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Unsteady Hydromagnetic Free Convective Flow and Mass Transfer of an Elastico-Viscous Fluid Past an InfiniteVertical Porous Plate in a Rotating Porous Medium

Archana Shukla

Department of Mathematics
Nehru Gram Bharati (Deemed To Be University), Prayagraj, India
Email: archanashuklaald@gmail.com

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Abstract: The work reported here is to study an unsteady hydromagnetic free convection flow and mass transfer of an elastic-viscous fluid in a rotating porous medium. The elastic-viscous fluid model considered here is reported in Oldroyd[1]. The flow analysis is made under fluctuating plate temperature which varies harmonically with time between limits $T_w \pm \in \left(T_w - T_\infty\right)$ as t varies from 0 to $2\pi/\omega$ and the species concentration at the free stream is assumed to be constant. The analytical expressions for velocity, temperature and concentration are derived and flow characteristic have been discussed.

Keywords: Unsteady, Hydromagnetic, Convection, Temperature, Concentration, Plate, Flow.

2010 AMS Classification Number: 76S05.

1. Introduction

Studies on free convection flow and mass transfer find extended theoretical and physical interest in view of their varied applications in the field of astrophysical, geothermal and geophysical science such as in the study of stars and planets, atmospheric and oceanic circulations, space flights, nuclear fusion and many other scientific and technical research, convection mechanism through porous media has been employed in get filtration processes, to maintain the temperature of a body and also render the heat insulation of the surface more effective.

The effect of free convection on viscous and elastic-viscous fluids with/without mass transfer has been studied by several workers. Raptis et al.[2] have analysed the affects of free convection currents on the flow of an electrically conducting fluid past an accelerated vertical infinite plate with variable suction. Singh and Singh[3] studied the MHD flow of an elastic-viscous fluid past an accelerated plate. Singh[4] estimated the effect of free convection and magnetic field on the flow of an electrically conducting fluid past an accelerated vertical porous plate in a rotating frame. Dash and Ojha[5] have disscussd the magneto hydrodynamic unsteady free convection effect on the flow past an exponentially accelerated vertical plate. Goyal and Bansal[6] reported the unsteady boundary layer flow of an electrically conducting fluid in presence of transverse magnetic field over a flat plate. Gholam and Singh[7] investigated the combined heat and mass transfer effects on unsteady free convection flow of a fluid through a porous medium with heat source and sink for low frequency. Hadim and Chen[8] reported the non-Darcy mixed convection in a vertical porous channel.

The hydromagnetic flow and heat transfer past a continuously moving porous boundary has been studied by Chandran et al.[9]. In a subsequent paper, Chandran et al.[10] extended the above problem to unsteady flow with heat flux and accelerated boundary motion. Dash and Das[11] investigated the effect of Hall current of MHD flow along an accelerated vertical porous flat plate with mass transfer and internal heat generation. Kim[12], Israel-cookey*et al.*[13] reported the free convection and mass transfer effects on a rotating electrically conducting fluid through a porous medium in presence of magnetic field. Makinde*et al.*[14] have discussed the unsteady free convection flow with suction on an accelerating porous plate. Panda *et al.*[15] have estimated the effect of free convection and mass transfer on unsteady flow of a rotating elastics-viscous liquid through porous media past a vertical porous plate.

The work reported here is to study an unsteady hydromagnetic free convection flow and mass transfer of an elastic-viscous fluid in a rotating porous medium. The elastic-viscous fluid model considered here is reported in Oldroyd[1]. The flow analysis is made under fluctuating plate temperature which varies harmonically with time between limits $T_w \pm \in (T_w - T_\infty)$ as t varies from 0 to $2\pi/\omega$ and the species concentration at the free stream is assumed to be constant. The analytical expressions for velocity, temperature and concentration are derived and flow characteristic have been discussed.

2. Formulation of the Problem

Consider the unsteady flow of a rotating elastic viscous electrically conducting fluid past a vertical infinite porous plate in presence of uniform transverse magnetic field B_0. The fluid under consideration is a very dilute solution of non-Newtonian fluid in water. We assume that the liquid has a short relaxation time. We shall do this by imposing a general constitute relation of Oldroyd liquid. The temperature and species concentration at the free stream are constant. Again we assume that the vertical infinite porous plate rotates in unison with an elastic-viscous fluid with a constant angular velocity Ω about an axis which is perpendicular to the vertical plane surface. A Cartesian co-ordinate system is chosen such that x and y-axis respectively are in the vertical upward and perpendicular directions on the plane of the vertical porous surface $z=0$ while z-axis is normal to it. With the above frame of reference and assumptions, the physical variables, except the pressure p, are function of z and time t only. Thus the equation of continuity gives $W=-W_0$, $W_0>0$ is the constant suction velocity normal to the plate. Now, taking into account the Boussinesq approximation, the equations which govern the problem are

$$\frac{\partial u}{\partial t}-w_0\frac{\partial u}{\partial z}-2\Omega v=\frac{1}{v}\frac{\partial p'_{13}}{\partial z}+g\beta\left(T-\bar{T}_\infty\right)+g\beta*\left(C-C_\infty\right)-\frac{\nu}{k^*}u-\frac{\sigma B_0^2 u}{\rho}, \tag{2.1}$$

$$\frac{\partial v}{\partial t}-w_0\frac{\partial v}{\partial z}+2\Omega u=\frac{1}{\rho}\frac{\partial p'_{23}}{\partial z}-\frac{\nu}{k^*}v-\frac{\sigma B_0^2 u}{\rho}, \tag{2.2}$$

