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Radiation Effects on Unsteady MHD Free Convective Flow with Hall Current and Mass Transfer Through Viscous Incompressible Fluid Past a Vertical Porous Plate Immersed in Porous Medium with Heat Source/ Sink

P. R. Sharma, Navin Kumar and Pooja Sharma
Department of Mathematics, University of Rajasthan,
Jaipur- 302055, Rajasthan, India
E.mail: profprsharma@yahoo.com; pooja_2383@yahoo.co.in

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Abstract: Aim of the present paper is to investigate the radiation effects on unsteady megenetohydrodynamic free convective flow with Hall current and mass transfer through a viscous incompressible electrically conducting fluid past an infinite vertical porous flat plate immersed in porous medium in the presence of a heat source/sink. The velocity, temperature and concentration distributions are derived, discussed numerically and their profiles for various values of physical parameters are shown through graphs. The coefficient of skin-friction and Nusselt number at the plate is derived discussed numerically and their numerical values for various values of physical parameters are presented through the tables.

Key words: Unsteady, MHD, Radiation, free convection, Hall current, mass transfer, porous medium, source/sink.
2000 Mathematics subject classification: 76S05 and 76W05

1. Introduction

Process involving coupled heat and mass transfers occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two in different geophysical cases etc. In many processes of industries, such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry, the cooling of threads or sheet of some polymer materials have importance in the production line. The rate of cooling can be controlled effectively to achieve final products of desired characteristics by drawing threads. The fabrication of microelectronic devices, micro lithography, creating efficient means of heat and mass transfer in micro scale devices, nuclear and aerospace applications are practical usages of such results.

The thermal radiation effects on free convection flow are important in context of space technology and in certain applications involving heat storage in aquifiers and gasification of oil and in the case of gasification large temperature gradient exist in the neighborhood of the combustion from the radiation. These types of problems are also extended in the case of magneto-hydrodynamics and if the strength of magnetic field is strong, then the effect of Hall-current cannot be neglected. Many researchers have studied radiative heat transfer in fluid flow through porous media.

In space technology, radiation effect at higher operating temperatures is quite complicated. Many aspects of its effect on free convention or combined convection with radiation and mass transfer have not been studied in recent years. However, Cogley et al[1] worked out that, in the optically thin limit for a gray gas near equilibrium, the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4\left(T - T_w\right) I \ , \tag{1.1}$$

where

$$I = \int_0^\infty k_{\lambda w} \left(\frac{\partial e_{b\lambda}}{\partial T} \right)_w d\lambda \, , \tag{1.2}$$

T is the temperature of the fluid, T_w is the wall temperature, q_r is the radiative heat flux, $k_{\lambda w}$ is the absorption coefficient, and $e_{b\lambda}$ is Plank's function. Grief et al[2] showed that for an optically thin limit, the fluid does not absorb its own emitted radiation this means that there is no self-absorption, but the fluid absorbs radiation emitted by the boundaries.

Free convection flow along a vertical flat plate embedded in a porous medium with application to heat transfer was discussed by Cheng and Minkowycz[3]. Bharali and Borkakati[4] analyzed the effects of Hall currents MHD flow and heat transfer between two parallel porous plates. Singh[5] investigated the Hall effects on MHD free convection flow past an accelerated vertical porous plate. The role of magnetic field on transient forced and free convection flow past an infinite vertical porous plate through porous medium with heat source was presented by Jha[6]. Chamkha[7] studied the MHD free convection from a vertical plate embedded in a thermally stratified porous medium with Hall current. The magnetic field effects on the free convection flow through porous medium due to an infinite vertical plate with uniform suction and constant heat flux was studied by Bodosa and Borkakati[8]. The mass transfer effect on unsteady three-dimensional flow and heat transfer along an infinite vertical porous surface bounded by porous medium was analyzed by Sharma and Mishra[9]. Badruddin et al[10] investigated the free convection and radiation characteristics for a vertical plate embedded in a porous medium. The unsteady flow and heat transfer along a porous vertical surface bounded by porous medium was studied by Sharma and Mishra[11]. The MHD flow past a steadily moving infinite vertical porous plate with mass transfer and constant heat flux was discussed by Raptis and Soundalgekar[12]. Mbeledogu et al[13] presented the unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. The Hall effect on MHD mixed convection flow of a viscous incompressible fluid past a vertical porous plate immersed in porous medium with heat source/sink was studied by Sharma et al[14]. The heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer was presented by Mbeledogu and Ogulu[15]. Effect of oscillatory suction and heat source on heat and mass transfer in MHD flow along a vertical moving porous plate bounded by porous medium was studied by Sharma and Sharma[14].

The aim of the present paper is to investigate the radiation effects on unsteady free convective flow with Hall current and mass transfer through a viscous incompressible electrically conducting fluid past an infinite vertical porous non-conducting flat plate immersed in porous medium, in the presence of transverse magnetic field with heat source/sink.

2. Mathematical Formulation

Consider the unsteady free convection flow of a viscous incompressible and electrically conducting fluid past an infinite vertical porous, plate coinciding with the plane $y^* = 0$. The x^*-axis is taken along the plate and y^*-axis normal to it and z^*-axis normal to $x^* y^*$–plane. A uniform magnetic field of intensity B_0 is assumed to be applied in the direction of y^*-axis, and the plate is taken as electrically non-conducting and this porous plate is immersed in porous medium.

The magnetic field $\vec{H}$ has components (H_{x^*}, H_{y^*}, H_{z^*}) and the solenoidal relation or divergence equation of magnetic field $\nabla.\vec{H} = 0$ (From Maxwell's electromagnetic field equation) gives, (2.1) $$\frac{\partial H_{y^*}}{\partial y^*} = 0 .$$

The induced magnetic field of the flow is negligible in comparison with the applied one which corresponds to very small magnetic Reynolds number. Hence

(2.2) $H_{x^*} = H_{z^*} = 0$ and $H_{y^*} = B_0$ (constant) .

Now using the equation of conservation of electric charge $\nabla.\vec{J} = 0$ for the current density $\vec{J} \equiv \left(J_{x^*}, J_{y^*}, J_{z^*}\right)$; we get

(2.3) J_{y^*} = constant.

Since the plate is non-conducting, $J_{y^*} = 0$ at the plate hence, it is zero everywhere in the flow.
Now, neglecting the polarization effect, we get

(2.4) $\vec{E} = \vec{0}$.

