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A New Goal Programming Approach for Multi-Objective Solid Transportation Problem with Interval-Valued Intuitionistic Fuzzy Logic

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Abstract: Many researchers considered fuzzy and intuitionistic fuzzy parameters in the transportation problem, but we deal with interval-valued intuitionistic fuzzy parameters, which is another type of uncertainty that covers the favourable as well as unfavourable cases. In a multi-objective transportation problem, it is difficult to find the optimal solution for all objectives simultaneously. So, we have used the Intuitionistic Fuzzy Goal Programming (IFGP) with deviational function $d_k = (1 - w_k)$ (where w_k is fixed numerical weight for d_k which decided the importance of highest acceptance level of k^{th} objective relative to other objectives) to find the compromise optimal solution. In this paper, we extend the model proposed by Nomani M.A. et al. in 2016 for Multi-Objective Solid Transportation Problem (MOSTP) with interval-valued intuitionistic fuzzy cost. We applied this approach to find the solution of the Solid Transportation Problem (STP) with satisfying all the constraints. To find the compromise optimal solution for all objectives simultaneously, we apply a new IFGP approach. The main focus of the proposed approach is to minimize the all objectives simultaneously and to obtain the solution nearly closed to the lower bounds of objectives. A numerical example is being carried out in favour of the proposed algorithm.
Keywords: Multi-Objective Transportation Problem (MOTP), Solid

Transportation Problem (STP), Interval-Valued Intuitionistic Fuzzy Numbers (IVIFN), Goal Programming (GP), Intuitionistic Fuzzy Goal Programming(IFGP).
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1. Introduction

Transportation problem is one of the linear programming problem, but it is a special type problem due to its structure. So it can not be solved by the usual simplex method. The main objective of transportation problem is to minimize the transport cost by satisfiying the demand of all costumers. Hitchcock[1] in 1941 introduced the concept of Classical Transportation Problem. Transportation cost depends on several conditions. In real life situation the parameters of the problem consists of various type of uncertainties. To deal with such uncertainties, Zadeh[2] in 1965 introduced the concept of fuzzy logic that helps to represent such type of imprecise data and Bellman[3] in 1970 used this theory in decision making problems. In current situation, if the transportation problem contains three kinds of constraints such as demand, supply and conveyance then these types of transportation problems are known as a Solid Transportation Problem (STP) introduced by Haley[4] in 1962. Classical transportation problem is a special type of Solid Transportation Problem when the number of conveyance in solid transportation problem is only one.

Transport cost is not only the objective of transportation problem but it also compromises to deal with more than one objective. There are more than one objectives of transportation problem which are conflicting with each other. Such type of transportation problems possessing this property are known as Multi-Objective Transportation Problems (MOTP). To find an optimal solution for all objectives of such type of problem is tedious task. Zimmermann[5] introduced fuzzy programming and linear programming with multi objectives and also[6] used fuzzy set theory in mathematical programming. Bit[7] et.al. gave a method to solve the multi-objective transportation problem with interval parameters. Then in 1961, Charnes and Cooper[8] introduced a powerful approach to deal with multi-objective known as “Goal Programming”, which is very helpful to find acompromise optimal solution for MOTP. It is also known as the generalization of linear programming problem dealing with multi-objectives. Narasimhan[9] introduced the Goal Programming approach in uncertain or fuzzy environment. Radhakrishnan and Anukokila[10] introduced the Fractional Goal Programming for solving Fuzzy Solid Transportation Problem with interval cost. Pal B.B. & Moitra[11] used Fuzzy Goal Programming (FGP) for Long Range Production Planning in Agricultural Systems by using fixed numerical weight $w_k = \dfrac{1}{U_k - L_k}$. El-Wahed and Lee[12] solved the MOTP by using Interactive Fuzzy Goal Programming. Also, Zangiabadi and Maleki[13] used FGP for solving MOTP. There are many approaches for solving multi-objective problems with fuzzy and intuitionistic fuzzy parameters, but Atanassov[14] and Atanasson and Gargov[15] introduced interval-valued intuitionistic fuzzy set in which membership and non-membership degree of any element is in interval form. Aggarwal and Gupta[16] used a new ranking method based on signed distance for solving Intuitionistic Fuzzy Solid Transportation Problem. In this research article we extended the model proposed by Nomani M.A. et. al.[17] for Multi-Objective Solid Transportation Problem (MOSTP) with interval-valued intuitionistic fuzzy cost.

The research paper is divided into eight sections. In the first section of the research paper deals with the introductory part of the problem and in the second section, we defined some basic concepts which are being used to our problem. The third section of the paper is related to deffuzzification of the Interval Valued Intuitionistic Fuzzy Numbers known as the the accuracy function for IVIFN. Fourth section of the paper we formulated the problem mathematically. We proposed our model in section fifth and the algorithm for getting the compromise solution of the multi-objective problems in the sixth section of the research paper. In the seventh section of the research paper a numerical is being carried out in support of our proposed model and algorithm to show the efficiency of our proposed model. In the last section of our research paper we gave the conclusion and interpretation of the results.

2. Preliminaries

2.1. Fuzzy Set: Let X be any universal set and $\bar{A}$ be any subset of X then $\bar{A}$ is fuzzy set if it is denoted by $\bar{A} = \{x, \mu_{\bar{A}}(x)\}$, where $x \in X, \mu_{\bar{A}}(x) : X \to [0,1]$ and $\mu_{\bar{A}}(x)$ is membership grade of x in $\bar{A}$.