$$\frac{\partial T}{\partial t}-w_0\frac{\partial T}{\partial z}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial z^2}, \tag{2.3}$$

$$\frac{\partial C}{\partial t}-w_0\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}-\lambda_n C, \tag{2.4}$$

along with the boundary conditions

$$\begin{aligned}&u=0,\ v=0,\ T=T_w+\in\left(T_2-T_\infty\right)e^{i\omega t}, C=C_w \text{ at } z=0,\\&u\to 0,\ v\to 0,\ T\to T_{\infty *}C\to C_\infty \text{ as } z\to\infty,\end{aligned} \tag{2.5}$$

where the components p'_{13} and p'_{23} are expressed by the implicit relations

$$(2.6)\qquad \left(1+\lambda_1\left(\frac{\partial}{\partial t}-w_0\frac{\partial}{\partial z}\right)\right)p'_{13}=\eta_0\left[\frac{\partial u}{\partial z}+\lambda_2\left(\frac{\partial^2 u}{\partial t\partial z}-w_0\frac{\partial^2 u}{\partial z^2}\right)\right],$$

$$(2.7)\qquad \left(1+\lambda_1\left(\frac{\partial}{\partial t}-w_0\frac{\partial}{\partial z}\right)\right)p'_{13}=\eta_0\left[\frac{\partial v}{\partial z}+\lambda_2\left(\frac{\partial^2 u}{\partial t\partial z}-w_0\frac{\partial^2 v}{\partial z^2}\right)\right].$$

In equation (2.6) and (2.7), η_0, λ_1 and λ_2 (which are all positive) denote the co-efficient of viscosity, stress relaxation and strain retardation time respectively with $\lambda_1 > \lambda_2$. Also in equations (2.1)-(2.5), the symbols have their usual meanings and subscripts w and ∞ mean the condition at the porous plane surface and far away from this surface.

Eliminating p'_{13} and p'_{23} from equations (2.1) and (2.2) with the help of equations (2.6) and (2.7) putting $U = u + iv$ and introducing the dimensionless quantities

$$U'=\frac{U}{w_0},\ z'=\frac{w_0 z}{v},\ \alpha_1=\frac{\lambda_1 w_0^2}{v},\ \alpha_2=\frac{\lambda_2 w_0^2}{v},$$

$$T'=(T-T_\infty)\frac{kw_0}{qv},\ C'=\frac{C-C_\infty}{C_w-C_\infty},\ R=\frac{\Omega v}{w_0^2},$$

$$G_r=\frac{g\beta v^2 q}{kw_0^4},\ G_m=\frac{vg\beta*(C_w-C_\infty)}{w_0^3}M=\sqrt{\frac{\alpha\beta_0^2 v}{\rho w_0^2}},$$

$$P_r=\frac{\rho v c_p}{k},\ S_c=\frac{v}{D},\ K_p=\frac{w_0^2 k}{v^2},\ K_n=\frac{v}{w_0^2}\frac{\lambda_n c^n}{C_w-C_\infty},$$

Equations (2.1)-(2.4) becomes (dropping the dashes)

$$(2.8)\qquad \alpha_2\frac{\partial^3 U}{\partial z^3}-\alpha_2\frac{\partial^3 U}{\partial t\partial z^2}+(\alpha_1-1)\frac{\partial^2 U}{\partial z^2}-\left(1+2i\alpha_1 R+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)\frac{\partial U}{\partial t}$$

$$+\left(2iR+\frac{1}{K_p}+M^2\right)U-2\alpha_1\frac{\partial^2 U}{\partial t\partial z}+\alpha_1\frac{\partial^2 U}{\partial t^2}-\left(1+2i\alpha_1 R+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)\frac{\partial U}{\partial Z}$$

$$=G_l T+G_m C-\alpha_1 G_r\frac{\partial T}{\partial Z}+\alpha_1 G_m\frac{\partial C}{\partial Z}-\alpha_1 G_m\frac{\partial C}{\partial t}-\alpha_1 G_r\frac{\partial T}{\partial t},$$

(2.9)
$$\frac{\partial^2 T}{\partial z^2}+P_r\frac{\partial T}{\partial z}=P_r\frac{\partial T}{\partial t},$$

(2.10)
$$\frac{\partial^2 C}{\partial z^2}+S_c\frac{\partial C}{\partial z}=S_c\frac{\partial C}{\partial t}-K_n.$$

Subject to the boundary conditions

(2.11)
$$U=0,\ \ T=1+\varepsilon\, e^{i\omega t},\ \ C=1\ \text{ at }\ z=0,$$
$$U\to 0,\ T\to 0,\ C\to 0\ \text{ as }\ z\to\infty.$$

3. Solution

If $\alpha_1=\alpha_2=0$ then the equation (2.8) reduces to the unsteady free convection flow and mass transfer of an incompressible viscous fluid bounded by a vertical porous plate in a rotating system. In order to solve the system of equation (2.8)-(2.10) under boundary conditions (2.11), we assume the velocity U, the temperature T and concentration C near the plate as

$$U(z,t)=U_0(z)+\in\ e^{i\omega t}U_1(z),$$

(3.1)
$$U(z,t)=T_0(z)+\in\ e^{i\omega t}T_1(z),$$

$$C(z,t)=C_0(z)+\in\ e^{i\omega t}C_1(z).$$

Substituting equation (3.1) into system of equations (2.8)-(2.10) and equating harmonic and non-harmonic terms, neglecting the coefficients of $\in^2$ we get

