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Rayleigh-Taylor Instability of Couple Stress Fluids in the Presence of Suspended Particles through Porous Medium

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Abstract: In this paper, we investigated Rayleigh-Taylor instability of couple stress fluids in the presence of suspended particles through porous medium.
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1. Introduction

Pradeep Kumar[1] has studied the stability of two superposed Rivlin-Ericksen elastic – viscose fluids permeated with suspended particles in porous medium. Stability of superposed viscous – visco-elastico fluids in the presence of suspended particles through porous medium has been studied by Kumar and Sharma[2]. The problem finds its usefulness in several geophysical situations and in chemical technology.

In this paper, we investigated Rayleigh-Taylor instability of couple stress fluids in the presence of suspended particles through porous medium.

2. Perturbation Equations

Here we consider a static state in which an incompressible couple stress fluids containing suspended particles is arranged in horizontal strata in a porous medium and the pressure p and the density ρ are functions of the vertical coordinate z only. The system is acted on by gravity force $\vec{g}(0,0,-g)$. Let p, ρ, μ, μ and $\vec{v}(u,v,w)$ denote respectively the pressure, density, kinematic viscosity, couple stress viscoelasticity and velocity of fluid; $\vec{u}(l,r,s)$, m and $N(\bar{x},t)$ denote the velocity, mass and number density of the particles respectively, ε is the medium porosity and $\bar{x}=(x,y,z)$.

Then the equations of motion and continuity for the couple-stress fluids containing suspended particals in a porous medium are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial\vec{v}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{v}.\nabla\right).\vec{v}\right]=-\nabla p+\rho\vec{g}+\left(v-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\vec{v}+\frac{KN}{\varepsilon}\left(\vec{u}-\vec{v}\right) \tag{2.1}$$

and

$$\nabla.\vec{v}=0, \tag{2.2}$$

where $K=6\pi\mu\eta$, η being the particle radius, is the stokes' drag coefficient.

Since the density of every fluid particle remains unchanged as we follow it with its motion, we have

$$\varepsilon\frac{\partial\rho}{\partial t}+\left(\vec{v}.\nabla\right)\rho=0. \tag{2.3}$$

Assuming uniform particle size spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term in the equation of motion (2.1), proportional to the velocity difference between particles and fluid. The force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reaction are ignored for we assume that the distances between particles are quite large compared with their diameter. The effect of pressure gravity and Dareian force on the suspended particles are negligibly small and therefore ignored under the above assumption, the equation of motion and continuity for the particles are

$$mN\left[\frac{\partial\vec{u}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{u}.\nabla\right)\vec{u}\right]=KN\left(\vec{v}-\vec{u}\right) \tag{2.4}$$

and

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.\left(N\vec{u}.\right)=0. \tag{2.5}$$

The initial state of the system is taken to be a quiescent layer (no settling) with uniform particles distribution N_0 i.e., $\vec{u}(0, 0, 0)$ and $\vec{v}(0, 0, 0)$ and $N = N_0$ is a constant.

Let $\delta\rho, \delta p, \vec{v}(u, v, w)$ and $\vec{u}(l, r, s)$ denote respectively, the pressure p, perturbations in density ρ, pressure p, of fluid and velocity of particle. Then the linearized perturbed forms of equations (2.1)-(2.5) become

(2.6) $$\frac{\rho}{\varepsilon}\frac{\partial \vec{v}}{\partial t} = -\nabla\delta p + \vec{g}\delta\rho + \left(v - w'\nabla^2\right)\nabla^2\vec{v} + \frac{KN}{\varepsilon}\left(\vec{u} - \vec{v}\right),$$

(2.7) $$\nabla.\vec{v} = 0,$$

(2.8) $$\varepsilon\frac{\partial}{\partial t}(\delta\rho) = -w(D\rho),$$

(2.9) $$\left(\frac{m}{K}\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v},$$