2.2. Fuzzy Number: A fuzzy set $\bar{A}$ on the real line is called a fuzzy number if it satisfies the following conditions;

(a) $\bar{A}$ is normal i.e. there exist any $x \in \bar{A}$ such that $\mu_{\bar{A}}(x) = 1$.

(b) $\bar{A}$ is convex set i.e.

$$\mu_{\bar{A}}(\lambda x_1+(1-\lambda)x_2)\geq \min(\mu_{\bar{A}}(x_1),\mu_{\bar{A}}(x_2)),\ \forall x_1,x_2\in \bar{A},\lambda\in[0,1].$$

(c) Membership function is piecewise continuous.

2.3. Intuitionistic Fuzzy Set: Let X be any universal set and $\bar{A}$ be any subset of X then $\bar{A}$ is intuitionistic fuzzy set if it is denoted by $\bar{A}=\{x,\mu_{\bar{A}}(x),\nu_{\bar{A}}(x)\}$, where $x\in X$, $\mu_{\bar{A}}(x):X\to[0,1]$, $\nu_{\bar{A}}(x):X\to[0,1]$ $\mu_{\bar{A}}(x)\,\&\,\nu_{\bar{A}}(x)$ denotes membership grade & non-membership grade of x in $\bar{A}$ with hesitation part, $\pi_{\bar{A}}(x)=1-(\mu_{\bar{A}}(x)+\nu_{\bar{A}}(x))$, where $0\leq\pi_{\bar{A}}(x)\leq 1$.

2.4. Intuitionistic Fuzzy Number: An Intuitionistic fuzzy set $\bar{A}$ on the real line is called an intuitionistic fuzzy number if it satisfies following conditions;

(a) $\bar{A}$ is normal i.e. there exist any $x\in\bar{A}$ such that $\mu_{\bar{A}}(x)=1$ or $\nu_{\bar{A}}(x)=0$.

(b) $\bar{A}$ is convex set for membership function

$$\mu_{\bar{A}}(\lambda x_1+(1-\lambda)x_2)\geq\min(\mu_{\bar{A}}(x_1),\mu_{\bar{A}}(x_2)),\ \forall x_1,x_2\in\bar{A},\ \lambda\in[0,1].$$

(c) $\bar{A}$ is concave set for non-membership function

$$\nu_{\bar{A}}(\lambda x_1+(1-\lambda)x_2)\leq\max(\nu_{\bar{A}}(x_1),\nu_{\bar{A}}(x_2)),\ \forall x_1,x_2\in\bar{A},\lambda\in[0,1].$$

(d) Membership and non-membership functions are piece-wise continuous.

2.5. Interval-Valued Intuitionistic Fuzzy Number (IVIFN): An interval-valued intuitionistic fuzzy number $\bar{A}$ is denoted by

$$\bar{A}=\{(a_1^L,a_3^U,a_2,a_4^L,a_5^U),(b_1^L,b_3^U,a_2,b_4^L,b_5^U)\}$$

with the lower and upper membership and non-membership is as follows

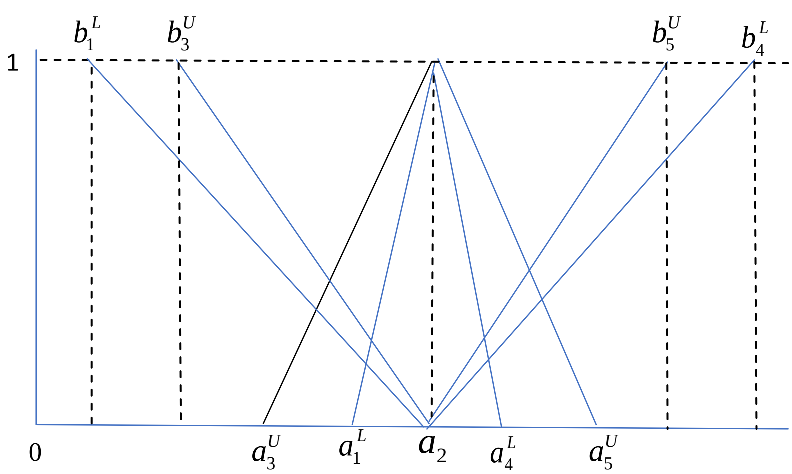

Figure 1.

Mathematically, the membership function and non-memebrship function for lower as well as upper bound will be formulated as follows:

2.5.1 Lower membership function: Lower bound for membership function is defined as

$$\mu_{\bar{A}}^{L}(x)=\begin{cases}1, & x=a_2,\\ \dfrac{x-a_1^L}{a_2-a_1^L}, & a_1^L<x<a_2,\\ \dfrac{a_4^L-x}{a_4^L-a_2}, & a_2<x<a_4^L,\\ 0, & a_1^L\geq x, x\geq a_4^L.\end{cases}$$

2.5.2. Upper membership function: Upper bound for membership function is defined as

$$\mu_{\bar{A}}^{U}(x)=\begin{cases}1, & x=a_2,\\ \dfrac{x-a_3^U}{a_2-a_3^U}, & a_3^U<x<a_2,\\ \dfrac{a_5^U-x}{a_5^U-a_2}, & a_2<x<a_5^U,\\ 0, & a_3^U\geq x, x\geq a_5^U.\end{cases}$$