Now, the Hall effect is taken into account, the generalized Ohm's law is given by

(2.5) $$\vec{J} + \frac{\omega_e \tau_e}{B_0}\left(\vec{J} \times \vec{H}\right) = \sigma\left(\vec{E} + \vec{V} \times \vec{H} + \frac{1}{e\eta_e}\nabla p_e\right),$$

where $\vec{V} = \left(u^*, v^*, w^*\right)$ is velocity vector, ω_e electron frequency , τ_e electron collision time, e electron charge, η_e number density of electron, p_e electron pressure, σ electrical conductivity, $\vec{E}$ electric field, $\vec{H}$ magnetic field vector and $\vec{J}$ is the current density vector.
Under the usual assumptions that the electron pressure p_e (for weak ionized gas), the thermo-electric pressure, the polarization effect and the ion slip are negligible. Further, it is also assumed that $\omega_e \tau_e \approx 0$ and $\omega_i \tau_i \leq 1$, where ω_i is the frequency of ions and τ_i the collision time of ions.
Now, the equation (2.5) gives

(2.6) $J_{x^*} = \frac{\sigma B_0}{1+m^2}\left(m u^* - w^*\right)$, and $J_{z^*} = \frac{\sigma B_0}{1+m^2}\left(u^* + m w^*\right)$,

where $m = \omega_e \tau_e$ is the Hall parameter.

The temperature in the fluid flowing is governed by the energy conservation equation involving radiative heat transfer with heat source/sink. Within the framework of these assumptions, the equations which govern the free convection flow of an electrically conducting fluid with radiation effects under usual Boussinesq approximation are

Equations of continuity

(2.7) $\nabla.\vec{V} = 0 \quad \Rightarrow \quad v^* = -V_0$ (constant),

Equations of Motion

(2.8) $$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho(1+m^2)}\left(u^* + m w^*\right) + g\beta\left(T^* - T_\infty^*\right) + g\beta^*\left(C^* - C_\infty^*\right) - \frac{\upsilon u^*}{K^*},$$

(2.9) $$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \upsilon \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\sigma B_0^2}{\rho(1+m^2)}\left(m u^* - w^*\right) - \frac{\upsilon w^*}{K^*}.$$

Equation of Energy

(2.10) $$\frac{\partial\left(T^* - T_\infty^*\right)}{\partial t^*} + v^* \frac{\partial\left(T^* - T_\infty^*\right)}{\partial y^*} = \frac{\kappa}{\rho\, C_p} \frac{\partial^2\left(T^* - T_\infty^*\right)}{\partial y^{*2}} + S^*\left(T^* - T_\infty^*\right) - \frac{1}{\rho\, C_p}\frac{\partial q^*}{\partial y^*},$$

Heat Flux Equation

(2.11) $$\frac{\partial q^*}{\partial y^*} = 4\left(T^* - T_\infty^*\right) I,$$

Equation of Concentration

(2.12) $$\frac{\partial\left(C^* - C_\infty^*\right)}{\partial t^*} + v^* \frac{\partial\left(C^* - C_\infty^*\right)}{\partial y^*} = D\, \frac{\partial^2\left(C^* - C_\infty^*\right)}{\partial y^{*2}},$$

where g is the acceleration due to gravity, C^* the species concentration, ρ the fluid density, υ the kinemetic viscosity, κ the thermal conductivity, C_p the specific heat at constant pressure, D the chemical and molecular diffusivity, K^* the permeability of porous medium, S^* the heat source/sink parameter, q^* the radiative heat flux. $T_\infty{}^*$ the temperature at equilibrium and t^* is the time.

Now from the equation (12) and (13), we get

$$\text{(2.13)}\quad \frac{\partial\left(T^* - T^*_\infty\right)}{\partial t^*} + v^* \frac{\partial\left(T^* - T^*_\infty\right)}{\partial y^*} = \frac{\kappa}{\rho\, C_p}\frac{\partial^2\left(T^* - T^*_\infty\right)}{\partial y^{*2}} + S^*\left(T^* - T^*_\infty\right) - \frac{4\, I\left(T^* - T^*_\infty\right)}{\rho\, C_p}\,.$$

In the energy equation, the viscous dissipation and Ohmic dissipation are neglected and in the concentration equation, the term due to chemical reactions assumed to be absent.

Now using,

$$T^*(y^*, t^*) - T^*_\infty = \theta^*(y^*, t^*) \quad \text{and} \quad C^*(y^*, t^*) - C^*_\infty = C'(y^*, t^*)\,,$$

the initial and boundary conditions are given by

when $t \le 0$: $\quad u^*(y^*, t^*) = w^*(y^*, t^*) = 0, \quad \theta^* = 0, \quad C' = 0\ ;\ \forall\ y^*,$

when $t \ \geq\ 0$: $\quad \boldsymbol{y^* = 0,}\ u^*(y^*, t^*) = w^*(y^*, t^*) = 0,\ \boldsymbol{\theta^* = \theta_w\, e^{i\omega^* t^*},}$

$\boldsymbol{C' = C_w\, e^{i\omega^* t^*}.}$

$$\text{(2.14)}\qquad y^* \to \infty : u^*(y^*, t^*) = w^*(y^*, t^*) = 0,\ \theta^*(y^*, t^*) = 0\,, C'(y^*, t^*) = 0\,.$$

Introducing the following non-dimensional quantities

$$\eta = \frac{V_0\, y^*}{\upsilon},\ t = \frac{V_0^2\, t^*}{4\upsilon},\ u = \frac{u^*}{V_0},\ w = \frac{w^*}{V_0},\ \theta = \frac{\theta^*}{\theta_w},\ C = \frac{C'}{C_w},\ K = \frac{V_0^2\, K^*}{4\upsilon^2},$$

$$\text{(2.15)}\qquad S = \frac{S^*\,\upsilon}{4V_0^2},\ \ \omega = \frac{4\omega^*\,\upsilon}{V_0^2},\ \ M = \frac{4\sigma B_0^2\,\upsilon}{V_0^2\,\rho},\ \ Gr = \frac{4 g\beta\theta_w\,\upsilon}{V_0^3},$$

$$Gc = \frac{4 g\,\beta^*\, C_w\,\upsilon}{V_0^3},\quad N = \frac{16\upsilon\, I}{V_0^2\ \rho C_p},\quad Pr = \frac{\rho C_p\,\upsilon}{\kappa},\ \ Sc = \frac{\upsilon}{D},$$

into the equations (2.8), (2.9), (2.12) and (2.13), we have

$$\frac{\partial u}{\partial t} - 4\frac{\partial u}{\partial \eta} = 4\frac{\partial^2 u}{\partial \eta^2} - \frac{M}{(1+m^2)}\left(u + mw\right) + Gr\,\theta + Gc\,C - \frac{u}{K}, \tag{2.16}$$

$$\frac{\partial w}{\partial t} - 4\frac{\partial w}{\partial \eta} = 4\frac{\partial^2 w}{\partial \eta^2} - \frac{M}{(1+m^2)}\left(mu - w\right) - \frac{w}{K}, \tag{2.17}$$

$$\frac{\partial \theta}{\partial t} - 4\frac{\partial \theta}{\partial \eta} = \frac{4}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + \theta(S - N), \tag{2.18}$$

$$\frac{\partial C}{\partial t} - 4\frac{\partial C}{\partial \eta} = \frac{4}{Sc}\frac{\partial^2 C}{\partial \eta^2}, \tag{2.19}$$

where ω is the frequency of vibration of the fluid particle, M the Hartmann number, Gr the Grashof number for heat transfer, Gc the Grashof number for mass transfer, N the radiation parameter, Pr the Prandtl number and Sc the Schmidt number.

