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MHD Flow through a Porous Walled Circular Cylinder Filled with Porous Medium in the Influence of Cosinusoidal Suction on the Wall

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Abstract: Two dimensional axisymmetric viscous incompressible electrically conducting fluid flows through a circular cylinder filled with porous medium in the presence of a static transverse magnetic field and a cosinusoidal suction in radial direction is studied. A domain Decomposition Method (ADM) is used to solve the non-linear coupled differential equation to obtain the velocity profiles in radial and axial direction. The effects of involved physical parameters are computed and discussed graphically. The skin-friction coefficient is also computed and effect of Reynolds number, Hartmann number and Darcy number are analyzed.

Keywords: MHD flow, porous walled cylinder, cosinusoidal suction, ADM.

1. Introduction

There are many frontier problems which exist in physics, engineering, medical and many other disciplines which dealt with the mathematical modeling. The formulation of these problems by means of Navier-Stokes equations gives rise to nonlinear ordinary or partial differential equations subject to the certain boundary conditions. The exact solutions of these problems are not always possible due to their nonlinear character. So, to find the solution of problems, we take advantage of numerical techniques, up to desired accuracy. Adomian[1] discussed the application of decomposition method to Navier-Stokes equations. Adomian[2-6] developed a powerful method known as decomposition method which provides analytic approximations to a wide class of nonlinear ordinary and partial differential equations. Halder[7] investigated two dimensional steady blood flow through a constructed artery in the presence of transverse magnetic field using Adomian decomposition method. Salas[8] obtained an exact solution of MHD boundary layer flow over a moving vertical cylinder. Boricic et al.[9] analyzed the unsteady two dimensional dynamic, unsteady, thermal and diffusion magneto hydrodynamic laminar boundary layer flow over a horizontal circular cylinder of incompressible and elecrically conducting fluid in a porous medium in the presence of a heat source or sink and chemical reaction. Bhattacharya et al.[10] studied axisymmetric boundary layer flow and heat transfer past a permeable shrinking cylinder subject to mass suction. Mukhopadhyay et al.[11]. studied axisymmetric laminar boundary layer flow of a viscous incompressible fluid and heat transfer towards a stretching cylinder under the influence of a magnetic field.

In the present study, a two dimensional steady fully developed flow through a horizontal porous walled circular cylinder in the influence of cosinusoidal suction and a transverse magnetic field is considered. The mathematical model of the problem is derived from Navier-Stokes equation and is non-linear partial differential equations. The solution of the problem is obtained by Adomian decomposition method.

2. Formulation of the Problem

In the present study a viscous incompressible electrically conducting fluid is flowing in a circular cylinder. The circular cylinder is of permeable wall filled with fluid saturated isotrpic porous medium. The z-axis is taken along the axis of the cylinder. The flow is considered as axisymmetric, therefore in the cylindrical coordinate system (r,θ,z) the velocity field can be defined by $\vec{q}(u,0,v)$. A static magnetic field $(B_0,0,0)$ is acting transversely to the flow and cosinusoidal suction is applied on the surface of the cylinder. In view of magneto hydrodynamics when an electrically conducting fluid flows in a magnetic field, an electromagnetic force generated due to the interaction of the current with magnetic field. The Maxwell equations for MHD flow are

(2.1) $$div\,\vec{B}=0$$

(2.2) $$Curl\,\vec{B} = \mu_m \vec{J}\,,$$

(2.3) $$Curl\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}\,,$$

(2.4) $$\vec{J} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right),$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, μ_m is the electric permeability, $\vec{J}$ is the current density and σ is the electrical conductivity.