2.5.3 Lower non-membership function: Lower bound non-membership function is defined as

$$v_{\bar{A}}^{L}=\begin{cases}0, & x=a_2,\\ \dfrac{a_2-x}{a_2-b_1^L}, & b_1^L<x<a_2,\\ \dfrac{x-a_2}{b_4^L-a_2}, & a_2<x<b_4^L,\\ 1, & x\leq b_1^L, x\geq b_4^L.\end{cases}$$

2.5.4. Upper non-membership function: Upper bound for non-membership function is defined as

$$v_{\bar{A}}^{U}\left(x\right)=\begin{cases}0, & x=a_2,\\ \dfrac{a_2-x}{a_2-b_3^U}, & b_3^U<x<a_2,\\ \dfrac{x-a_2}{b_5^U-a_2}, & a_2<x<b_5^U,\\ 1, & x\le b_3^U, x\ge b_5^U,\end{cases}$$

3. Defuzzification (Accuracy) Function for Interval Valued Intuitionistic Fuzzy Number

Let $\bar{A}=\{x,(a_1^L,a_3^U,a_2,a_4^L,a_5^U),(b_1^L,b_3^U,a_2,b_4^L,b_5^U)\}$ be an IVIFN. Then these inte- rval valued intuotionistic fuzzy numbers will be converted into closed inte- rvals using $\alpha-cut$ method as follows:

$$u^L(\alpha)=[a_1^L+\alpha(a_2-a_1^L),a_4^L-\alpha(a_4^L-a_2)],$$

$$u^U(\alpha)=[a_3^U+\alpha(a_2-a_3^U),a_5^U-\alpha(a_5^U-a_2)],$$

$$v^L(\alpha)=[b_1^L-(1-\alpha)(a_2-b_1^L),a_2+(1-\alpha)(b_4^L-a_2)],$$

$$v^U(\alpha)=[b_3^U-(1-\alpha)(a_2-b_3^U),a_2+(1-\alpha)(b_5^U-a_2)],$$

where $0<\alpha<1$.

Then by using parametric form of IVIFN, Helipern[18] introduced the defuzzified(accuracy) value for IVIFN. Which is as follows:

$$R(\bar{A})=\frac{a_1^L+a_3^U+b_1^L+b_3^U+8*a_2+a_4^L+a_5^U+b_4^L+b_5^U}{16}$$

4. Mathematical Formulation of the Problem

4.1. Multi-Objective Transportation Problem (MOTP): Let there be m sources and n destination in the transportation problem and a_i be available quantity of items at i^{th} sources and b_j be the demand of j^{th}

destination. Let x_{ij} be the number of items which have to be transported from i^{th} source to j^{th} destination. Let c_{ij} be the cost per unit of the item which has to be transported from i^{th} source to j^{th} destination, where $i = 1,2.....m$ and $j = 1,2.....n$. The aim of the problem is to find the quantity of items x_{ij} at which the all objectives of problem are optimized.

Let there be K number of objectives in transportation problem, then the mathematical formulation of the problem is:

$$MinZ_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \text{ , where } k = 1,2....K \text{ .}$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \text{ , where } i = 1,2.....m, \ \sum_{i=1}^{m} x_{ij} = b_j \text{ . where } j = 1,2.....n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ where } i = 1,2.....m, \ j = 1,2.....n, \ x_{ij} \geq 0 \text{ .}$$

4.2. Multi-Objective Solid Transportation Problem (MOSTP): Let there be m sources, n destination and D no. of conveyance in the problem. Let a_{jd} be available quantity of items which transported to j^{th} destination by d^{th} conveyance, b_{id} be the demand of the product which transported from i^{th} source by d^{th} conveyance and e_{ij} be the quantity of product which transported from i^{th} source to j^{th} destination. Let x_{ijd} be the no. of products transport from i^{th} source to j^{th} destination by d^{th} conveyance and c_{ijd} be the cost of per unit of product which transport from i^{th} source to j^{th} destination d^{th} conveyance, where $i = 1,2.....m$, $j = 1,2.....n$ and $d = 1,2.....D$. Then the mathematical formulation of MOSTP:

$$MinZ_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{d=1}^{D} c_{ijd}^{k} x_{ijd} \text{ , where } k = 1,2....K \text{ .}$$

Subject to the constraints:

$$\sum_{i=1}^{m} x_{ijd} = a_{jd} \text{ , } \sum_{j=1}^{n} x_{ijd} = b_{id} \text{ , } \sum_{d=1}^{D} x_{ijd} = e_{ij}$$

where, $$\sum_{j=1}^{n} a_{jd} = \sum_{i=1}^{m} b_{id}\ , \quad \sum_{j=1}^{n} e_{ij} = \sum_{d=1}^{D} b_{id}\ , \quad \sum_{i=1}^{m} b_{ij} = \sum_{d=1}^{D} a_{jd}\ ,$$

$$\sum_{j=1}^{n}\sum_{d=1}^{D} a_{jd} = \sum_{d=1}^{D}\sum_{i=1}^{m} b_{di} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\ ,\ x_{ijd} \geq 0\ ,$$

where $i = 1, 2.....m$, $j = 1, 2.....n$ and $d = 1, 2.....D$.