The initial and boundary conditions are given by

$$\text{when } t \le 0: \quad u = w = 0,\ \theta = 0,\ C = 0; \quad \forall \eta,$$

$$\text{when } t \ge 0: \quad \eta \to 0 : u = w = 0,\ \theta = e^{i\omega t},\ C = e^{i\omega t}; \tag{2.20}$$

$$\eta \to \infty : u \to 0,\ w \to 0, \theta \to 0, C \to 0.$$

3. Method of Solution

On combining the equations (2.16)and (2.17) and using the complex variable, $\psi = u + i\,w$,

We have

$$\frac{\partial^2 \psi}{\partial \eta^2} - \frac{1}{4}\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial \eta} - \frac{1}{4}\left[\frac{M}{(1+m^2)}\left(1 - i\,m\right) + \frac{1}{K}\right]\psi = -\frac{1}{4}Gr\,\theta - \frac{1}{4}\,Gc\,C. \tag{3.1}$$

Now the corresponding boundary conditions (2.20) are converted to

$$\eta = 0 : \psi = 0,\ \theta = e^{i\omega t},\ C = e^{i\omega t}; \tag{3.2}$$

$$\eta \to \infty : \psi \to 0, \theta \to 0\ C \to 0.$$

Substituting $\theta(\eta,t)=e^{i\omega t} f(\eta)$ and $C(\eta,t)=e^{i\omega t} g(\eta)$ into the equations (2.18) and (2.19), respectively, we get

(3.3) $$f''(\eta) + \Pr\, f'(\eta)+\left\{\frac{\Pr}{4}(S-N)-\frac{i\,\omega\Pr}{4}\right\} f(\eta)=0,$$

and

(3.4) $$g''(\eta) + Sc\, g'(\eta)+\frac{i\,\omega Sc}{4} g(\eta)=0.$$

Hence, the corresponding boundary conditions are reduced to

(3.5) $$\eta=0\ :\ f(\eta)=1,\ g(\eta)=1;$$

$$\eta\to\infty : f(\eta)\to 0,\ g(\eta)\to 0.$$

Equations (3.3) and (3.4) are linear ordinary differential equations with constant coefficients and solved by usual method using boundary conditions (3.5). Through straight forward algebra, the solutions of f(η) and g(η) are known and given by

(3.6) $$f(\eta)=e^{\frac{1}{2}\left\{-\Pr-\sqrt{\Pr^2+i\omega\Pr-(S-N)\Pr}\right\}\eta},$$

(3.7) $$g(\eta)=e^{\frac{1}{2}\left\{-Sc-\sqrt{Sc^2+i\omega Sc}\right\}\eta}.$$

Hence $\theta(\eta,t)$ and $C(\eta,t)$ are known, finally their expressions are obtained in the form given below

(3.8) $$\theta(\eta,t)=\theta_r(\eta,t)+i\,\theta_i(\eta,t) \quad \text{and} \quad C(\eta,t)=C_r(\eta,t)+i\,C_i(\eta,t),$$

where $$\theta_r(\eta,t)=e^{-\frac{\eta}{2}\left\{\Pr+A_1\cos\frac{\theta_1}{2}\right\}}\cos(\omega t-\frac{\eta}{2}A_1\sin\frac{\theta_1}{2}),$$

$$\theta_i(\eta,t)=e^{-\frac{\eta}{2}\left\{\Pr+A_1\cos\frac{\theta_1}{2}\right\}}\sin(\omega t-\frac{\eta}{2}A_1\sin\frac{\theta_1}{2}),$$

$$C_r(\eta,t)=e^{-\frac{\eta}{2}\left\{Sc+A_2\cos\frac{\theta_2}{2}\right\}}\cos(\omega t-\frac{\eta}{2}A_2\sin\frac{\theta_2}{2}),$$

$$C_i(\eta,t)=e^{-\frac{\eta}{2}\left\{Sc+A_2\cos\frac{\theta_2}{2}\right\}}\cos(\omega t-\frac{\eta}{2}A_2\sin\frac{\theta_2}{2}),$$

$$\theta_1=\tan^{-1}\frac{\omega}{\{\Pr-(S-N)\}},\qquad \theta_2=\tan^{-1}\frac{\omega}{Sc},$$

$$A_1=\Pr^{1/2}\left[\{\Pr-(S-N)\}^2-\omega^2\right]^{1/4},\qquad A_2=Sc^{1/2}\left[Sc^2-\omega^2\right]^{1/4}.$$

Substituting

$$\psi(\eta,t)=e^{i\omega t}F(\eta), \tag{3.9}$$

into equation (3.1), we have

$$F''(\eta)+F'(\eta)-\frac{1}{4}(A_3+iA_4)F(\eta)=-\frac{1}{4}Gr\,\theta e^{i\omega t}-\frac{1}{4}Gc\,C\,e^{i\omega t}, \tag{3.10}$$

where $A_3=\dfrac{M}{(1+m^2)}+\dfrac{1}{K}$ and $A_4=\omega-\dfrac{Mm}{(1+m^2)}$.

The corresponding boundary conditions (3.2) are reduced to

$$\begin{aligned}&\eta=0\ :\ F(\eta)=0,\\&\eta\to\infty : F(\eta)\to 0.\end{aligned} \tag{3.11}$$

Equations (3.10) is a linear ordinary differential equation with constant coefficients and solved by usual method under the boundary conditions (3.11). Hence solving the equation (3.10) with the use of (3.8), we have

$$\begin{aligned}u(\eta,t)=&e^{-\lambda_{2r}\eta}\left\{A_{21}\cos(\omega t-\lambda_{2i}\eta)+A_{22}\sin(\lambda_{2i}\eta-\omega t)\right\}\\&-A_{15}e^{-A_7\eta}\left\{A_9\cos(3\omega t-\eta A_8)+A_{10}\sin(3\omega t-\eta A_8))\right\}\\&-A_{18}e^{-A_{11}\eta}\left\{A_{13}\cos(3\omega t-\eta A_{12})+A_{14}\sin(3\omega t-\eta A_{12}))\right\},\end{aligned} \tag{3.12}$$

and

$$\begin{aligned}w(\eta,t)=&e^{-\lambda_{2r}\eta}\left\{A_{21}\sin(\omega t-\lambda_{2i}\eta)+A_{22}\cos(\lambda_{2i}\eta-\omega t)\right\}\\&-A_{15}e^{-A_7\eta}\left\{A_9\sin(3\omega t-\eta A_8)-A_{10}\cos(3\omega t-\eta A_8))\right\}\\&-A_{18}e^{-A_{11}\eta}\left\{A_{13}\sin(3\omega t-\eta A_{12})+A_{14}\cos(3\omega t-\eta A_{12}))\right\},\end{aligned} \tag{3.13}$$

where

$$A_5=\left\{\left(1+A_3\right)^2+A_4^2\right\}^{1/4}\cos\frac{\theta_3}{2},\quad A_6=\left\{\left(1+A_3\right)^2+A_4^2\right\}^{1/4}\sin\frac{\theta_3}{2},$$