(3.2)
$$\alpha_2 U_0''' +(\alpha_1-1)U_0'' -\left(1+2i\alpha_1 R+\alpha_1/K_p+\alpha_1 M^2\right)U_0'$$

$$+\left(2iR+1/K_p+M^2\right)U_0-G_mC_0+\alpha_1G_mC_0^{'}-G_rT_0+\alpha_1G_rT_0^{'}=0,$$

(3.3

$$\alpha_2U_1^{'''}+\left(\alpha_1-1-i\omega\alpha_2\right)U_1^{''}-\left(1+2i\alpha_1R+2i\omega\alpha_1+\alpha_1/K_p+\alpha_1M^2\right)U_1^{'}$$

$$+\left[i\left(\omega+2R+\alpha_1\omega/K_p+\alpha_1\omega M^2\right)-\omega\alpha_1\left(\omega+2R\right)+1/K_p+M^2\right]U_1$$

$$+\alpha_1G_mC_p^{'}-G_m\left(1+i\omega\alpha_1\right)C_1+\alpha_1G_rT_1^{'}-G_r\left(1+i\omega\alpha_1\right)T_1=0,$$

(3.4) $$T_0^{''}+P_rT_0^{'}=0,$$

(3.5) $$T_1^{''}+P_rT_1^{'}-P_ri\omega T_1=0,$$

(3.6) $$C_0^{''}+S_cC_0^{'}=K_n,$$

(3.7) $$C_1^{''}+S_cC_0^{'}-S_ci\omega C_1=0,$$

(3.8)

$$U_0(0)=0, T_0(0)=1,\ C_0(0)=1, U_1(0)=0, T_1(0)=1, C_1(0)=0,$$

$$U_0(\infty)\to 0, T_0(\infty)\to 0, C_0(\infty)\to 0, U_1(\infty)\to 0, T_1(\infty)\to 0, C_1(\infty)\to 0,$$

where the primes indicate the differentiation with respect to z.

Solving the equations (3.4)-(3.7) under the boundary conditions (3.8), we get

(3.9) $$T(z,t)=e^{-Prz}+\in\ e^{i\omega t}e^{-\lambda z},$$

where

$$\lambda=1/2\left[P_r+\left(P_r^2+4i\omega P_r\right)^{1/2}\right],$$

(3.10) $$C(z,t)=e^{-S_cz}.$$

Since, the equations (3.2)-(3.3) are of order three, an additional boundary condition will be required in order to get a unique solution. But, no boundary condition is physically plausible, we impose the requirement that the solution of the equation (2.8) reduces to the classical viscous case as $\alpha_1, \alpha_2 \to 0$. As will be seen, this enables to determine all the arbitrary constants appearing in the solution of equations (3.2)-(3.3). Now solving the differential equations (3.2) along with the boundary conditions of (3.8) and considering the limiting case as a viscous fluid, we get

$$U_0 = c_1 e^{m_1 z} + c_2 e^{m_2 z} + c_3 e^{m_2 z} + \frac{H_1}{M_{11}} e^{-p_1 z} + \frac{G_1}{M_{12}} e^{-S_c z}, \tag{3.11}$$

where

$$H_1 = \left(\alpha_1 P_r + 1\right) G_r, G_1 = G_m\left(1 + \alpha_1 S_c\right),$$

$$a_1 = -\frac{1}{\alpha_2}\left(1 + \frac{\alpha_1}{K_p} + \alpha_1 M^2\right) + 2iR\frac{\alpha_1}{\alpha_2},$$

$$Q = \frac{\left(1 + 2i\alpha_1^R + \frac{\alpha_1}{K_p} + \alpha_1 M^2\right)}{3\alpha_2} - \frac{\left(\alpha_1 - 1\right)^2}{9\alpha_2^2},$$

$$R_1 = \left(1 - \alpha_1\right)\frac{\left(1 + 2i\alpha_1^R + \frac{\alpha_1}{K_p} + \alpha_1 M^2\right)}{6\alpha_2^2} - \frac{\left(2iR + \frac{1}{K_p} + M^2\right)}{2\alpha_2} - \frac{\left(\alpha_1 - 1\right)^3}{27\alpha_2^3},$$

$$S_1 = -\left[\frac{1}{3\alpha_2} + \frac{\alpha_1}{3\alpha_2 K_p} + \frac{\alpha_1 M^2}{3\alpha_2} + \frac{\left(\alpha_1 - 1\right)^2}{9\alpha_2^2}\right]^3$$

$$+ \frac{4\alpha_1^2 R_1^2}{3\alpha_2^2}\left[\frac{1}{3\alpha_2} + \frac{\alpha_1}{3\alpha_2 K_p} + \frac{\alpha_1 M^2}{3\alpha_2} + \frac{\left(\alpha_1 - 1\right)^2}{9\alpha_2^2}\right]$$

$$-\left[\frac{\left(1 - \alpha_1\right)\alpha_1 R_1}{3\alpha_2^2} - \frac{R_1}{\alpha_2}\right]^2 + \left[\frac{1 - \alpha}{6\alpha_2^2} + \left(1 + \frac{\alpha_1}{K_p} + \alpha_1 M^2\right)\right.$$

$$\left. - \frac{1}{2\alpha_2 K_p} - \frac{M^2}{2\alpha_2} - \frac{\left(\alpha_l - 1\right)^3}{27\alpha_2^3}\right]^2$$