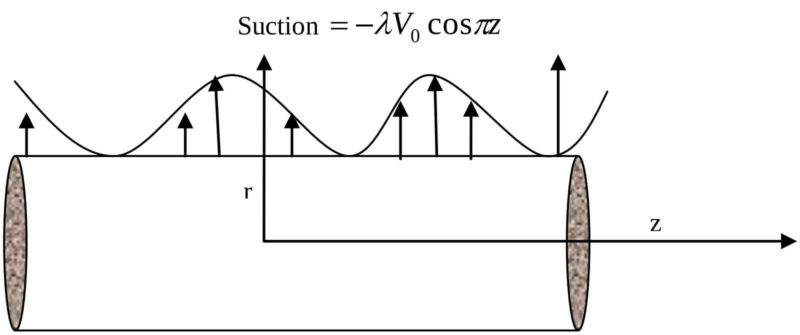

Figure 1. Physical model of the problem

The governing equations of motion describing the flow and the prescribed boundary conditions are defined as
The equation of continuity

(2.5) $$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0\,.$$

The equations of motion are in radial direction

(2.6) $$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] - \frac{\nu}{K}u\,.$$

In axial direction

(2.7) $$u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right] - \frac{\sigma \beta_0{}^2 v}{\rho} - \frac{\nu}{K}v\,.$$

The corresponding boundary conditions are

(2.8) $r = 0 \ : \quad \frac{\partial v}{\partial r} = 0,$

(2.9) $r = a \ : \quad v = 0, \ \ u = -\lambda V_0 \cos \pi z$

3. Method of Solution

Introducing following dimensionless quantities

$$u^* = \frac{u}{V_0}, \ v^* = \frac{v}{V_0}, \ r^* = \frac{r}{a}, \ \ z^* = \frac{z}{a}, \ p^* = \frac{p}{\rho V_0^{\ 2}},$$

where p pressure, λ suction\injection parameter, ρ density, μ viscosity K permeability, V_0 maximum suction\ injection, a radius of cylinder.
The non-dimensional form of equations of motion when the asterisk is dropped for the sake of simplicity, are given by

(3.1) $$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right] - \frac{1}{Da\mathrm{Re}}u,$$

(3.2) $$u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right] - \frac{M^2}{\mathrm{Re}}v - \frac{1}{Da\,\mathrm{Re}}v,$$

where $\mathrm{Re} = aV_0/\nu$ the Reynolds number, $M = \sqrt{\sigma\beta_0^{\ 2}a^2/\mu}$ the Hartmann number, $Da = \mu/K$ the Darcy number.
Differentiate (3.1) with respect to z and (3.2) with respect to r and then taking their difference to eliminate the pressure term, we obtained

(3.3) $$u\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r}\frac{\partial v}{\partial r} + v\frac{\partial^2 v}{\partial r\partial z} + \frac{\partial v}{\partial r}\frac{\partial v}{\partial z} - u\frac{\partial^2 u}{\partial r\partial z} - \frac{\partial u}{\partial z}\frac{\partial u}{\partial r} - v\frac{\partial^2 u}{\partial z^2} - \frac{\partial v}{\partial z}\frac{\partial u}{\partial z}$$
$$= \frac{1}{\mathrm{Re}}\left[\frac{\partial^3 v}{\partial r^3} - \frac{1}{r^2}\frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial^2 v}{\partial r^2} + \frac{\partial^3 v}{\partial r\partial z^2} - \frac{\partial^3 u}{\partial r^2\partial z} - \frac{1}{r}\frac{\partial^2 u}{\partial r\partial z} + \frac{1}{r^2}\frac{\partial u}{\partial z} - \frac{\partial^3 u}{\partial z^3}\right]$$
$$-\frac{M^2}{\mathrm{Re}}\frac{\partial v}{\partial r} - \frac{1}{Da\,\mathrm{Re}}\frac{\partial v}{\partial r} + \frac{1}{Da\,\mathrm{Re}}\frac{\partial u}{\partial z}$$

Taking the stream function $\psi(r,z)$ that related with u and v as

(3.4) $$u=\frac{1}{r}\frac{\partial\psi}{\partial z},\ v=-\frac{1}{r}\frac{\partial\psi}{\partial r}$$