4.3. Goal Programming Problem (GPP): Goal Programming was introduced by Charnes and Cooper in 1961. The aim of GP is to minimize the deviational of objectives Z_k from aspiration levels $\overline{Z}_k$ of the objectives which are to be decided by decision maker. When the given problem is of minimizing nature then the over achievement deviational variable d_k^+ will be minimized whenever problem is of maximizing nature then under achievement deviational variable d_k^- will be maximized, where the mathematical form of the over achievement deviational variable is of the form

$$d_k^+ = \frac{1}{2}\{(Z_k - \overline{Z}_k) + \left|Z_k - \overline{Z}_k\right|\},\ Z_k \geq \overline{Z}_k$$

and the mathematical form of the under achievement deviational variable is of the form

$$d_k^- = \frac{1}{2}\{(\overline{Z}_k - Z_k) + \left|\overline{Z}_k - Z_k\right|\}, \overline{Z}_k \geq Z_k\ .$$

The mathematical formulation of goal programming:

$$\text{Min } Z_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{d=1}^{p} c_{ijd}^k x_{ijd} - \overline{Z}_k \quad .$$

Subject to $$\sum_{i=1}^{m} x_{ijd} = a_{jd}\ ,\ \sum_{j=1}^{n} x_{ijd} = b_{id}\ ,\ \sum_{d=1}^{D} x_{ijd} = e_{ij}\ ,$$

where,

$$\sum_{j=1}^{n} a_{jd} = \sum_{i=1}^{m} b_{id}\ ,\ \sum_{j=1}^{n} e_{ij} = \sum_{d=1}^{D} b_{id}\ ,\ \sum_{i=1}^{m} b_{ij} = \sum_{d=1}^{D} a_{jd}\ ,$$

$$\sum_{j=1}^{n}\sum_{d=1}^{D} a_{jd} = \sum_{d=1}^{D}\sum_{i=1}^{m} b_{di} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} \ , \ x_{ijd} \geq 0 \ ,$$

where $i = 1, 2.....m$, $j = 1, 2.....n$ and $d = 1, 2.....D$.

Where Z_k and $\bar{Z}_k$ are k^{th} objective function and aspiration level of k^{th} objective function respectively. Now, the mathematical formulation is as follows:

$$\text{Min} \sum_{k=1}^{K} (d_k^{+} - d_k^{-})$$

Subject to $$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{d=1}^{p} c_{ijd}^{k} x_{ijd} - \bar{Z}_k = d_k^{+} - d_k^{-},$$

$$\sum_{i=1}^{m} x_{ijd} = a_{jd} \ , \ \sum_{j=1}^{n} x_{ijd} = b_{id} \ , \ \sum_{d=1}^{D} x_{ijd} = e_{ij} \ , d_k^{+} - d_k^{-} \geq 0, \ k = 1, 2...K$$

4.3.1. Fuzzy Goal Programming (FGP): Let L_k and U_k be lower and upper bounds of Z_k objective function and let $\mu_k(Z_k)$ be a membership function for Z_k objective function in minimizing problem. We consider $\mu_k(Z_k)$ function as linear function with the highest membership grade 1, which defines as;

$$\mu_k(Z_k(x)) = \begin{cases} 1, & \text{if } Z_k \leq L_k, \\ \dfrac{U_k - Z_k(x)}{U_k - L_k} & \text{if } L_k \leq Z_k(x) \leq U_k \ , \\ 0, & \text{if } Z_k \geq U_k. \end{cases}$$

By applying min-max approach, mathematical model of fuzzy goal programming;

$$\text{Min}\,\theta \ ,$$

Subject to

$$\mu_k(Z_k(x)) - d_k^{+} + d_k^{-} = 1$$

$$\sum_{i=1}^{m} x_{ijd} = a_{jd} \ , \ \sum_{j=1}^{n} x_{ijd} = b_{id} \ , \ \sum_{d=1}^{D} x_{ijd} = e_{ij}$$

$$\theta \geq d_k^-,\ k=1,2...K,\ d_k^+ d_k^- = 0,\ \theta \geq 0, \theta \leq 1,\ x_{ijd} \geq 0,$$

where $i=1,2.....m$, $j=1,2.....n$ and $d=1,2.....D$.
Now, if DM is not satisfied then choose new upper or lower bounds of the objectives and again apply same process.

5. Proposed Approach for the MOTP Model

For MOTP, there are several approaches to solve the problem and find the solution for all objectives simultaneously. In our proposed method, we used the new fuzzy goal programming approach with the deviational function $d_k = (1-w_k)$ in the place of deviational variables d_k^- and d_k^+ where $w_k = \dfrac{1}{U_k - L_k}$ is fixed numerical weight associated with d_k (under ach-evement deviational variable) which decided the importance of the highest acceptance level of k^{th} objective relative to other objectives. In the existing paper[13], the model is used to find the compromise optimal solution for Multi-Objective Transportation Problem (MOTP) with crisp data, we extend it for Multi-Objective Solid Transportation Problem (MOSTP) with Interval-Valued Intuitionistic Fuzzy Cost (IVIFC) and also take underachievement deviational variable d_k for membership function of each objective, used fixed $w_k = \dfrac{1}{U_k - L_k}$ associated to each d_k. By using the linear membership function with deviational function $d_k = (1-w_k)$, the membership goal with highest acceptance of membership grade 1 is represented as:

$$\mu_k(z_k(x)) + d_k(1-w_k) = 1,$$

We know that highest acceptance of membership goal is 1, there exist only underachievement deviational variable d_k. Because, if overachievement deviational variable exist that means the aspiration level of fuzzy goal is more than 1 which is not possible. So only under achievement deviational variable is required for achieving the highest aspired level for membership goal. Now, by applying the min-max approach of goal programming, the proposed model is as follows;