$$T_l = -\frac{2\alpha_l R_l}{\alpha_2^3}\left[\frac{1}{3}+\frac{\alpha_l}{3K_p}+\frac{\alpha_l M^2}{3}+\frac{(\alpha_l-1)^2}{9\alpha_2}\right]^2+\frac{8\alpha_1^3 R_1^3}{27\alpha_2^3}$$

$$+\left[\frac{(1-\alpha_l)\alpha_l R_l}{3\alpha_2^2}-\frac{R_l}{\alpha_2}\right]\left[\frac{1-\alpha}{3\alpha_2^2}\left(1+\frac{\alpha_l}{K_p}+\alpha_l M^2\right)\right.$$

$$\left.-\frac{1}{\alpha_2 K_p}-\frac{M^2}{\alpha_2}-2\frac{(\alpha_l-1)^3}{27\alpha_2^3}\right],$$

$$Q_l = \frac{1}{\sqrt{2}}\sqrt{S_l+\sqrt{S_1^2+T_1^2}}+\frac{1}{\sqrt{2}}\sqrt{S_l-\sqrt{S_1^2+T_1^2}},$$

$$I = (R_l+Q_l)^{1/3},$$

$$I = (R_l-Q_l)^{1/3},$$

$$m_l = I + J - a_l/3,$$

$$m_2 = -\frac{1}{2}(I+J)+\frac{\sqrt{3}}{2}i\sqrt{I-J}-\frac{a_l}{3},$$

$$m_3 = -\frac{1}{2}(I+J)-\frac{\sqrt{3}}{2}i\sqrt{I-J}-\frac{a_l}{3},$$

$$M_{ll} = -\alpha_2 P_r^3+(\alpha_l-1)P_r^2+\left(1+2iR\alpha_l+\frac{\alpha_l}{K_p}+\alpha_l M^2\right)P_r$$

$$+2iR+\frac{1}{K_p}+M^2,$$

$$M_{l2} = -\alpha_2 S_r^3+(\alpha_l-1)S_3^2+\left(1+2iR\alpha_l+\frac{\alpha_l}{K_p}+\alpha_l M^2\right)S_c$$

$$+2iR+\frac{1}{K_p}+M^2,$$

In the limit when $\alpha_1,\ \alpha_2 \to 0$, we have

$$m_1 \to \infty,$$

$$m_2 = -1/2 + 1/2\left(1 + 8iR + 4/K_p + 4M^2\right)^{1/2},$$

and

$$m_3 = -1/2 - 1/2\left(1 + 8iR + 4/K_p + 4M^2\right)^{1/2}.$$

It can be seen from the above that's he roots m_2 and m_3 reduce to the classical viscous case, while m_1 is the additional root. Hence we take the arbitrary constants $c_1 = 0$ in the complementary function. Using the boundary condition (3.8), the order two arbitrary constants are obtained and the final solution of equation (3.2) can be written as

$$U_0 = \frac{H_1}{M_{11}}\left(e^{-p_r z} - e^{m_3 z}\right) + \frac{G_1}{M_{12}}\left(e^{-S_c z} - e^{m_3 z}\right). \tag{3.12}$$

From equation (3.3) we get

$$U_1 = c_4 e^{m_4 z} + c_5 e^{m_5 z} + c_6 e^{m_6 z} + e^{-\lambda z}\frac{Gr}{M_3}\left(1 + \alpha_1\lambda + \alpha_1 i\omega\right). \tag{3.13}$$

With similar argument as above and using the boundary condition (3.8) we get,

$$U_1 = \frac{Gr}{M_3}\left(1 + \alpha_1\lambda + \alpha_1 i\omega\right)\left(e^{-\lambda z} - e^{m_3 z}\right), \tag{3.14}$$

where $\lambda = 1/2\left[P_r + \left(P_r^2 + 4i\omega P_r\right)^{1/2}\right]$.

$$m_5 = \frac{1}{2}\left(-1 + \sqrt{1 + \frac{4}{K_p} + 4M^2 + i\left(4\omega + 8r\right)}\right), \tag{3.15}$$

(3.16) $$m_6 = \frac{1}{2}\left(-1-\sqrt{1+\frac{4}{K_p}+4M^2+i(4\omega+8r)}\right),$$

(3.17) $$M_3 = -\alpha_2\lambda^3 + (\alpha_1 - 1 - i\omega\alpha_2)\lambda^2 + \left(1+2i\alpha_1(\omega+R)+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)\lambda$$
$$+\left[\frac{1}{K_p}+M^2-\omega\alpha_1(\omega+2R)+i\left(\omega+2R+\frac{\alpha_1\omega}{K_p}+\alpha_1\omega M^2\right)\right].$$

The primary and secondary velocity fields in terms of the fluctuating parts are

(3.18) $$\frac{u}{w_0} = U_0 + \varepsilon(N_r\cos\omega t - N_l\sin\omega t),$$

(3.19) $$\frac{v}{w_0} = V_0 + \varepsilon(N_r\cos\omega t + N_l\sin\omega t),$$

where

$$u_0 + iv_0 = U_0 \text{ and } N_r + i\, N_i = U_l,$$

$$N_r = \frac{G_r\begin{bmatrix} B_6B_4(1+\alpha_lB_l)+\alpha_1B_5B_6(B_2+\omega) \\ +\alpha_1B_3B_4(B_2+\omega)-B_3B_5(1+\alpha_1B_l) \end{bmatrix}}{B_6^2+B_3^2},$$