In view of (3.4), The equation (3.3) reduces to

(3.5) $$\mathrm{Re}\left[\frac{J}{r^2}-\frac{2}{r^3}\nabla^2\psi\frac{\partial\psi}{\partial z}\right]=-\nabla^4\psi+\left(M^2+\frac{1}{Da}\right)\left(\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}\right)+\frac{1}{Da}\frac{\partial^2\psi}{\partial z^2},$$

where, the Jacobian J is defined by

(3.6) $$J=\frac{\partial\left(\nabla^2\psi,\psi\right)}{\partial(r,z)}=\begin{vmatrix}\frac{\partial}{\partial r}\left(\nabla^2\psi\right) & \frac{\partial\psi}{\partial r}\\ \frac{\partial}{\partial z}\left(\nabla^2\psi\right) & \frac{\partial\psi}{\partial z}\end{vmatrix}$$

and

(3.7) $$\nabla^2\equiv\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}.$$

The corresponding boundary conditions reduces to

(3.8) $$r=0\ :\quad \frac{\partial}{\partial r}\left(-\frac{1}{r}\frac{\partial\psi}{\partial r}\right)=0,$$

(3.9) $$r=1\ :\quad -\frac{1}{r}\frac{\partial\psi}{\partial r}=0\ ,\ \frac{1}{r}\frac{\partial\psi}{\partial z}=-\lambda\cos\pi z\,.$$

The solution of the non-linear partial differential equation (3.5) is obtained with the Adomian decomposition method (ADM). On applying ADM

Taking $L\equiv\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}$ a linear operator and $N_\psi=\frac{J}{r^2}-\frac{2}{r^3}\nabla^2\psi\frac{\partial\psi}{\partial z}$ then the equation (3.5) can be written as

(3.10) $$L^2\psi=-\mathrm{Re}\,N_\psi-2\frac{\partial^4\psi}{\partial r^2\partial z^2}+\frac{2}{r}\frac{\partial^3\psi}{\partial r\partial z^2}-\frac{\partial^4\psi}{\partial z^4}+\left(M^2+\frac{1}{Da}\right)L\psi+\frac{1}{Da}\frac{\partial^2\psi}{\partial z^2}.$$

Applying inverse operator L^{-2} on the equation (3.10) it reduces into

(3.11) $$\psi = \psi_0 + L^{-2}\left(-\mathrm{Re}\,N_\psi - 2\frac{\partial^4\psi}{\partial r^2\partial z^2} + \frac{2}{r}\frac{\partial^3\psi}{\partial r\partial z^2} - \frac{\partial^4\psi}{\partial z^4} + \left(M^2 + \frac{1}{Da}\right)L\psi\right),$$

where ψ_0 is the solution of the equation

(3.12) $$L^2\psi_0 = 0$$

Now decomposing ψ and N_ψ in the following form

(3.13) $$\psi = \sum_{n=0}^{\infty}\psi_n\zeta^n,$$

(3.14) $$N_\psi = \sum_{n=0}^{\infty}P_n\zeta^n,$$

where P_n are Adomian' special polynomials. The parameter ζ is used only for grouping the terms of different order. The regular decomposition of ψ in the recurrence form is given by

(3.15) $$\psi_{n+1} = \psi_0 + L^{-2}\left(-\mathrm{Re}\,P_n - 2\frac{\partial^2 L\psi_n}{\partial z^2} - \frac{\partial^4\psi_n}{\partial z^4} + \left(M^2 + \frac{1}{Da}\right)L\psi_n\right)$$

$$+\frac{1}{Da}\frac{\partial^2\psi_n}{\partial z^2},$$

where, n=0, 1, 2 . . . Once the component ψ_0 is determined, the other components of ψ such as ψ_1, ψ_2 etc. can be determined from (3.14).