$$A_7=\frac{1}{2}\left\{\Pr+A_1\cos\frac{\theta_1}{2}\right\},\quad A_8=\frac{A_1}{2}\sin\frac{\theta_1}{2},\ A_9=-A_8^2+A_7^2-A_7-\frac{A_3}{4},$$

$$A_{10}=2A_7A_8-A_8-\frac{A_4}{4},\quad A_{11}=\frac{1}{2}\left\{Sc+A_2\cos\frac{\theta_2}{2}\right\},$$

$$A_{12}=\frac{A_2}{2}\sin\frac{\theta_2}{2},\quad A_{13}=-A_{12}^2+A_{11}^2-A_{11}-\frac{A_3}{4},$$

$$A_{14}=2A_{11}A_{12}-A_{12}-\frac{A_4}{4},\quad A_{15}=\frac{1}{4}\frac{Gr}{\left(A_9^2+A_{10}^2\right)},$$

$$A_{16}=A_9\cos(2\omega t)+A_{10}\sin(2\omega t),$$

$$A_{17}=A_9\sin(2\omega t)-A_{10}\cos(2\omega t),\ A_{18}=\frac{1}{4}\frac{Gc}{\left(A_{13}^2+A_{14}^2\right)},$$

$$A_{19}=A_{13}\cos(2\omega t)+A_{14}\sin(2\omega t),\quad A_{20}=A_{13}\sin(2\omega t)-A_{14}\cos(2\omega t),$$

$$A_{21}=A_{15}A_{16}+A_{18}A_{19},\ A_{22}=A_{15}A_{17}+A_{18}A_{20},$$

$$\theta_3=\tan^{-1}\frac{A_4}{\left(1+A_3\right)},\ \lambda_{2r}=\frac{1}{2}+\frac{A_5}{2},\ \lambda_{2i}=\frac{A_6}{2}.$$

4. Skin-Friction Coefficient

The coefficient of the skin-friction at the plate along the x-axis is given by

(4.1)
$$\begin{aligned}C_{fx}=\left(\frac{\partial u}{\partial\eta}\right)_{\eta=0}=&-\lambda_{2r}\left\{A_{21}\cos\omega t+A_{22}\sin\left(-\omega t\right)\right\}\\&+\lambda_{2i}\left\{A_{21}\sin\omega t+A_{22}\cos\left(-\omega t\right)\right\}\\&+A_7A_{15}\left\{A_9\cos 3\omega t+A_{10}\sin 3\omega t\right\}\\&-A_8A_{15}\left\{A_9\sin 3\omega t+A_{10}\cos 3\omega t\right\}\\&+A_{18}A_{11}\left\{A_{13}\cos 3\omega t+A_{14}\sin 3\omega t\right\}\\&-A_{18}A_{12}\left\{A_{13}\sin 3\omega t-A_{14}\cos 3\omega t\right\}.\end{aligned}$$

And the coefficient of the skin-friction at the plate along the z-axis is given by

$$(4.2) \qquad C_{fz} = \left(\frac{\partial w}{\partial \eta}\right)_{\eta=0} = -\lambda_{2r}\left\{A_{21}\sin\omega t + A_{22}\cos(-\omega t)\right\}$$
$$+\lambda_{2i}\left\{-A_{21}\cos\omega t - A_{22}\sin(-\omega t)\right\}$$
$$+A_7 A_{15}\left\{A_9 \sin 3\omega t - A_{10}\cos 3\omega t\right\}$$
$$-A_8 A_{15}\left\{-A_9\cos 3\omega t - A_{10}\sin 3\omega t\right\}$$
$$+A_{18}A_{11}\left\{A_{13}\sin 3\omega t - A_{14}\cos 3\omega t\right\}$$
$$-A_{18}A_{12}\left\{-A_{13}\cos 3\omega t - A_{14}\sin 3\omega t\right\}.$$

Table- 1. Numerical values of coefficient of skin-friction at plate along the x-axis and z-axis when N=1, M=1, K=1, ω=10, ωt= π/6, and S=1.

	Pr	Gr	Gc	Sc	m	C_{fx}	C_{fz}
I	0.71	5	5	0.78	0.5	0.558861	0.670311
II	7.0	5	5	0.78	0.5	0.321004	0.462168
III	0.71	10	5	0.78	0.5	0.844143	1.009675
IV	0.71	5	10	0.78	0.5	0.832441	1.001259
V	0.71	5	5	2.62	0.5	0.416167	0.559361
VI	0.71	5	5	0.78	1.0	0.566389	0.669505

Table- 2. Numerical values of coefficient of skin-friction at plate along the x-axis and z-axis when N=1, M=1, K=1, ω=10, ωt= π/6, and S = -1.

	Pr	Gr	Gc	Sc	m	C_{fx}	C_{fz}
I	0.71	5	5	0.78	0.5	0.540616	0.679850
II	7.0	5	5	0.78	0.5	0.315988	0.469695
III	0.71	10	5	0.78	0.5	0.807652	1.028753
IV	0.71	5	10	0.78	0.5	0.814196	1.010799
V	0.71	5	5	2.62	0.5	0.397921	0.568900
VI	0.71	5	5	0.78	1.0	0.548100	0.679403

5. Nusselt Number

The rate of heat transfer in terms of Nusselt number at the vertical, porous plate is given by

$$Nu = -\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} = -\left[-\frac{1}{2}\left(\Pr + A_1\cos\frac{\theta_1}{2}\right)\cos\omega t + \frac{1}{2}A_1\sin\frac{\theta_1}{2}\sin\omega t\right] - i\left[-\frac{1}{2}\left(\Pr + A_1\cos\frac{\theta_1}{2}\right)\sin\omega t - \frac{1}{2}A_1\sin\frac{\theta_1}{2}\cos\omega t\right]. \tag{5.1}$$

Table-3. Numerical values of Nusselt number at the plate for various values of physical Parameters

	Pr	N	ω	ωt	Nu (S = 1)	Nu (S = -1)
I	0.71	1	10	π/6	0.69904	0.829036
II	7.0	1	10	π/6	5.517515	5.886100
III	0.71	3	10	π/6	0.829036	0.957460
IV	0.71	1	20	π/6	0.828658	0.920403
V	0.71	1	10	π/3	-0.119958	0.004383

6. Results and Discussion

A study of the velocity field, variations of temperature, skin-friction coefficient at the plate in unsteady MHD free convective flow with Hall current and mass transfer through a viscous incompressible electrically conducting fluid in the presence of thermal radiation and heat source/sink is carried out in the present paper. Approximate solutions of velocity and temperature distributions are obtained for viscous flow parameters. In order to set physical insight of the flow into the problem, the velocity, temperature, skin-friction coefficient at the plate are discussed by assigning numerical values of the Hall parameter m, Grashof number for heat transfer Gr, modified Grashof number for mass transfer Gm, Prandtl number Pr, Schmidt number Sc, Radiation parameter N and Hartmann number M. The values of Pr are taken 0.71 and 7.0 of the fluid for air and water respectively. The values of Sc are taken 0.78 and 2.62 for NH_3 and propel benzene in air, respectively. The positive and negative values of S represent heat source and sink, respectively.