$$N_r = \frac{-G_r\begin{bmatrix} B_3B_4(1+\alpha_lB_l)+\alpha_1B_5B_3(B_2+\omega) \\ -\alpha_1B_6B_4(B_2+\omega)+B_6B_5(1+\alpha_1B_l) \end{bmatrix}}{B_6^2+B_3^2},$$

$$u_0 = G_{ll}\left[A_3\left(e^{-S_cz}-e^{A_lz}\cos A_2z\right)-A_4e^{A_lz}\sin A_2z\right]$$
$$+H_{ll}\left[\left(e^{-P_cz}-e^{A_lz}\cos A_2z\right)A_5-A_6e^{A_lz}\sin A_2z\right],$$

$$v_0 = G_{ll}\left[-A_4\left(e^{-S_cz}-e^{A_lz}\cos A_2z\right)-A_3e^{A_lz}\sin A_2z\right]$$

$$+H_{ll}\left[-A_6\left(e^{-P_c z}-e^{A_1 z}\cos A_2 z\right)-A_5 e^{A_1 z}\sin A_2 z\right],$$

where

$$M_4=\left(1+\frac{4}{K_p}+4M^2\right)^2+64R^2,$$

$$A_1=-\frac{1}{2}-\frac{1}{2}\left[0.5\left(1+\frac{4}{K_p}+4M^2+M_4\right)\right]^{\frac{1}{2}},$$

$$A_2=-\frac{1}{2}\left[0.5\left(M_4-1-\frac{4}{K_p}-4M^2\right)\right]^{\frac{1}{2}},$$

$$A_3=-\alpha_2 S_c^3+\left(\alpha_1-1\right)S_c^2+\left(1+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)S_c+\frac{1}{K_p}+M^2,$$

$$A_4=2R\left(1+\alpha_1 S_c\right),$$

$$A_5=-\alpha_2 P_r^3+\left(\alpha_1-1\right)P_r^2+\left(1+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)P_r+\frac{1}{K_p}+M^2,$$

$$A_6=2R\left(1+\alpha_1 P_r\right),\quad G_{ll}=\frac{G_l}{\left(A_3^2+A_4^2\right)},\ H_{ll}=\frac{H_l}{\left(A_5^2+A_6^2\right)},$$

$$N=\sqrt{P_r^4+16P_r^2\omega^2},\ B_1=\frac{P_r}{2}+\frac{1}{2}\sqrt{\frac{\left(N+P_r^2\right)}{2}},\ B_2=\frac{1}{2}\sqrt{\frac{\left(N-P_r^2\right)}{2}},$$

$$B_6=-\alpha_2 B_1\left(B_1^2-3B_2^2\right)+\left(\alpha_1-1\right)\left(B_1^2-B_2^2\right)+2B_1B_2\omega\alpha_2$$

$$+B_1\left(1+\frac{\alpha_1}{K_p}+\alpha_1 M^2\right)-2\alpha_1 B_{21}\left(\omega+R\right)+\frac{1}{K_p}+M^2$$

$$-\omega\alpha_1\left(\omega+2R\right)+2\alpha_1B_1\left(\omega+R\right)+2R+\omega+\alpha_1\frac{\omega}{K_p}+\alpha_1\omega M^2,$$

$$B_3=-\alpha_2B_2\left(B_2^2-3B_1^2\right)+2B_1B_2\left(\alpha_1-1\right)-\omega\alpha_2\left(B_1^2-B_2^2\right)$$

$$+B_{21}\left(1+\frac{\alpha_1}{K_p}+\alpha_1M^2\right)+2\alpha_1B_1\left(\omega+R\right)+2R+\omega+\alpha_1\frac{\omega}{K_p}+\alpha_1\omega M^2,$$

$$B_4=\bar{e}^{B_2z}\cos B_2z-e^{B_6z}\cos B_3z,$$

$$B_5=\bar{e}^{B_2z}\mathrm{Sin}\,B_2z+e^{B_6z}\mathrm{Sin}\,B_3z.$$

4. Results and Discussion

The problem of hydro magnetic unsteady free convective flow and mass transfer of an elastic-viscous fluid past an infinite vertical porous plate in a rotating porous medium under fluctuating temperature has been studied. An interesting feature of the present study is to discuss case of non-conducting flow as a special case of Panda *et al.*[15]. The effects of flow parameters on the primary and secondary velocity of the flow field are discussed below with the help of figures 1-8.

The nature of variation of primary and secondary velocity profiles with the change of elastic parameters $\left(\alpha_1, \alpha_2\right)$, porosity parameter $\left(K_p\right)$ and magnetic parameter $\left(M\right)$ is as depicted in figures (1) and (2) respectively. From curves (5) and (4) of both the figures, it is observed that α_1 enhance both the components of velocity of the flow field while α_2 shows the reverse effect. Curves (2) and (3) of both the figures show that the porosity parameter $\left(K_p\right)$ has an accelerating effect (magnitude) on both the components of velocity. Further, curves (2) and (6) of both the figures report the retarding effect (magnitude) of magnetic parameter $\left(M\right)$ on both the components of velocity of the flow field. Curves (7) is in good agreement with Panda *et al.*[15].

Figures (3) and (4) point out the effect of magnetic parameter $\left(M\right)$ separately on the primary and secondary velocity of the flow field respectively. The curves of both the figures clearly depict the retarding effect of magnetic parameter (M) on both the components of the velocity of the flow field. This retardation of velocity may be attributed to the magnetic pull caused by the Lorentz Force on the conducting fluid due to the applied magnetic field. It is further observed that the velocity components assumes higher values in non-MHD flow (case of Panda *et al.*[15]) than that in MHD flow.