Again substitution of (3.13) and (3.14) into the boundary conditions mentioned in (3.8) and (3.9) gives the boundary conditions for the respective components ψ_0, ψ_1 etc. as follows

(3.16) $$r = 0: \quad \frac{\partial}{\partial r}\left(-\frac{1}{r}\frac{\partial\psi_0}{\partial r}\right) = 0\,, \quad \frac{\partial}{\partial r}\left(-\frac{1}{r}\frac{\partial\psi_n}{\partial r}\right) = 0,$$

(3.17) $$r = 1: \; -\frac{1}{r}\frac{\partial\psi_0}{\partial r} = 0,\; \frac{1}{r}\frac{\partial\psi_0}{\partial z} = -\lambda\cos\pi z\,, -\frac{1}{r}\frac{\partial\psi_n}{\partial r} = 0\,, \frac{1}{r}\frac{\partial\psi_n}{\partial z} = 0\,.$$

Invoking double decomposition of ψ_0 defined by

(3.18) $$\psi_0 = \sum_{n=0}^{\infty} \psi_{0,n} \zeta^n .$$

In the equation (3.15), the double decomposition components of ψ are given by

(3.19) $$\psi_{n+1} = \psi_{0,n+1} + L^{-2}\left(-\operatorname{Re} P_n - 2\frac{\partial^2 L\psi_n}{\partial z^2} - \frac{\partial^4 \psi_n}{\partial z^4} + \left(M^2 + \frac{1}{Da} \right) L\psi_n + \frac{1}{Da}\frac{\partial^2 \psi_n}{\partial z^2} \right),$$

where n being any non-negative integer.
In view of the definition of linear operator L, the inverse operator L^{-2} is defined by

(3.20) $$L^{-2} \equiv L_1^{-1}\, r\, L_1^{-1}\, r^{-1} L_1^{-1}\, r\, L_1^{-1}\, r^{-1},$$

therefore, the solution of the equation (3.12) is given by

3.21) $$\psi_0 = A\frac{r^4}{16} + B\left(\frac{r^2 \log r}{2} - \frac{r^2}{4} \right) + C\frac{r^2}{2} + D .$$

Since the expression for ψ_0 contains the constants A, B, C and D, therefore, the parameterized decomposition forms of all these constants will be of the form

(3.22) $$A = \sum_{n=0}^{\infty} A_n \zeta^n ,\ B = \sum_{n=0}^{\infty} B_n \zeta^n ,\ C = \sum_{n=0}^{\infty} C_n \zeta^n ,\ D = \sum_{n=0}^{\infty} D_n \zeta^n ,$$

Substituting these in the equation (3.21) and comparing the like power terms of ζ on the both sides of the resulting expression we get

(3.23) $$\psi_{0,n+1} = A_{n+1}\frac{r^4}{16} + B_{n+1}\left(\frac{r^2 \log r}{2} - \frac{r^2}{4} \right) + C_{n+1}\frac{r^2}{2} + D_{n+1} .$$

The constants involved in each ψ_n will be determined by their respective boundary conditions.

The Adomian polynomials $P_0, P_1, \ldots P_n$ are defined in such a way that $P_0 \equiv P_o(\psi_0)$, $P_1 \equiv P_1(\psi_0, \psi_1)$, $P_2 \equiv P_2(\psi_0, \psi_1, \psi_2)$ ………. $P_n \equiv P_n(\psi_0, \psi_1, ..\psi_n)$, here

$$P_0 = \frac{1}{r}\frac{\partial\left(\nabla^2\psi_0, \psi_0\right)}{\partial(r,z)} - \frac{2}{r^2}\frac{\partial\psi_0}{\partial z}\nabla^2\psi_0,$$

$$P_1 = \frac{1}{r}\left[\frac{\partial\left(\nabla^2\psi_1, \psi_0\right)}{\partial(r,x)} + \frac{\partial\left(\nabla^2\psi_0, \psi_1\right)}{\partial(r,z)}\right]$$

$$= \frac{2}{r^2}\left[\frac{\partial\psi_0}{\partial z}.\nabla^2\psi_1 + \frac{\partial\psi_1}{\partial z}\nabla^2\psi_0\right].$$