Fig.1 shows that the axial fluid velocity increases with the increase of Hall parameter, modified Grashof number or Grashof number, while it decreases with the increase of Schmidt number or Prandtl number in the presence of heat source. Fig.2 depicts that the axial fluid velocity increases with the increase of Hall parameter, modified Grashof number or Grashof number, while it decreases with the increase of Schmidt number or Prandtl number in the presence of heat sink.

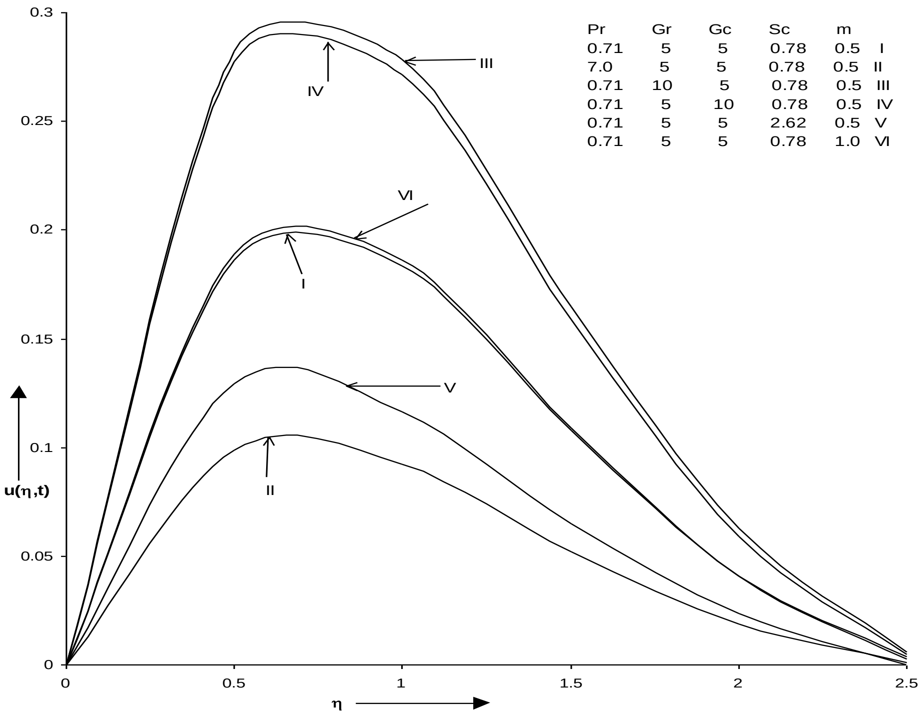

Fig.1. Axial velocity distribution versus η when N =1, M =1, K = 1, ω = 10, $\omega t = \pi/6$ and S =1.

Fig.3 reflects that the axial fluid velocity increases with the increase of Hall parameter or the permeability of porous medium, while it decreases with the increase of radiation parameter, Hartmann number or frequency of vibration of the fluid particles in the presence of heat source. Further, the axial velocity of the fluid decreases near the plate and then increases due to increase in the phase angle.Fig.4 depicts that axial fluid velocity increases with the increase of Hall parameter or the permeability of porous medium, while it decreases with the increase of radiation parameter, Hartman number or frequency of vibration of fluid particles in the presence of heat sink. Further, the axial velocity of the fluid decreases near the plate and then increases due to the increase in the phase angle.

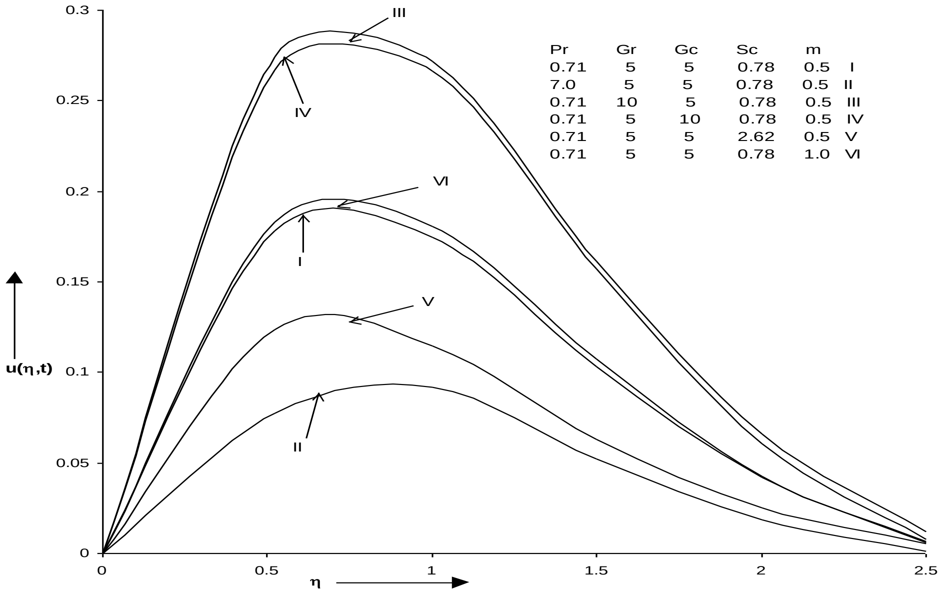

Fig.2. Axial velocity distribution versus η when N =1, M =1, K = 1, ω = 10, ωt = π /6 and S = -1.

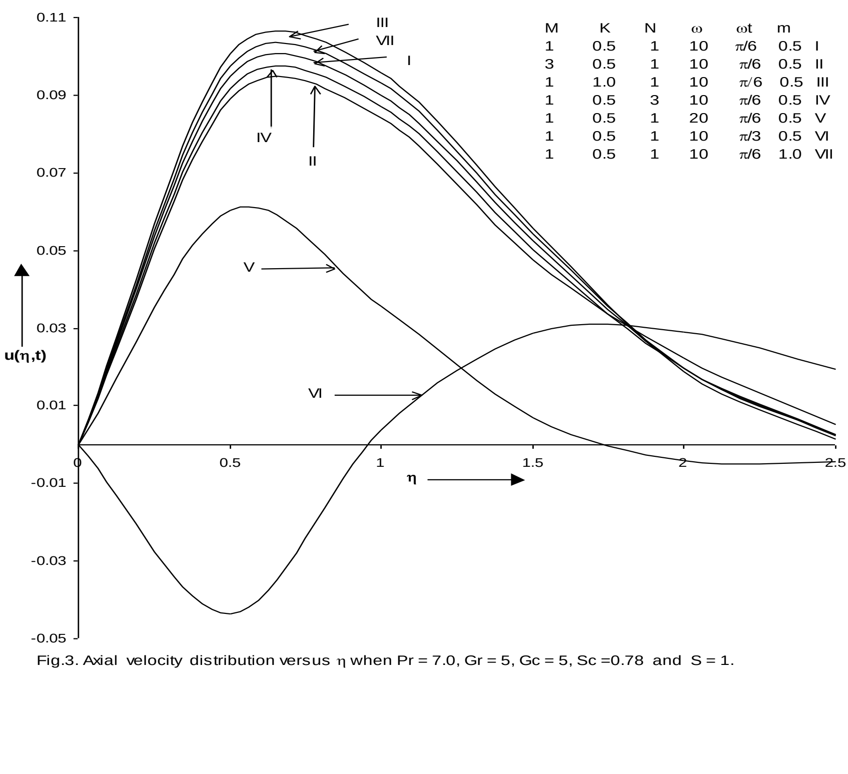

Fig.3. Axial velocity distribution versus η when Pr = 7.0, Gr = 5, Gc = 5, Sc =0.78 and S = 1.

