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On *gs-Closed Sets in Bitopological Spaces

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Abstract: In this paper, we introduce *gs-closed sets in bitopological spaces. Properties of these sets are investigated and we introduce new bitopoligical spaces (i, j)-T_s, (i, j)-T^*_s and (i, j)-$^*T_s$ spaces as applications. Further we introduce and study *gs-continuity in bitopological spaces.

Key Words: (i, j)-*gs-closed sets; (i, j)-T_s spaces; (i, j)-T^*_s spaces and (i, j)*T_s spaces; *DS (i, j)-continuity.

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1. Introduction

A triple (X, τ_1, τ_2) where X is non empty set and τ_1, τ_2 are two topologies on X is called a bitopological space. Kelly[1] initiated and study such spaces in 1985. Fukutaka[2] introduced the concept of g-closed sets[3] in bitopological spaces. Sheik John and Sundram[4] introduced the concept of g*-closed sets in bitopological spaces and after that many authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Recently Agarwal, Garg and Goel[5] introduced and studied the concepts of *gs-closed sets and *gs-continuity in topological spaces. In this paper, we introduce the concepts of *gs-closed sets and *gs-continuity for bitopological spaces and then investigate three new bitopological spaces (i, j)-T_s space, (i, j)-T^*_s space and (i, j)-$^*T_s$ space.

2. Preliminaries

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A is denoted by A^C.

Definition 2.1: **(i)** A subset A of a topological space (X, τ) is called semi-open[6]. (resp. regular open[7], pre-open[5]) if $A \subseteq cl(int(A))$ (resp. $A = int(Cl(A))$, $A \subseteq int(cl(A))$).

(ii) Subset A of a topological space (X, τ) is called a generalized closed set[3] (briefly g-closed set) (resp. $\hat{g}$-closed set[8], *g-closed set[9], *gs-closed set[10]) if $cl(A) \subseteq U$ (resp. $cl(A) \subseteq U$, $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open (resp. semi-open, $\hat{g}$-open, $\hat{g}$-open) in X.

Definition 2.2: The intersection of all pre-closed (resp. semi-closed) sets containing A is called the pre-closure (resp. semi-closure) of A and it is denoted by pcl(A) (resp.scl(A)).

Throughout this paper, X and Y represent non-empty bi-topological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise explicitly mentioned and integers i, j, k $\in \{1, 2\}$. For a subset A of X, τ_i-cl(A) (resp. τ_i-int(A), τ_i-pcl(A)) denote the closure (resp. interior, preclosure) of A w.r.t. topology τ_i. We denote the family of all $\hat{g}$-open subsets of X w.r.t. topology τ_i by $\hat{G}O(X, \tau_i)$ and the family of all τ_j-closed sets is denoted by F_j. By (i, j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.3: A subset A of a bitopological space (X, τ_1, τ_2) is called :

(i) (i, j)-g-closed[2] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and $U \in \tau_i$.

(ii) (i, j)-rg-closed[11] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.

(iii)(i, j)-gpr-closed[12] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.

(iv)(i, j)-Wg-closed[13] if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever $A \subseteq U$ and $U \in \tau_i$.

(v) (i, j)-ω-closed[12] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is semi-open in τ_i.

(vi)(i, j)-g*-closed[4] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is generalized-open in τ_i.

(vii)(i, j)-*g-closed[14] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\hat{g}$-open in τ_i.

(viii)(i, j)- $\hat{\hat{g}}$ -closed[15] if τ_j-cl(A)$\subseteq$U whenever A$\subseteq$U andU is sg-open in τ_i.

The family of all (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-Wg-closed, (i, j)-ω-closed, (i, j)-g*-closed, (i, j)-*g-closed, (i, j)-$\hat{\hat{g}}$-closed) subsets of a bitopological space (X, τ_1, τ_2) is denoted by D(i, j) (resp. Dr(i, j), ζ(i, j), W(i, j), C(i, j) and D*(i, j), *D(i, j), $\hat{\hat{D}}$(i, j)).

Definition 2.4

(i) A bitopological space (X, τ_1, τ_2) is said to be (i, j)-$T_{1/2}$[2] (resp. (i, j)-$T^*_{1/2}$, (i, j)-*$T_{1/2}$, (i, j)-*g$T^*_{1/2}$, (i, j)-*g*$T_{1/2}$, (i, j)- $\hat{\hat{T}}_{1/2}$ space) if every (i, j)-g-closed (resp. (i, j)-g*-closed, (i, j)-g-closed, (i, j)-*g-closed, (i, j)-g-closed, (i, j)- $\hat{\hat{g}}$ -closed) sets is τ_j-closed (resp. τ_j-closed, (i, j)-g*-closed, τ_j-closed, (i, j)-*g-closed, τ_j-closed)[4,14,15].