$$\text{Min } \theta$$

$$\mu_k(Z_k(x)) + d_k(1-w_k) = 1,\ \theta \geq d_k(1-w_k),\ k=1,2....K$$

$$\sum_{i=1}^{m} x_{ijd} = a_{jd}\ ,\ \sum_{j=1}^{n} x_{ijd} = b_{id}\ ,\ \sum_{d=1}^{D} x_{ijd} = e_{ij}\ ,$$

$$x_{ijd} \geq 0\,,\ d_k \geq 0\,,\ 0 \leq \theta \leq 1$$

where,

$$\mu_k(z_k(x)) = \begin{cases} 1, & \text{if } Z_k \leq L_k, \\ \dfrac{U_k - Z_k(x)}{U_k - L_k}, & \text{if } L_k \leq Z_k(x) \leq U_k, \\ 0, & \text{if } Z_k \geq U_k. \end{cases}$$

linear function for minimize problem.

6. Proposed Algorithm for MOSTP

Step-1: First, Consider a Multi-Objective Solid Transportation Problem (MOSTP) with Interval-Valued Intuitionistic Fuzzy Cost (IVIFC).
Step-2: Convert the problem into crisp form by using defuzzification (accuracy) function.
Step-3: Check whether the given problem is balanced otherwise convert into balanced problem by adding a dummy row or Column with zero cost.
Step-4: Now, solve the problem by Vogel's Approximation Method taken single objective at a time for each j respectively. Then, we get basic feasible solutions for all objectives of the problem.
Step-5: Now, apply the method[16] proposed to interval valued intuitionistic fuzzy cost to find the optimal solution for each objective. The method will be as follows:

(i) After find the solution by vogel's approximation method, for each $j=1$, check allocation of c_{i1d} exceeds b_{id} or not. If not exceeds then find the solution for each j by vogel's approximation method.

(ii) If exceeds by θ units, then making a loop starting with c_{i1d} and move horizontally and vertically with allocation filled up cells. Now, assign $+\theta$ and $-\theta$ sign alternatively at each corner started $-\theta$ at c_{i1d}. All cells with must be filled up with minimum θ allocation.

(iii) Now, if any entry at c_{i1d} with +ve sign in loop is not filled up, then minimum value at b_{id} must be θ. If it is filled up with β value then minimum value at b_{id} must be $\theta+\beta$. Do the same process till all three constraints are not satisfied.

Step-6: Now, make the payoff matrix for all objectives as shown in the table 1, where $X_1, X_2.....X_k$ be the Solution obtained by step-5 for $Z_1, Z_2.....Z_k$ respectively then;

Table 1. Payoff matrix for all objectives

	X_1	X_2	…..	X_k
Z_1				
Z_2				
. . .	. . .	. . .	. . .	. . .
Z_k				

Step-7: Determine the lower bound L_k and upper bound U_k for k^{th} objective function given in the problem.

Step-8: Now, to get a compromise optimal solution for all objectives simultaneously, built the model for given problem then solve by using Lingo software[19].

7. Numerical Example

We consider a Multi-objective solid transportation problem as shown in table with two objectives.

Table 2. Solid Transportation Problem

	D_1	D_2	D_3	Supply
O_1	$e_{11}=0$ $\bar{c}_{111}, \bar{c}_{112}, \bar{c}_{113}$	$e_{12}=6$ $\bar{c}_{121}, \bar{c}_{122}, \bar{c}_{123}$	$e_{13}=9$ $\bar{c}_{131}, \bar{c}_{132}, \bar{c}_{133}$	$b_{11}\, b_{12}\, b_{13}$ 6, 9, 10
O_2	$e_{21}=21$ $\bar{c}_{211}, \bar{c}_{212}, \bar{c}_{213}$	$e_{22}=9$ $\bar{c}_{221}, \bar{c}_{222}, \bar{c}_{223}$	$e_{23}=14$ $\bar{c}_{231}, \bar{c}_{232}, \bar{c}_{233}$	$b_{21}\, b_{22}\, b_{23}$ 13, 14, 17

O_3	$e_{31}=21$ $\overline{c}_{311},\overline{c}_{312},\overline{c}_{313}$	$e_{32}=13$ $\overline{c}_{321},\overline{c}_{322},\overline{c}_{323}$	$e_{33}=12$ $\overline{c}_{331},\overline{c}_{332},\overline{c}_{333}$	$b_{31}\ b_{32}\ b_{33}$ 15, 13, 18
Demand	$a_{11}\ a_{12}\ a_{13}$ 15, 17, 20	$a_{21}\ a_{22}\ a_{23}$ 8, 11, 9	$a_{31}\ a_{32}\ a_{33}$ 11, 8, 16	

Now we will apply our proposed algorithm for this numerical problem

Step 1& 2:

Where for First objective:

$$\overline{c}_{111}=(5,7,9,11,13)(4,5,9,13,14),\quad R(\overline{c}_{111})=9;$$
$$\overline{c}_{112}=(4,6,7,8,9)(3,4,7,9,10),\quad R(\overline{c}_{112})=6.8;$$
$$\overline{c}_{113}=(6,7,9,11,12)(5,6,9,12,13),\quad R(\overline{c}_{113})=9;$$
$$\overline{c}_{211}=(8,9,10,12,13)(7,8,10,13,14),\quad R(\overline{c}_{211})=10.3;$$
$$\overline{c}_{212}=(3,5,6,7,9)(2,3,6,9,10),\quad R(\overline{c}_{212})=6;$$
$$\overline{c}_{213}=(2,3,5,7,8)(1,2,5,8,9),\quad R(\overline{c}_{213})=5;$$
$$\overline{c}_{311}=(5,6,7,8,9)(3,4,7,10,11),\quad R(\overline{c}_{311})=7;$$
$$\overline{c}_{312}=(2,3,5,7,8)(1,2,5,8,9),\quad R(\overline{c}_{312})=5;$$
$$\overline{c}_{313}=(3,4,5,6,7)(1,2,5,7,8),\quad R(\overline{c}_{313})=4.8;$$
$$\overline{c}_{121}=(2,3,5,7,8)(1,2,5,8,9),\quad R(\overline{c}_{121})=5;$$
$$\overline{c}_{122}=(8,9,10,12,13)(7,8,10,13,14),\quad R(\overline{c}_{122})=10.3;$$
$$\overline{c}_{123}=(2,3,5,7,8)(1,2,5,8,9),\quad R(\overline{c}_{123})=5;$$
$$\overline{c}_{221}=(3,5,6,7,9)(2,3,6,9,10),\quad R(\overline{c}_{221})=6;$$
$$\overline{c}_{222}=(5,7,9,11,13)(4,5,9,13,14),\quad R(\overline{c}_{222})=9;$$
$$\overline{c}_{223}=(7,8,10,11,12)(6,7,10,12,13),\quad R(\overline{c}_{223})=9.8;$$
$$\overline{c}_{321}=(3,4,6,7,8)(2,3,6,8,9),\quad R(\overline{c}_{321})=5.8;$$
$$\overline{c}_{322}=(10,11,12,13,14)(8,9,12,15,16),\quad R(\overline{c}_{322})=12;$$
$$\overline{c}_{323}=(6,7,10,11,12)(4,5,10,12,13),\quad R(\overline{c}_{323})=9;$$
$$\overline{c}_{131}=(10,11,12,13,14)(8,9,12,15,16),\quad R(\overline{c}_{131})=12;$$
$$\overline{c}_{132}=(8,9,10,12,13)(7,8,10,13,14),\quad R(\overline{c}_{132})=10.3;$$
$$\overline{c}_{133}=(6,7,10,11,12)(4,5,10,12,13),\quad R(\overline{c}_{133})=9;$$

$\bar{c}_{231} = (3,4,5,6,7)(1,2,5,7,8),\qquad R(\bar{c}_{231}) = 4.8;$
$\bar{c}_{232} = (5,6,7,8,9)(3,4,7,10,11),\qquad R(\bar{c}_{232}) = 7;$
$\bar{c}_{233} = (2,3,5,7,8)(1,2,5,8,9),\qquad R(\bar{c}_{233}) = 5;$
$\bar{c}_{331} = (6,7,10,11,12)(4,5,10,12,13),\qquad R(\bar{c}_{331}) = 9;$
$\bar{c}_{332} = (5,7,9,11,13)(4,5,9,13,14),\qquad R(\bar{c}_{332}) = 9;$
$\bar{c}_{333} = (5,6,7,8,9)(3,4,7,10,11),\qquad R(\bar{c}_{333}) = 7;$

And for second objective:

$\bar{c}_{111} = (5,7,9,11,13)(4,5,9,13,14),\qquad R(\bar{c}_{111}) = 9;$
$\bar{c}_{112} = (10,11,12,13,14)(8,9,12,15,16),\qquad R(\bar{c}_{112}) = 12;$
$\bar{c}_{113} = (8,9,10,12,13)(6,7,10,14,15),\qquad R(\bar{c}_{113}) = 10.3;$
$\bar{c}_{211} = (8,9,10,12,13)(6,7,10,14,15),\qquad R(\bar{c}_{211}) = 10.3;$
$\bar{c}_{212} = (6,7,9,11,12)(5,6,9,12,13),\qquad R(\bar{c}_{212}) = 9;$
$\bar{c}_{213} = (4,6,7,8,9)(3,4,7,9,10),\qquad R(\bar{c}_{213}) = 6.8;$
$\bar{c}_{311} = (4,6,7,8,9)(3,4,7,9,10),\qquad R(\bar{c}_{311}) = 6.8;$
$\bar{c}_{312} = (6,7,9,11,12)(5,6,9,12,13),\qquad R(\bar{c}_{312}) = 9;$
$\bar{c}_{313} = (2,3,5,7,8)(1,2,5,8,9),\qquad R(\bar{c}_{313}) = 5;$
$\bar{c}_{121} = (3,4,6,7,8)(2,3,6,8,9),\qquad R(\bar{c}_{121}) = 5.8;$
$\bar{c}_{122} = (6,7,9,11,12)(5,6,9,12,13),\qquad R(\bar{c}_{122}) = 9;$
$\bar{c}_{123} = (4,5,7,8,9)(2,3,7,10,11),\qquad R(\bar{c}_{123}) = 6.8;$
$\bar{c}_{221} = (7,8,10,11,12)(6,7,10,12,12),\qquad R(\bar{c}_{221}) = 9.8;$
$\bar{c}_{222} = (10,11,12,13,14)(8,9,12,15,16),\qquad R(\bar{c}_{222}) = 12;$
$\bar{c}_{223} = (8,9,10,12,13)(6,7,10,14,15),\qquad R(\bar{c}_{223}) = 10.3;$
$\bar{c}_{321} = (4,5,7,8,9)(2,3,7,10,11),\qquad R(\bar{c}_{321}) = 6.8;$
$\bar{c}_{322} = (8,9,10,12,13)(6,7,10,14,15),\qquad R(\bar{c}_{322}) = 10.3;$
$\bar{c}_{323} = (7,8,10,11,12)(6,7,10,12,13),\qquad R(\bar{c}_{323}) = 9.8;$
$\bar{c}_{131} = (5,7,9,11,13)(4,5,9,13,14),\qquad R(\bar{c}_{131}) = 9;$
$\bar{c}_{132} = (4,5,7,8,9)(3,4,7,9,10),\qquad R(\bar{c}_{132}) = 6.8;$
$\bar{c}_{133} = (6,7,9,11,12)(5,6,9,12,13),\qquad R(\bar{c}_{133}) = 9;$
$\bar{c}_{231} = (3,4,6,7,8)(2,36,8,9),\qquad R(\bar{c}_{231}) = 5.8;$
$\bar{c}_{232} = (5,7,9,11,13)(4,5,9,13,14),\qquad R(\bar{c}_{232}) = 9;$
$\bar{c}_{233} = (7,8,10,11,12)(6,7,10,12,13),\qquad R(\bar{c}_{233}) = 9.8;$
$\bar{c}_{331} = (3,5,6,7,9)(2,3,6,9,10),\qquad R(\bar{c}_{331}) = 6;$