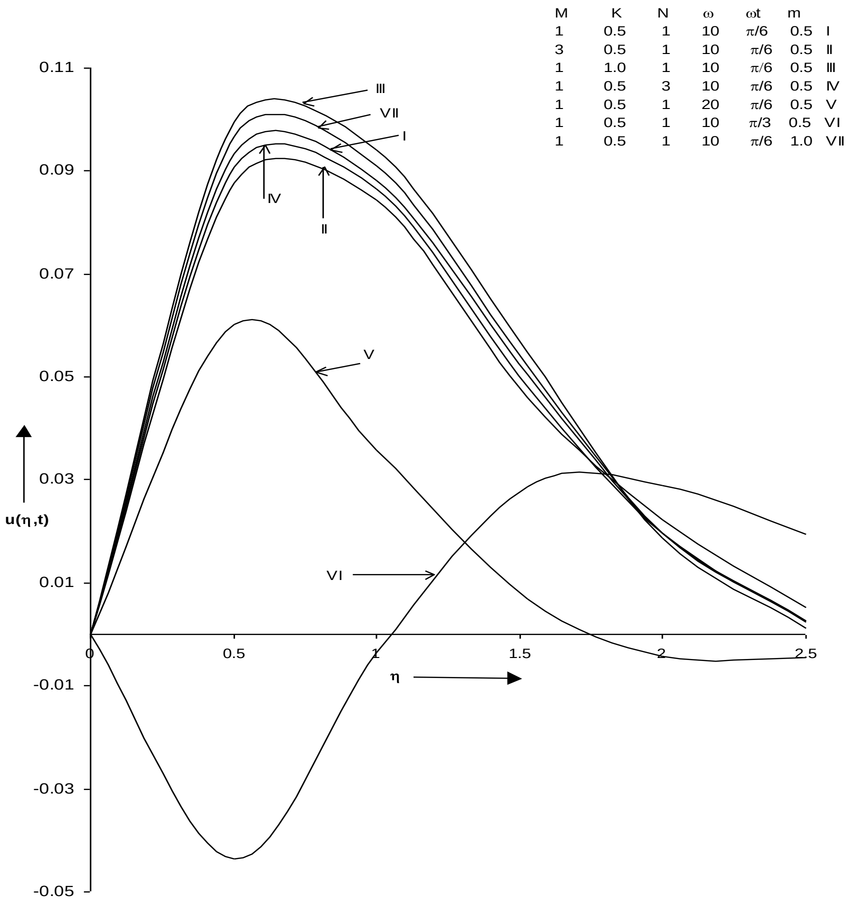

Fig.4. Axial velocity distribution versus η when Pr = 7.0, Gr = 5, Gc = 5, Sc =0.78 and S = -1.

Fig.5 shows that the normal fluid velocity increases with the increase of modified Grashof number or Grashof number, while it decreases with the increase of Hall parameter, Schmidt number or Prandtl number in the presence of heat source. Fig.6 depicts that normal fluid velocity increases with the increase of modified Grashof number or Grashof number, while it decreases due to increase of Hall parameter, Schmidt number or Prandtl number in the presence of heat sink.

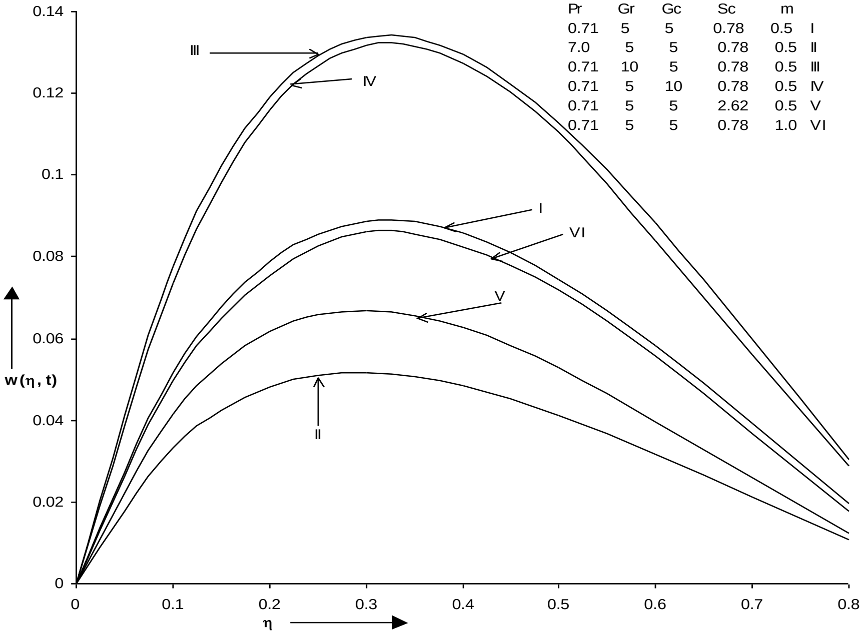

Fig.5 Normal velocity distribution versus η when N =1, M =1, K = 1, ω = 10, ωt = π/6 and S =1.

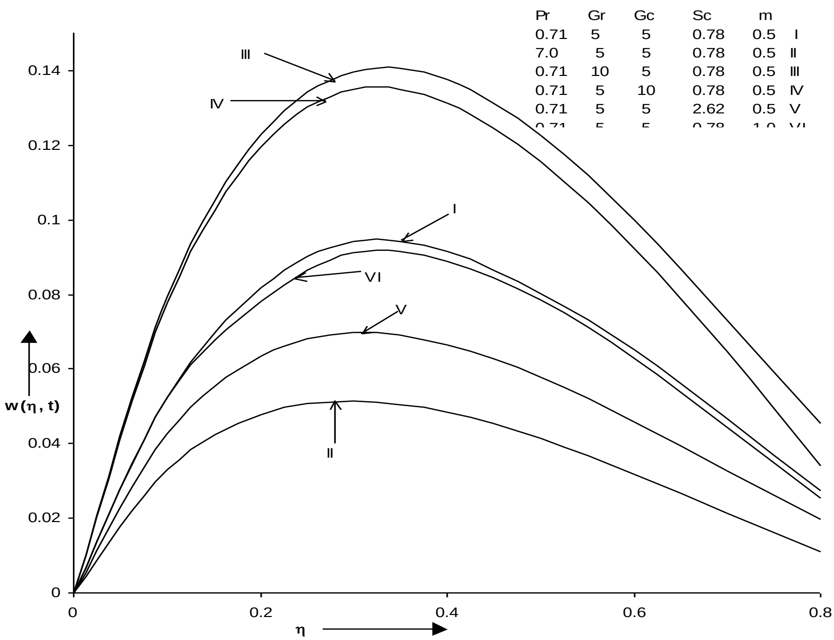

Fig.6. Normal velocity distribution versus η when N =1, M =1, K = 1, ω = 10, ωt = π/6 and S = -1.