Under the prescribed boundary conditions, the solution of the equation (3.21) will be

$$\psi_0 = \left(\lambda\frac{\sin\pi z}{\pi}\right)\left(r^4 - 2r^2\right). \tag{3.24}$$

Using ψ_0, the value of Adomian polynomial P_0 is obtained and given by

$$P_0 = \left(2\lambda^2\,\pi\, r^6 - \left(\frac{32\lambda^2}{\pi} + 8\lambda^2\pi\right)\left(r^4 - r^2\right)\right)\sin(\pi z)\cos\left(\pi z\right). \tag{3.25}$$

Now, for $n = 1$, the equation (3.19) gives

$$\psi_1 = \psi_{0,1} + L^{-2}\left(\operatorname{Re} P_0 - 2\frac{\partial^2 L\psi_0}{\partial z^2} - \frac{\partial^4\psi_0}{\partial z^4} + \left(M^2 + \frac{1}{Da}\right)L\psi_0 + \frac{1}{Da}\frac{\partial^2\psi_0}{\partial z^2}\right), \tag{3.26}$$

where , $\psi_{0,1}$ is the solution of the equation

$$L^2\psi_{0,1} = 0.$$

Letting

$$\operatorname{Re} P_0 - 2\frac{\partial^2 L\psi_0}{\partial z^2} - \frac{\partial^4 \psi_0}{\partial z^4} + \left(M^2 + \frac{1}{Da}\right) L\psi_0 + \frac{1}{Da}\frac{\partial^2 \psi_0}{\partial z^2}$$
$$= \beta_1 r^6 + \beta_2 r^4 + \beta_3 r^2 .$$

On using values of ψ_0 and P_0 the values of $\beta_i(z), i = 1,2,3$ are defined as follows.

where $\beta_1(z) = \operatorname{Re}\lambda^2 \pi \sin 2\pi z$,

$$\beta_2(z) = -\operatorname{Re}\left(\frac{16\lambda^2}{\pi} + 4\lambda^2\pi\right)\sin 2\pi z - \lambda\pi^3 \sin \pi z - \lambda\pi \sin \pi z$$

$$\beta_3(z) = \operatorname{Re}\left(\frac{16\lambda^2}{\pi} + 4\lambda^2\pi\right)\sin 2\pi z + \begin{pmatrix} 16\lambda\pi + 2\lambda\pi^3 + \\ 8\dfrac{\lambda}{\pi}\left(M^2 + \dfrac{1}{Da}\right) + 2\lambda\pi \end{pmatrix}\sin \pi z$$

Then ψ_1 can be expressed as

$$\psi_1 = \frac{r^4}{16} A_1(z) + C_1(z)\frac{r^2}{2} + \frac{\beta_1(z)}{3840} r^{10} + \frac{\beta_2(z)}{1152} r^8 + \frac{\beta_3(z)}{192} r^6, \tag{3.27}$$

Substitute values of ψ_0 and ψ_1 in the equation (3.13) and using ψ in the equation (3.4) , we have The radial velocity profile

$$u = \lambda \cos \pi z \cdot \left(r^3 - 2r\right) + A_1'(z)\frac{r^3}{16} + C_1'(z)\frac{r}{2} + \beta_1'(z)\frac{r^9}{3840} + \beta_2'(z)\frac{r^7}{1152} + \beta_3'(z)\frac{r^5}{192} \tag{3.28}$$

and the axial velocity profile

$$v = -\begin{pmatrix} \dfrac{\lambda \sin \pi z}{\pi}\left(4r^2 - 4\right) + A_1(z)\dfrac{r^2}{4} + C_1(z) \\ +\beta_1(z)\dfrac{r^8}{384} + \beta_2(z)\dfrac{r^6}{144} + \beta_3(z)\dfrac{r^4}{32} \end{pmatrix}, \tag{3.29}$$