Figures (5) and (6) depict the effect of Grash number for heat transfer (G_r) on the primary and secondary velocity profiles respectively under different physical situations. It is found from curves (1) and (2) of both the figures that the effect of G_r is to enhance the magnitude of both the components of velocity of the flow field due to the flow of free convection current. Comparing curves (2) and (3) of both the figures, it is observed that in presence of constant free convection current both the components of velocity assumes higher values in non-MHD flow $(M = 0)$ than that of MHD flow. Further, under similar physical conditions of G_r and K_p (curves (2) and (3)), the velocity components assumes higher values in non-MHD case.

Figures (7) and (8) report the nature of primary and secondary velocity profiles respectively due to variations of different parameters (G_m, R, α_2, K_p, M and ω). The Grash of number for mass transfer (G_m) has negligible effect on the secondary velocity (curves (4) and (5)). The rotation parameter (R) reduces the primary velocity components and enhances the secondary velocity component (curves (1) and (4)).

The frequency parameter (ω) has a retarding effect on both the components of velocity (curves (3) and (4)). Comparing curve (6) with the curve (7) for non-MHD flow, it is observed that magnitude of velocity components is higher in non-MHD flow than its counterpart in MHD flow except the case of rotation parameter. In this case the secondary velocity possess higher magnitude than its counterpart in non-MHD flow. Here the effect of rotation parameter dominates over the effect of magnetic parameter.

5. Conclusion

The above study brings out the following results of physical interest.

1. The magnetic parameter (M) has a decelerating effect on both the components of velocity (primary and secondary) due to the magnetic

pull caused by the Lorentz force on the conducting fluid. In comparison to MHD flow, the velocity components take higher values in non-MHD flow.

2. The elastic parameter α_1 enhance the magnitude of both the components of velocity while the other elastic parameter α_2 has a retarding effect on both the components of velocity.
3. The porosity parameter has an accelerating effect on both the components of velocity.
4. The porosity parameter has an accelerating effect on both the components of velocity.
5. The Grash of number for mass transfer (G_m) has a retarding effect on the primary velocity and a very negligible effect on the secondary velocity.
6. The rotation parameter (R) reduces the magnitude of primary velocity component and enhances the secondary velocity components.
7. The angular amplitude (ω) has a retarding effect on both the components of velocity.

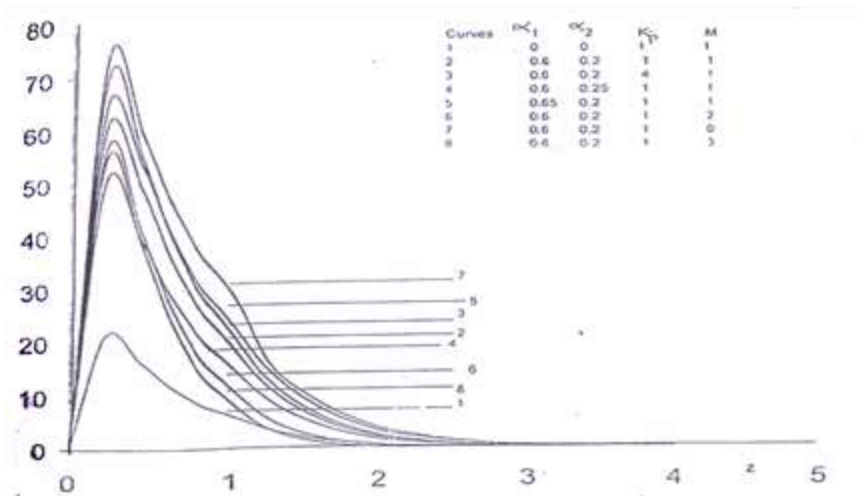

Figure 1. Effect of Elastic Paramenters on Primary velocity profiles when $G_r = 2, G_m = 2, P_r = 7, S_c = 900$. $R = 0.4$, $\varepsilon = 0.01$, $\omega = 5$

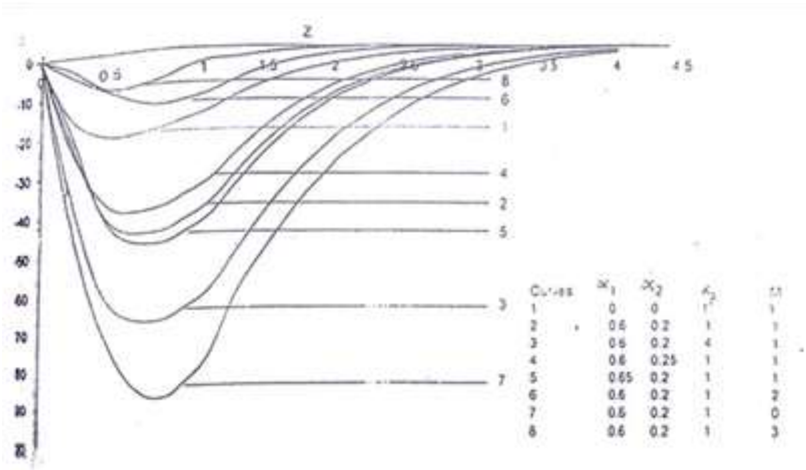

Figure 2. Effect of Elastic Paramenters on Secondary velocity profiles when $G_r = 2$, $G_m = 2$, $P_r = 7$, $S_c = 900$. $R = 0.4$, $\varepsilon = 0.01$, $\omega = 5$