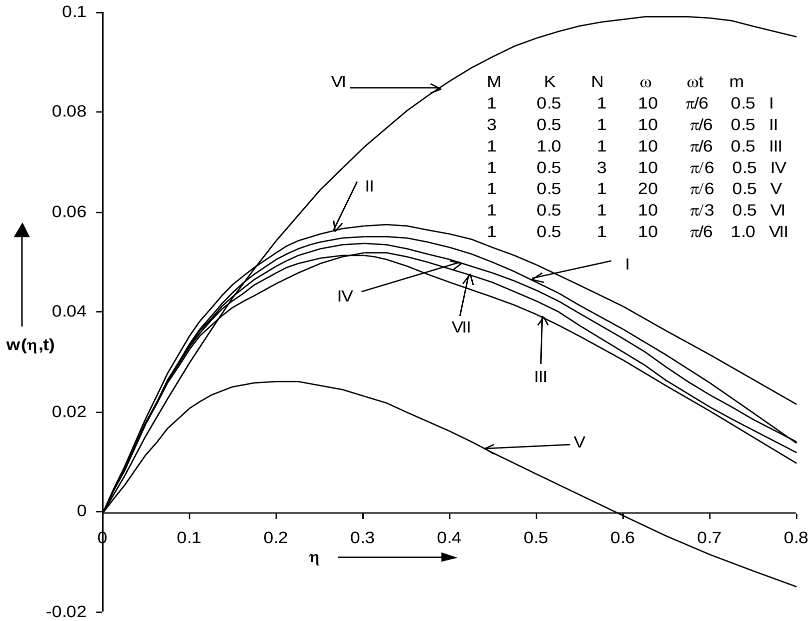

Fig.7. Normal velocity distribution versus η when Pr = 7.0, Gr = 5, Gc = 5, Sc =0.78 and S =1.

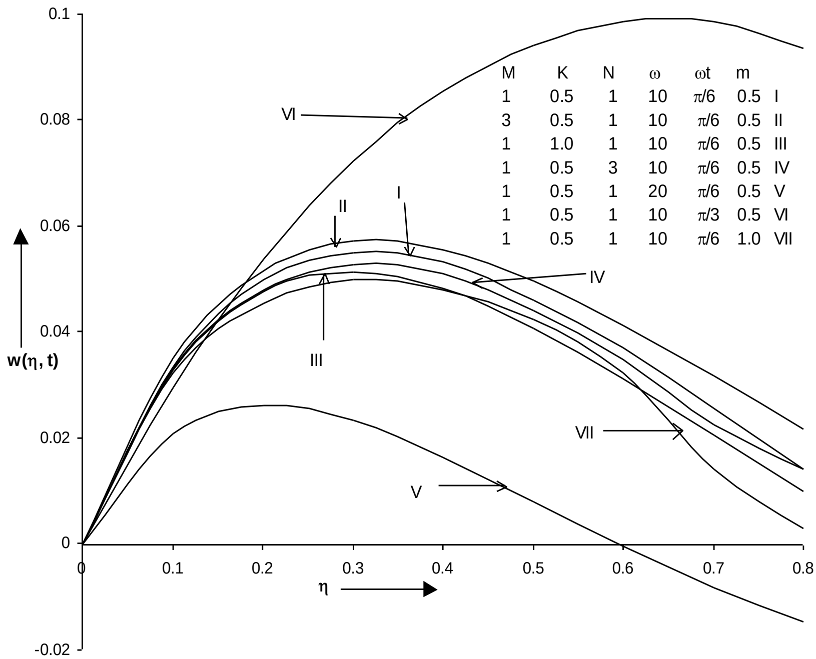

Fig.8.Normal velocity distribution versus η when Pr = 7.0, Gr = 5, Gc = 5, Sc =0.78 and S = -1.

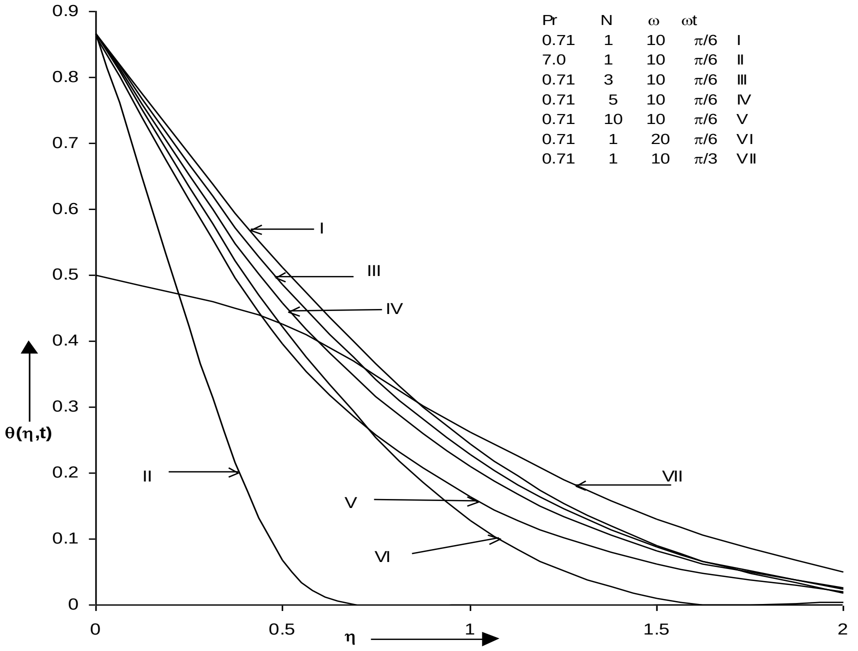

Fig.9. Temperature distribution versus η when S = 1.