(2.10) $$\frac{\partial M}{\partial t} + \nabla.\vec{u} = 0,$$

where $M = \frac{\varepsilon N}{N_0}$, $F = \frac{\mu'}{\rho_0 d^2 v} = \frac{v'}{d^2}$, $D = \frac{d}{dz}$ and N_0, N, $v(= \mu/\rho)$, $v'(\mu'/\rho)$ stand for initial uniform number density, perturbations in number density, kinematic viscosity kinematic-viscoelastic respectively

Analyzing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is given by

(2.11) $$\exp\left(ik_x x + ik_y y + nt\right),$$

where k_x, k_y are horizontal wave numbers, $k^2 = k_x^2 + k_y^2$, and n is a complex constant.

For a perturbation of the form (2.11), equations (2.6)-(2.9) after eliminating $\vec{u}$, give

(2.12) $$\left[n' - \frac{1}{\rho}\left\{v - vv'\left(D^2 - k^2\right)\right\}\left(D^2 - k^2\right)\right]\rho u = ik_x\delta p,$$

(2.13) $$\left[n'-\frac{1}{\rho}\left\{v-vv'\left(D^2-k^2\right)\right\}\left(D^2-k^2\right)\right]\rho\nu=-ik_y\delta p\,,$$

(2.14) $$\left[n'-\frac{1}{\rho}\left\{v-vv'\left(D^2-k^2\right)\right\}\left(D^2-k^2\right)\right]\rho w=-D\delta p-g\delta\rho\,,$$

(2.15) $$ik_x u+ik_y\nu+Dw=0$$

and

(2.16) $$\varepsilon n\delta\rho=-wD\rho\,,$$

where $n'=\frac{n}{\varepsilon}\left(1+\frac{mN_0K/\rho}{mn+K}\right)$.
Eliminating δp between equation (2.12)-(2.14) and using (2.15) and (2.16), we obtain

(2.17) $$n'\left[D\left(\rho Dw\right)-k^2\rho w\right]+\frac{1}{\rho}\left(k^2-D^2\right)\left[D\rho vIw-k^2\rho vw\right]$$
$$+\frac{1}{\rho}\left(D^4+k^4-2D^2k^2\right)+\left[D^2\rho v\nu'w-k^2\rho v\nu'w\right]=-\frac{gk^2}{n\varepsilon}w\left(D\rho\right)$$

3. Two Superposed Couple-Stress Viscous Fluids Separated by a Horizontal Boundary

Consider the case when two superposed couple stress fluids of densities ρ_1 and ρ_2, uniform viscosities μ_1 and μ_2, uniform couple stress viscosity μ_1' and μ_2' are separated by a horizontal boundary $z=0$. The subscript 1 and 2 distinguish the upper and lower fluids, respectively. Then in each region of constant ρ and constant μ, μ' equation (2.17) becomes

(3.1) $$\left(D^2-k^2\right)w=0\,.$$

The general solution of equation (3.1) is

(3.2) $$w=Ae^{kz}+Be^{-kz}\,,$$

where A and B are arbitrary constants.
The boundary conditions to be satisfied in the present problem are:
(i) The velocity should vanish when $z \to \infty$ (for the upper fluid) and $z \to -\infty$ (for the lower fluid).
(ii) $w(z)$ is continuous at $z = 0$.
(iii) The jump condition at the interface.
Applying the boundary conditions (i) and (ii), we have

(3.3) $$\mathrm{w}_1 = \mathrm{A}e^{+kz}, (z < 0),$$

(3.4) $$w_2 = Ae^{-kz}, (z > 0),$$

where the same constant A has been chosen to ensure the continuity of w at $z = 0$.