where

$$A_1(z) = -\frac{\beta_1(z)}{60} - \frac{\beta_2(z)}{24} - \frac{\beta_3(z)}{6} \qquad A_1'(z) = -\frac{\beta_1'(z)}{60} - \frac{\beta_2'(z)}{24} - \frac{\beta_3'(z)}{6}$$

$$C_1(z) = \frac{\beta_1(z)}{640} + \frac{\beta_2(z)}{288} + \frac{\beta_3(z)}{96} \qquad C_1'(z) = \frac{\beta_1'(z)}{640} + \frac{\beta_2'(z)}{288} + \frac{\beta_3'(z)}{96}$$

$$\beta_1'(z) = 2\,\mathrm{Re}\,\lambda^2\pi^2\cos 2\pi z\,,\ \beta_2'(z) = -\,\mathrm{Re}\left(32\lambda^2 + 8\lambda^2\pi^2\right)\cos 2\pi z$$
$$-\lambda\pi^4\cos\pi z - \lambda\pi^2\cos\pi z$$

$$\beta'_3(z) = \mathrm{Re}\left(32\lambda^2 + 4\lambda^2\pi^2\right)\cos 2\pi z + \begin{pmatrix} 16\lambda\pi^2 + 2\lambda\pi^4 + \\ 8\lambda\left(M^2 + \dfrac{1}{Da}\right) + 2\lambda\pi^2 \end{pmatrix}\cos\pi z$$

The simulation of velocities for various physical parameters the computation has been carried out with MATLAB programming for $\lambda=2$.

4. Skin Friction Coefficients

The non-dimensional shearing stress on the wall and the interface in terms of the local skin-friction coefficient is derived as follows and computed values are given in table 1.

$$C_f = \frac{2}{\mathrm{Re}}\left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right)_{r=1}$$

Table 1. Skin friction coefficient at the wall of cylinder

Re	M	D_a	C_f
10	3	0.1	1.76968
20	3	0.1	2.43382
30	3	0.1	2.6552
40	3	0.1	2.76588
10	1	0.1	1.96925
10	5	0.1	1.37054
10	8	0.1	0.39763
10	3	1	1.9942
10	3	0.01	-0.4755
10	3	1E-3	-22.92724

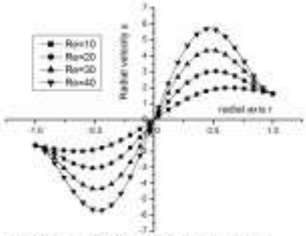

Figure 2. The effect of Reynolds number on radial velocity at M=2, Da=0.1, z=0.8

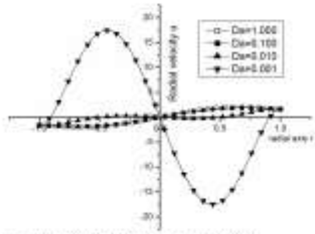

Figure 3. The effect of Darcy number on radial velocity at M=2, Re=10, z=0.8

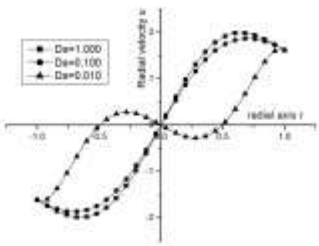

Figure 4. The effect of Darcy number on radial velocity at M=2, Re=10, z=0.8

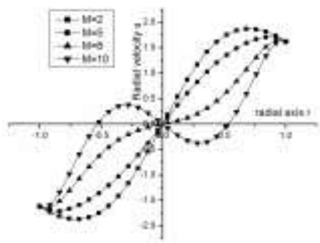

Figure 5. The effect of Hartmann number on radial velocity on Re=10, Da=0.1, z=0.8

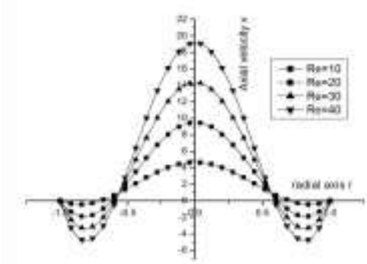

Figure 6. The effect of Reynolds number on axial velocity at M=2 Da=0.1 and z=0.8.