$$\bar{c}_{332} = (2,3,5,7,8)(1,2,5,8,9), \qquad R(\bar{c}_{332}) = 5;$$
$$\bar{c}_{333} = (3,5,6,7,9)(2,3,6,9,10), \qquad R(\bar{c}_{333}) = 6;$$

Step 3: Since

$$\sum_{j=1}^{n} a_{jd} = \sum_{i=1}^{m} b_{id}\ , \qquad \sum_{j=1}^{n} e_{ij} = \sum_{d=1}^{D} b_{id}\ , \qquad \sum_{i=1}^{m} b_{ij} = \sum_{d=1}^{D} a_{jd}\ ,$$

$$\sum_{j=1}^{n}\sum_{d=1}^{D} a_{jd} = \sum_{d=1}^{D}\sum_{i=1}^{m} b_{di} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\ ,$$

Thus, problem is balanced.

Step 4 & 5: Now, by applying step 4 and 5 we get the optimal solution for both objectives such as:
For first objective solution X_1 as follows:

$$x_{112} = 7, x_{113} = 3, x_{212} = 4,\ x_{213} = 17, x_{311} = 15, x_{312} = 6,\ x_{121} = 1, x_{122} = 2, x_{123} = 3,$$

$$x_{221} = 7, x_{222} = 2, x_{322} = 7,\ x_{323} = 6, x_{131} = 5, x_{133} = 4,\ x_{231} = 6, x_{232} = 8,\ x_{333} = 12.$$

For second objective solution X_2 as follows:

$$x_{111} = 6, x_{112} = 4, x_{211} = 4,\ x_{212} = 13, x_{213} = 4,\ x_{311} = 5, x_{313} = 16, x_{122} = 5,$$

$$x_{123} = 1, x_{222} = 1, x_{223} = 8,\ x_{321} = 8, x_{322} = 5, x_{133} = 9,\ x_{231} = 9, x_{233} = 5,$$

$$x_{331} = 2, x_{332} = 8, x_{333} = 2.$$

Step 6: Make a payoff matrix as shown in table 3.

Table 3. Payoff matrix

	X_1	X_2
Z_1	822	835.7
Z_2	950	899.7

Step 7: Now, L_1 =822, U_1 =835.7; L_2 =899.7, U_2 =950, where L_1 and U_1 are lower and upper bounds of Z_1(first objective function) and L_2, U_2 are lower and upper bounds of Z_2 respectively (second objective function).

Step 8: Built Model for the MOSTP:

$$\text{Min } \theta,$$

$$\mu_1(Z_1(x)) + d_1(1-w_1) = 1; \text{ where } w_1 = 0.073,$$

i.e., $$Z_1(x) - 12.7d_1 = 822;$$

$$\mu_2(Z_2(x)) + d_2(1-w_2) = 1; \text{ where } w_2 = 0.02,$$

i.e., $$Z_2(x) - 49.29d_2 = 899.7;$$

$$\theta \geq 0.927d_{1,}, \ \theta \geq 0.980d_{2,}$$

$$\begin{aligned}
&x_{111}+x_{112}+x_{113}=10;\ x_{211}+x_{212}+x_{213}=21;\ x_{311}+x_{312}+x_{313}=21;\\
&x_{111}+x_{211}+x_{311}=15;\ x_{112}+x_{212}+x_{312}=17;\ x_{113}+x_{213}+x_{313}=20;\\
&x_{121}+x_{122}+x_{123}=6;\ x_{221}+x_{222}+x_{223}=9;\ x_{321}+x_{322}+x_{323}=13;\\
&x_{121}+x_{221}+x_{321}=8;\ x_{122}+x_{222}+x_{322}=11;\ x_{123}+x_{223}+x_{323}=9;\\
&x_{131}+x_{132}+x_{133}=9;\ x_{231}+x_{232}+x_{233}=14;\ x_{331}+x_{332}+x_{333}=12;\\
&x_{131}+x_{231}+x_{331}=11;\ x_{132}+x_{232}+x_{332}=8;\ x_{133}+x_{233}+x_{333}=16;\\
&x_{111}+x_{121}+x_{131}=6;\ x_{112}+x_{122}+x_{132}=9;\ x_{113}+x_{123}+x_{133}=10;\\
&x_{211}+x_{221}+x_{231}=13;\ x_{212}+x_{222}+x_{232}=14;\ x_{213}+x_{223}+x_{233}=17;\\
&x_{311}+x_{321}+x_{331}=15;\ x_{312}+x_{322}+x_{332}=13;\ x_{313}+x_{323}+x_{333}=18;\\
&0 \leq d_1 \leq 1, 0 \leq d_2 \leq 1, 0 \leq \theta \leq 1,
\end{aligned}$$