Equation (2.17) gives the jump condition at the interface $z = 0$ as

(3.5) $$n'\Delta_0(\rho Dw) + \frac{1}{\rho}(k^2 - D^2)\Delta_0(\rho\nu Dw) + \frac{1}{\rho}(D^4 + k^4 - 2D^2k^2) \times \Delta_0(\rho\nu\nu' Dw) + \frac{gk^2}{n\varepsilon}\nabla_0(\rho)w_0 = 0.$$

Applying the condition (3.5) to the solutions (3.3) and (3.4), we obtain

(3.6) $$n^3 + n^2\left[\frac{K}{m} + \frac{N_0K}{\rho_1+\rho_2} + \frac{\varepsilon k^2}{\rho}(\alpha_2\nu_2 + \alpha_1\nu_1) - \frac{\varepsilon D^2}{\rho}(\alpha_2\nu_2 + \alpha_1\nu_1) + \varepsilon k^2 + \frac{\varepsilon D^4}{\rho}(\alpha_2\nu_2 + \alpha_1\nu_1)(\alpha_2\nu_2' + \alpha_1\nu_1') + \frac{\varepsilon k^4}{\rho}(\alpha_2\nu_2 + \alpha_1\nu_1) \times (\alpha_2\nu_2' + \alpha_1\nu_1') - \frac{2\varepsilon D^2k^2}{\rho}(\alpha_2\nu_2 + \alpha_1\nu_1)(\alpha_2\nu_2' + \alpha_1\nu_1')\right]$$
$$+ n\left[\frac{\varepsilon Kk^2}{m\rho}(\alpha_2\nu_2 + \alpha_1\nu_1) - \frac{\varepsilon KD^2}{m\rho}(\alpha_2\nu_2 + \alpha_1\nu_1) + \frac{\varepsilon KD^4}{m\rho}, \times (\alpha_2\nu_2 + \alpha_1\nu_1)(\alpha_2\nu_2' + \alpha_1\nu_1') + \frac{\varepsilon Kk^4}{m\rho}(\alpha_2\nu_2 + \alpha_1\nu_1)(\alpha_2\nu_2' + \alpha_1\nu_1')\right.$$

$$-\frac{2\varepsilon KD^2k^2}{m\rho}\left(\alpha_2 v_2+\alpha_1 v_1\right)\left(\alpha_2 v_2'+\alpha_1 v_1'\right)+gk^2\left(\alpha_1-\alpha_2\right)\Bigg]$$

$$+\frac{gKk^2}{m}\left(\alpha_1-\alpha_2\right)=0,$$

where $\alpha_{12}=\frac{\rho_1\rho_2}{\rho_1+\rho_2}, \nu_{12}=\frac{\mu_1\mu_2}{\rho_1\rho_2}, \nu_{12}'=\frac{\mu_{12}'}{\rho_{12}}$.

(a) Stable Case $(\alpha_2<\alpha_1)$**:** For the potentially stable arrangement $\alpha_2<\alpha_1$, equation (3.6) does not allow any positive root as there is no change of sign. The system is therefore stable.

(b) Unstable Case $(\alpha_2>\alpha_1)$**:** For the potentially unstable configuration, $(\alpha_2>\alpha_1)$the constant term in equation (3.6) is negative. Equation (3.6) therefore allows one positive root of n and so the system is unstable.

4. The Case of Exponentially Varying Density

Here we consider the density stratification in the fluid of depth d as

(7.4) $$\rho(z)=\rho_0 e^{\beta z},$$

where ρ_0 and β are constants. Assume that $\beta d<<1$, i.e. the variation of density at two neighboring points in the velocity field, which is much less than the average density has a negligible effect on the inertia of the fluid.

Then boundary conditions for the case of two free surfaces are

(4.2) $$w=0,\ D^2w=0 \text{ at } z=0 \text{ and } z=d$$

The proper solution of equation (2.17) satisfying (4.2) is

(4.3) $$w=w_0\sin\frac{s\pi z}{d},$$

where w_0 is a constant and s is an integer.