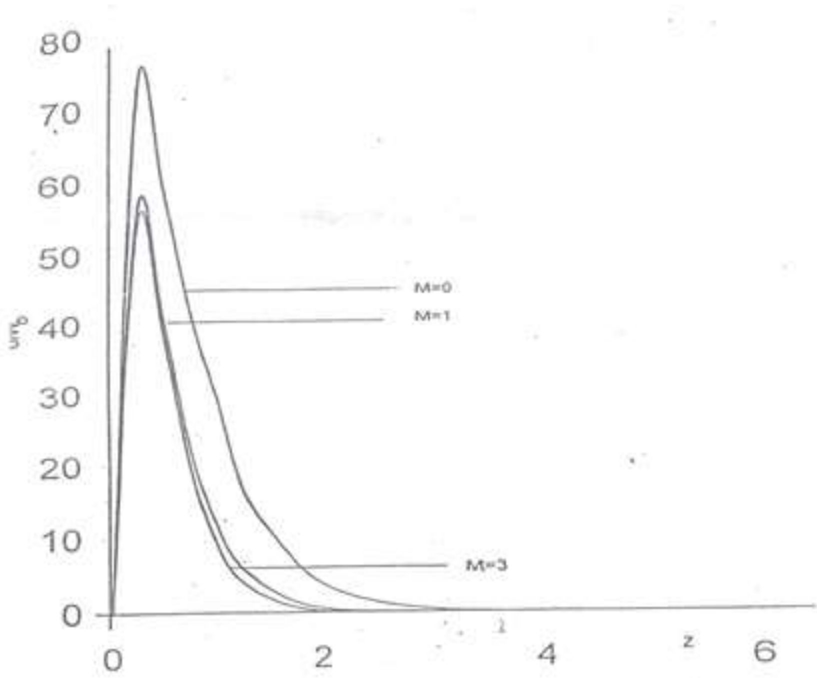

Figure 3. Effect of Magnetic parameter on primary velocity profile

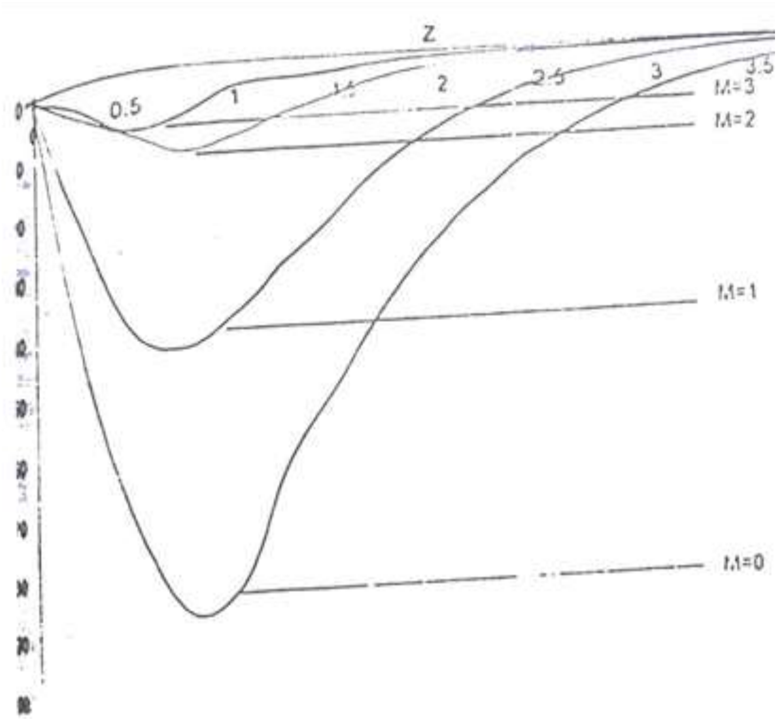

Figure 4. Effect of Magnetic Parameter on Secondary velocity profile

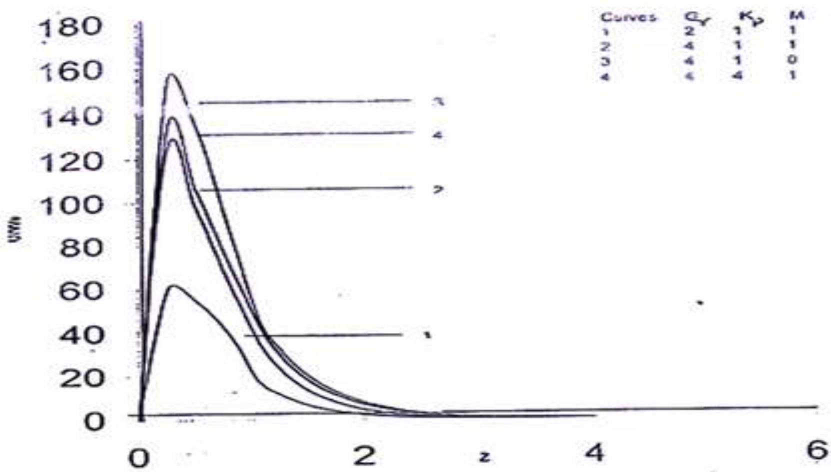

Figure 5. Effect of G_r on Primary velocity profiles when $G_m = 2$, $P_r = 7$, $S_c = 900$, $R = 0.4$, $\varepsilon = 0.01$, $\omega = 5$