Then, global solution of the problem obtained by Lingo 18:

x_{111} =6, x_{112} =4, x_{211}=1.696429, x_{212} =8.151786, x_{213} =11.151786, x_{311} =7.303571, x_{312} =4.848214, x_{313}=8.848214, x_{112} =2.848214, x_{123} =3.151786, x_{221} =3.151786, x_{223} =5.848214, x_{321} =4.848214, x_{322} =8.151786, x_{132} =2.151786, x_{133} =6.848214, x_{231}=8.151786, x_{232} =5.848214, x_{331} =2.848214, x_{333}=9.151786.

Z_1 =822 and Z_2 = 909.5.

8. Conclusion

We applied new approach to find the compromise solution of the Multi-Objective Transportation Problem (MOTP). We take special transportation problem known as the Solid Transportation Problem (STP) in which three constraints are included. We use Lingo software to find the compromise optimal solution of the problem. It is a new approach to find the global solution for Multi-Objective Solid Transportation Problem (MOSTP) with Interval-Valued Intuitionistic Fuzzy Cost (IVIFC). In this approach, we take a different deviational function $d_k = (1 - w_k)$ where w_k is fixed and $w_k = 1/U_k - L_k$ associated to each d_k. By this proposed method, we found best solution for multi-objective solid transportation problem. We compared our solution with Fuzzy Goal Programming (FGP) approach. Our solution is more optimal than the solution obtained by FGP with linear membership function. The solution obtained by FGP Z_1 = 835.7 and Z_2 = 947.39 which is more than Z_1 =822 and Z_2 = 909.5 solution obtained by our proposed algorithm. This proves that our proposed algorithm is better than the previous algorithm.

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References

1. F. L. Hitchcock, The Distribution of Product from Several Sources to Numerous Localities, *MIT Journal of Mathematics and Physics*, **20** (1941), 224-230.

2. L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338-353.

3. R. Bellman and L. A. Zadeh, Decision Making in Fuzzy Environment, *Management Science*, **17(B)** (1970), 141–164.

4. K. Haley, The Solid Transportation Problem, *Operations Research*, **10** (1962), 448-463.

5. H. J. Zimmermann, Fuzzy Programming and Linear Programming with Several Objective Functions, *Fuzzy Sets and Systems*, **1**(1978), 45-55.

6. H. J. Zimmermann, Application of Fuzzy Set Theory to Mathematical Programming, *Information Sciences*, **36** (1985), 29-58.

7. A. K. Bit, M. P. Biswal and S. S. Alam, Multi-objective Transportation Problem with Interval Cost Source and Destination Parameters, *European Journal of Operational Research*, **117** (1999), 100-112.

8. A. Charnes and W. W. Cooper, Programming with Linear Fractional Functionals, *Naval Research Logistics Quarterly*, **9** (1962), 181-186.

9. R. Narasimhan, Goal Programming in Fuzzy Environment, *Decision Science*, **111** (1980), 325-336.

10. B. Radhakrishnan and P. Anukokila, Fractional Goal Programming for Fuzzy Solid Transportation problem with Interval Cost, *Fuzzy Information and Engineering*, **6** (2014), 359-377.

11. B. B. Pal and B. N. Moitra, Using Fuzzy Goal Programming for Long Range Production Planning in Agricultural Systems, *Indian Journal of Agriculture Economics*, **59(1)** (2004), 75-90.

12. F. Abd El-Wahed Waiel and S. M. Lee, Interactive Fuzzy Goal Programming for Multiobjective Transportation Problems, *Omega*, **34** (2006), 158-166.

13. M. Zangiabadi and H. R. Maleki, Fuzzy Goal Programming for Multiobjective Transportation Problems, *Journal of Applied Mathematics and computing*, **24(1-2)** (2007), 449-469.

14. K. T. Atanassov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, **20(1)** (1986), 87-96.

15. K. T. Atanassov and G. Gargov, Interval-valued Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, **31** (1989), 343-349.

16. S. Aggarwal and C. Gupta, Solving Intuitionistic Fuzzy Solid Transportation Problem Via New Ranking Method Based on Signed Distance, *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, **24(4)** (2016), 483-501.

17. M. A. Nomani, I. Ali and A. Ahmad, A New Approach for Solving Multi-Objective Transportation Problems, *International Journal of Management Science and Engineering Management*, **12(3)** (2016), 165-173.

18. S. Helipern, The Expected Value of Fuzzy Number, *Fuzzy Sets and Systems*, **47** (1992), 81-86.

19. L. Schrage, *Lingo User's Guide*, LINDO Systems, Inc., Chicago, 2006.