Substituting (4.3) in equation (2.17) and neglecting the effect of heterogeneity on the inertia, we obtain

$$
\begin{aligned}
(4.4) \qquad & n^3 + n^2 \left[\frac{K}{m}\left(1+\frac{mN_0}{\rho}\right) + \frac{\varepsilon v k^2}{\rho} + \frac{\varepsilon v\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho} + \frac{\varepsilon v v' k^4}{\rho} \right. \\
& \left. \frac{\varepsilon v v'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}^2}{\rho} + \frac{2\varepsilon v v' k^2\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho} \right] \\
& + n \left[\frac{\varepsilon K v k^2}{m\rho} + \frac{\varepsilon K v\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho} + \frac{\varepsilon K v v'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}^2}{m\rho} \right. \\
& \left. + \frac{\varepsilon K v v' k^4}{m\rho} + \frac{2k^2 \varepsilon K v v' k\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho} - - \frac{g\beta k^2}{\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}} \right] \\
& - \frac{g\beta K k^2/m}{\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}} = 0 .
\end{aligned}
$$

For the stable stratifications ($\beta < 0$), equation (4.4) does not admit of any positive root and hence the system is stable. For the unstable stratifications ($\beta > 0$), the constant term in equation (4.4) is negative. Equation (4.4), therefore, allows one positive root of n and hence the system is unstable.

Let n_0 denote the positive root of equation (4.4). Then

$$n_0^3+n_0^2\left[\frac{K}{m}\left(1+\frac{mN_0}{\rho}\right)+\frac{\varepsilon vk^2}{\rho}+\frac{\varepsilon v\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho}+\frac{\varepsilon vv'k^4}{\rho}\right.$$
$$\left.\frac{\varepsilon vv'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}^2}{\rho}+\frac{2\varepsilon vv'k^2\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho}\right]$$
$$+n_0\left[\frac{\varepsilon Kvk^2}{m\rho}+\frac{\varepsilon Kv\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho}+\frac{\varepsilon Kvv'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}^2}{m\rho}\right.$$
$$\left.+\frac{\varepsilon Kvv'k^4}{m\rho}+\frac{2k^2\varepsilon Kvv'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho}-\frac{g\beta k^2}{\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}\right]$$
$$-\frac{g\beta Kk^2/m}{\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}=0\,. \tag{4.5}$$

To study the effect of particle number density concerning the growth rates of unstable modes, we examine the natures of $\frac{dn_0}{dN_0}$ analytically.

Equation (4.5) yields

$$\frac{dn_0}{dN_0}=\frac{(K/\rho)n_0^2}{\frac{g\beta k^2}{\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}-\left[3n_0^2+2n_0\left\{\frac{K}{m}\left(1+\frac{mN_0}{\rho}\right)\right.\right.} \tag{4.6}$$

$$\frac{\varepsilon\nu k^2}{\rho}+\frac{\varepsilon\nu\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho}+\frac{\varepsilon\nu\nu' k^4}{\rho}+\frac{\varepsilon\nu\nu'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho}+\frac{2\varepsilon\nu\nu' k^2\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{\rho}\Bigg]$$

$$+\Bigg[\frac{\varepsilon K\nu k^2}{m\rho}+\frac{\varepsilon K\nu\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho}+\frac{\varepsilon K\nu\nu' k^4}{m\rho}+\frac{\varepsilon K\nu\nu'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho}$$

$$+\frac{2k^2\varepsilon K\nu\nu'\left\{k^2+\left(\frac{s\pi}{d}\right)^2\right\}}{m\rho}\Bigg].$$

It is clear from (4.6) that $\frac{dn_0}{dN_0}$ may be positive or negative.

6. Conclusion

In this chapter the stability of two superposed visco-elastic fluids is discussed. The case of suspended particles in porous medium is considered. Stable and unstable cases are discussed by equation (3.6). Further the case of exponentially varying density $\rho(z)=\rho_0 e^{\beta z}$ is considered. It has been shown that for stable stratifications ($\beta<0$) the system is stable and for unstable stratification ($\beta>0$), the system is unstable.

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