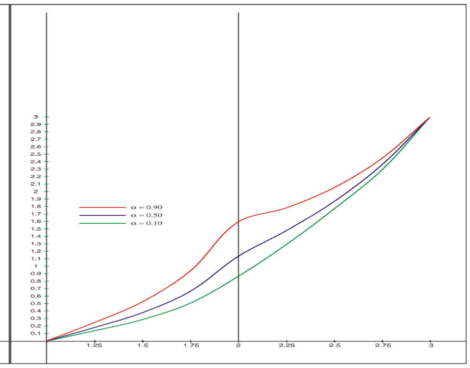


Fig-10Varitation of velocities with slip parameter

The velocities for various values of α and σ for fixed value of Mo. are shown to decrease in the figures 9 and 10. These figures show that the velocity at the interface depicts a maxima.

3. References

1. H. Darcy, *The Flow of Fluids Through Porous Media*, Mc Graw Hill Book.Co., New York, 1937.
2. G. Ahmadi and R. Manvi, Equation of Motion for Viscous fluid through a rigid porous medium, *Ind. J. tech.*, **9**(1971) 491-496.
3. K. R. Cramer and S. I. Pai, *Magnetofluiddynamics for engineers and applied Physicists*, McGraw Hill, New York, 1973.
4. B. C.Chandrasekhara, R. Kantha and N. Rudraiah, Effect of Slip on Porous Walled Squeeze Films in the Presence of a Transverse Magnetic Field, *Appl. Sci. Res.*, **34**(1978) 393-411.
5. A. C.Srivastava and B. Barman, The Flow of a Non-Newtonian Fluid Confined Between an Impervious Rotating Disk and a Porous Medium, *Proc. Nat. Acad. Sic. India*, **67(A)2 (**1997) 165-174.
6. V. K. Sud, P. K. Suri, and R. K. Mishra, Effects of magnetic field on oscillating blood flowing arteries, Studia Biophysica. , **46** (1974)163.
7. M. P. Perry, *A Survey of Ferromagnetic Liquid Applications*, Proceedings of the International Advanced Course and Workshop on Thermodynamics of Magnetic Fluids, (1983) 555-556.
8. V. M. Soundalgekar and I. POP, On hydromagnetic flow in a rotating fluid past an infinite porous wall, *ZAMM*, **53**(1993)718-19.

9. B. K. Sharma, A. K .Jha and R. C. Chaudhary, MHD Forced Flow of a Conducting Viscous Fluid Through a Porous Medium Induced by an Imprevious Rotating Disk, *Rom. Journ. Phys.*, **52**(1-2) (2007) 73-84.
10. P. K. Sharma and Sabbir Khan, MHD Flow in porous media by torsionally oscilating disk, *Computers and Fluids*, **20**(2010) 1256 -1260.
11. A. C.Srivastava and P. K. Srivastava, Flow due to rotation of a cylinder in a Viscous Fluid Bounded by a Porous Medium, *Ganita*, **57**(2006) 1-10.
12. H. C. Brinkman, Calculation of Viscous Forces Exerted by a Flow in Fluids on a Dense Swarm of Particles, *Appl. Sci. Res.*, **A1** (1947) 27-36.
13. J. A. Ochoa-Tapia, and S. Whitaker, Momentum Transfer at the Boundary Between a Porous Medium and a Homogeneous Fluid-1, Theoretical Development, *Int. J. Heat Mass Transfer*, **38**(1995) 2635-2646.