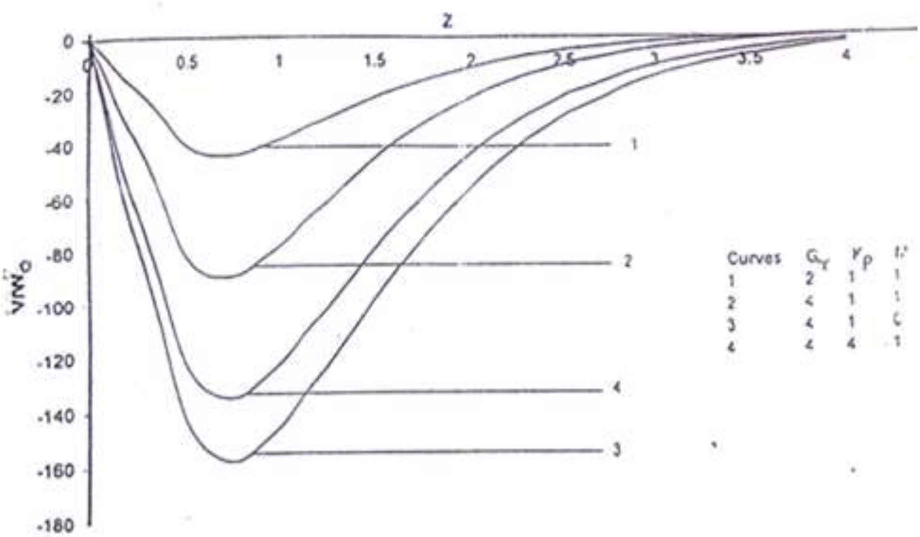

Figure 6. Effect of G_r on Secondary velocity profiles when G_m = 2, P_r = 7, S_c = 900, R = 0.4, ε = 0.01, ω = 5 α_1 = 0.6, α_2 = 0.6

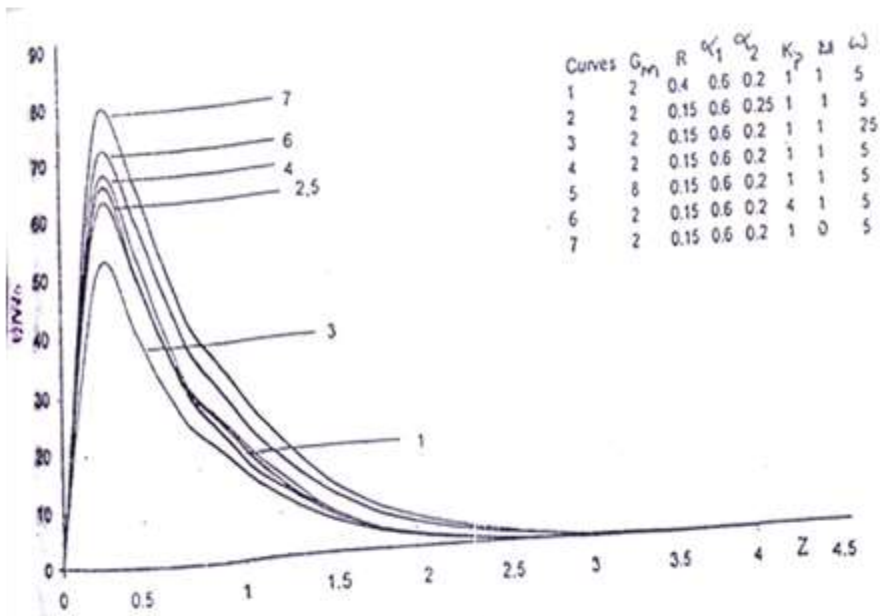

Figure 7. Effect of different parameters on Primary velocity profiles when G_r = 2, P_r = 7, S_c = 900, ε = 0.01

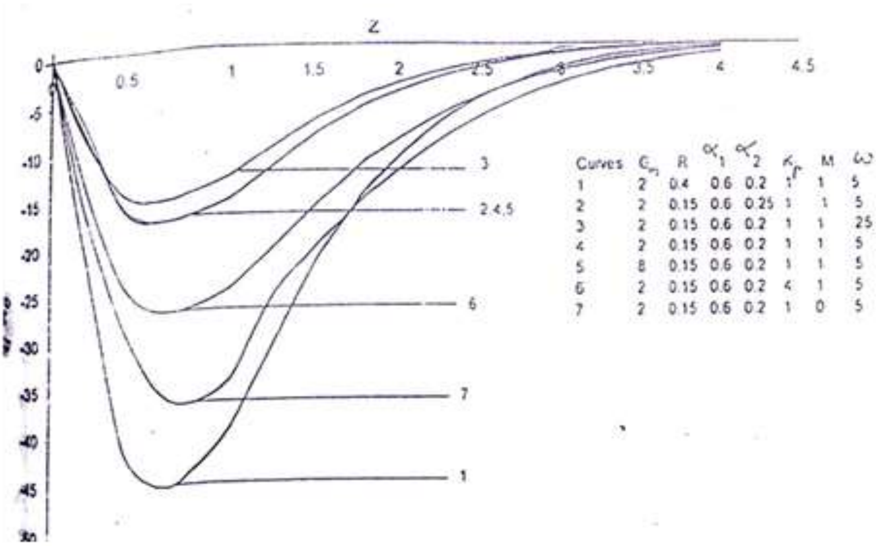

Figure 8. Effect of different parameters on Secondary velocity profiles when $G_r = 2$, $P_r = 7$, $S_c = 900$, $\varepsilon = 0.01$

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