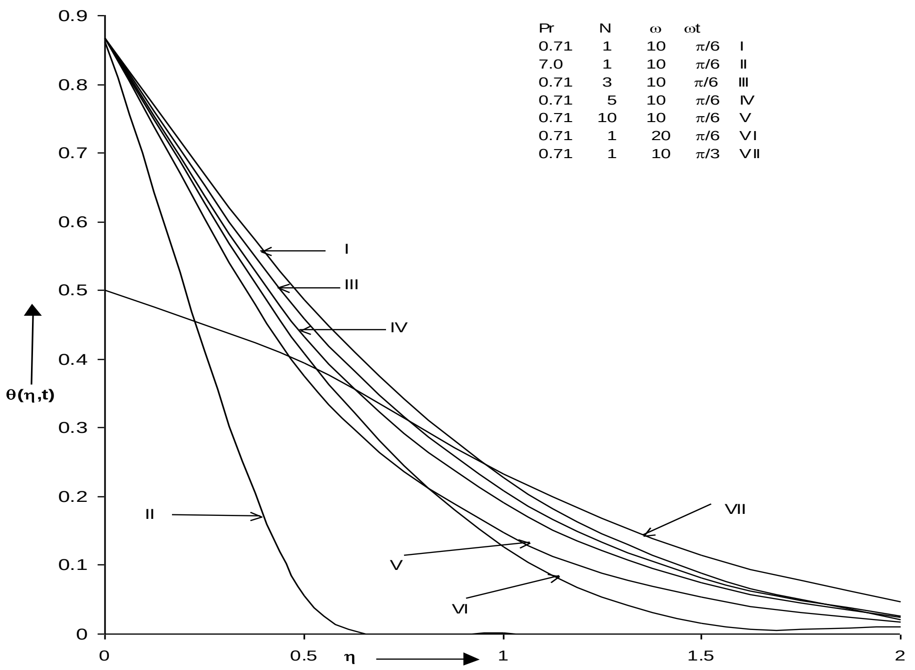

Fig.10. Temperature distribution versus η when S = -1.

It is observed from Fig.7 that the normal fluid velocity increases near the plate with the increase of Hartman number, while it decreases with the increase of radiation parameter, Hall parameter, permeability of porous medium or frequency of vibration of the fluid particles in the presence of heat source. Also sudden increase in magnitude of fluid velocity is noticed with an increase of phase angle. Fig.8 depicts that normal fluid velocity increases with the increase of Hartman number or phase angle, while it decreases with the increase of radiation parameter, Hall parameter permeability of porous medium or frequency of vibration of the fluid particles in the presence of heat sink.

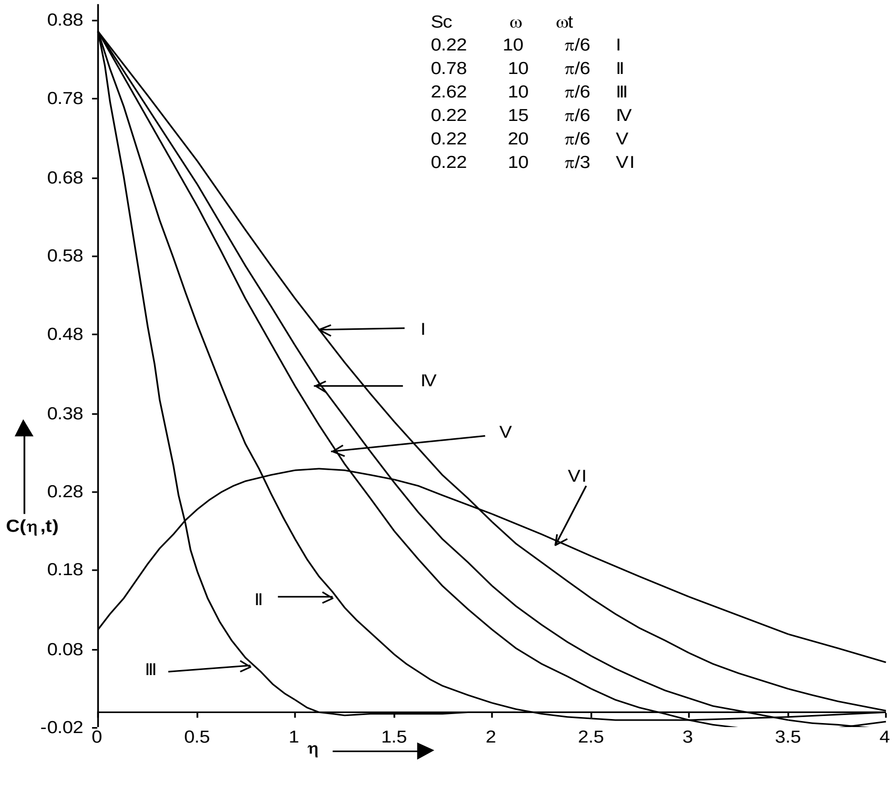

Fig.11. Concentration distribution versus η.

It is observed from Fig.9 that the fluid temperature decreases with the increase of radiation parameter, frequency of vibration of the fluid particles or Prandtl number. It is also noticed that the phase angle plays an important role to decrease or increase the fluid temperature near or away the plate in the presence of heat source. It is noticed from Fig.10 that the fluid temperature decreases with the increase of radiation parameter, frequency of vibration of the fluid particles or Prandtl number and it behaves differently as phase angle increases in the presence of heat sink.

It is observed from Fig. 11 that the fluid concentration decreases with an increase of frequency of vibration of the fluid particles or Schmidt number. The phase angle plays an important role to decrease or increase the fluid concentration near or away from the plate.

The numerical values of the coefficient of skin-friction at plate along the x-axis and z-axis in the presence of heat source or sink are shown in Table-1 and Table-2, respectively. Table-1 shows that coefficient of skin-friction at plate along the x-axis increases with the increase of the Grashof number, modified Grashof number or Hall parameter, while it decreases due to increase of Prandtl number or Schmidt number in the presence of heat source. Further the coefficient of skin-friction at plate along the z-axis increases with the increase of Grashof number or modified Grashof number, while it decreases with an increase of Prandtl number, Schmidt number or Hall- parameter in the presence of heat source.

It is noticed from Table-2 that coefficient of skin-friction at plate along the x-axis increases with the increase of the Grashof number, modified Grashof number or Hall parameter, while it decreases with an increase of Prandtl number and Schmidt number in the presence of heat sink. Further, the coefficient of skin-friction at plate along the z-axis increases with the increase of Grashof number or modified Grashof number while it decreases with the increase of Prandtl number, Schmidt number or Hall- parameter in the presence of heat sink.

The numerical values of the Nusselt number at plate in the presence of heat source or sink are shown in Table-3. It shows the Nusselt number at the plate increases with the increase of Prandtl number, frequency of vibration of the fluid particles and radiation parameter, while it decreases due to increase of phase angle in the presence of heat source. Further the Nusselt number at the plate increases with the increase of Prandtl number, frequency of vibration of the fluid particles and radiation parameter, while it decreases with the increase of phase angle in the presence of heat sink.

7. Conclusions

(i) Axial fluid velocity decreases with an increase of radiative heat or intensity of magnetic field, while it increases due to increase in buoyancy force parameter or the modified Grashof number.

(ii) The magnitude of fluid velocity as well as of temperature for air is more in comparison to water.

(iii) The phase angle plays an important role in axial velocity and temperature profiles. On increase of phase angle, the magnitude of axial velocity and temperature of the fluid near the plate decreases, while adverse behavior is observed far away from the plate.

(iv) The temperature and concentration of the fluid at the plate is maximum and approach towards boundary conditions.

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