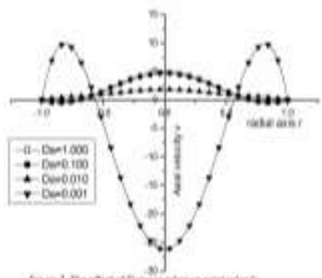

Figure 7. The effect of Darcy number on axial velocity at Re=10, M=2, z=0.8

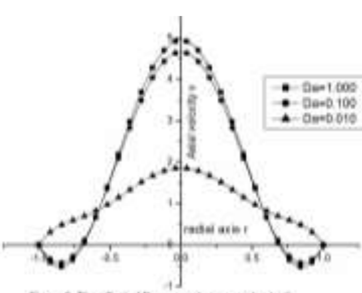

Figure 8. The effect of Darcy number on axial velocity at Re=10, M=2, z=0.8

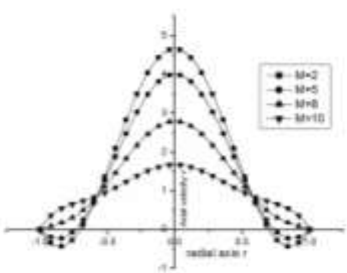

Figure 9. The effect of Hartmann number on Axial velocity at Re=10, Da=0.1, z=0.8.

5. Results and Discussion

The radial velocity of the fluid in the porous media filled circular cylinder is significantly increases in magnitude with the increase of Reynolds number as observed in figure 2.The radial velocity of the fluid is comparatively very high in magnitude for the small value of Darcy number. With the increase of Darcy number the magnitude of radial velocity decreases significantly as shown in the figures 3 and 4. The flow field is also affected by the magnetic field as shown in Figure 5. The radial velocity decreases in magnitude with the increase of Hartmann number. When Hartmann number is small there is no back flow while for large value of Hartmann number e.g. $M=10$, the backflow near the centerline is observed.In figure 6, it is observed that the axial velocity of the fluid flow in the central region of the cylinder is in forward direction and increases with the increase of Reynolds number. There is back flow in the axial velocity profile in the vicinity of the cylinder's surface which is caused by suction at the surface. The effects of permeability on the flow profile are shown in terms of the Darcy number in figure 7 and 8. Due to suction there is back flow in the vicinity of the surface of the cylinder for comparatively large value of the Darcy number. With the decrease of the Darcy number the magnitude of the back flow reduces and diminished for $D_a < 0.1$ as in figure 8. Also with the decrease in Darcy number the axial velocity reduces in the central region of the cylinder. The axial velocity can be controlled with the external magnetic field is plausible in figure 9. The magnitude of axial flow velocity decreases with the increase of Hartmann number which is in agreement that the Lorengian force produced by magnetic field retarded the flow. An interesting result is observed that the back flow in the vicinity of the cylinder can be diminished by increasing the strength of magnetic field. In table1,it is observed that the skin friction at the wall of the cylinder increases with the increase in the value of Reynolds number. The skin friction reduces with the increasing value of Hartmann number. The magnitude of skin friction decreases with the increase of Darcy number.

6. Conclusion

(1) Radial velocity increases with increase in values of Darcy number and Reynolds number but decreases with the increase in Hartmann number
(2) Back flow is observed in the vicinity of the circumference of the cylinder .

(3) The magnetic field can be used as controlling device for the skin friction on the surface of the cylinder.

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