(ii) Bitopological space (X, τ_1, τ_2) is said to be strongly pairwise-$T_{1/2}$ (resp. strongly pairwise $T^*_{1/2}$, strongly pairwise *g$T^*_{1/2}$) space if it is (1, 2)-$T_{1/2}$ and (2, 1)-$T_{1/2}$ (resp. (1, 2)-$T^*_{1/2}$ and (2, 1)-$T^*_{1/2}$, (1, 2)-*g$T^*_{1/2}$ and (2, 1)-*g$T^*_{1/2}$ space[2,4,14].

Definition 2.5: A map f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called :

(i) τ_j-σ_k-continuous[16] if $f^{-1}(V) \in \tau_j$ for every V $\in \sigma_k$.

(ii) D (i, j)-σ_k-continuous[16] (resp.Dr(i, j)-σ_k-continuous[11], ζ(i, j)-σ_k-continuous[12], W(i, j)-σ_k-continuous[13], C(i, j)-σ_k-continuous[12] and D*(i, j)-σ_k-continuous[4], *D(i, j)-σ_k-continuous[14], $\hat{\hat{D}}$(i, j)-σ_k-continuous[15]) if the inverse image of every σ_k-closed set is (i, j)-g-closed (resp. (i, j)-rg-closed, (i, j)-gpr-closed, (i, j)-wg-closed, (i, j)-ω-closed and (i, j)-g*-closed, (i, j)-*g-closed, (i, j)- $\hat{\hat{g}}$ -closed,) set in (X, τ_1, τ_2).

2. (i, j)-*gs-Closed Sets

In this section we introduce the concepts of (i, j)-*gs-closed sets in bitopological spaces.

Definition 3.1: A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j)-*gs-closed set if τ_j-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U$\in \hat{G}$O (X, τ_i).

We denote the family of all (i, j)-*gs-closed sets in (X, τ_1, τ_2) by *DS (i, j).

Remark 3.2: By setting $\tau_1 = \tau_2$ in definition (3.1.1), an (i, j)-*gs-closed set is a *gs-closed set.

Proposition 3.3: *If A is τ_j-sclosed and so τ_j-closed subset of (X, τ_1, τ_2), then A is (i, j)-*gs-closed.*

The converse of the above proposition is not necessarily true as seen from following example.

Example 3.4: Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Here the subset {a, b} is (1, 2)-*gs-closed but not τ_2-sclosed and τ_2-closed in (X, τ_1, τ_2).

Proposition 3.5: *In a bitopological space every (i) (i, j)-g*-closed set (ii) (i, j)-*g-closed set (iii) (i, j)-$\ddot{g}$-closed set is (i, j)-*gs-closed set.*

The following examples show that the reverse implications of the above proposition are not true.

Example 3.6: Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{b, c\}, X\}$ then the subset {b} is (2, 1)-*gs-closed but not (2, 1)-g*-closed.

Example 3.7: Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$ then the subset {b} is (1, 2)-*gs-closed but not (1, 2)-*g-closed.

Example 3.8: Let X = {a, b, c, d}, $\tau_1 = \{\phi, \{a, b\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b, c\}, X\}$ then the subset {b, d} is (2, 1)-*gs-closed but not (2, 1)-$\ddot{g}$-closed.

Remark 3.9: The following examples show that (i, j)-*gs-closed set is independent from (i) (i, j)-g-closed set (ii) (i, j)-rg-closed set (iii) (i, j)-gpr-closed set (iv) (i, j)-ω-closed set (v) (i, j)-s-closed set (vi) (i, j)-wg-closed set, (vii) (i, j)-sg-closed set.

Example 3.10: In example (3.4), the subset {c} is (1, 2)-*gs-closed but not (1, 2)-g-closed.

Example 3.11: In example (3.7), the subset {a, b} is (1, 2)-g-closed but not (1, 2)-*gs-closed.

Example 3.12: In example (3.6), the subset {a} is (2, 1)-*gs-closed but not (2, 1)-rg-closed. Also {a, b} is (2, 1)-rg-closed but not (2, 1)-*gs-closed.

Example 3.13: Let X = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{a, c\}, X\}$, $\tau_2 = \{\phi, \{a, b\}, X\}$, the subset {a, c} is (1, 2)-gpr-closed but not (1, 2)-*gs-closed.

Example 3.14: In example (3.6), the subset {a} is (2, 1)-*gs-closed but not (2, 1)-gpr-closed.

Example 3.15: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {b}, X} the subset {a, b} is (1, 2)-*gs-closed but not (1, 2)-ω-closed.

Example 3.16: Let X = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = {ϕ, {b}, {a, c}, X}, the subset {a, b} is (2, 1)-ω-closed but not (2, 1)-*gs-closed.

Example 3.17: In example (3.4), the subset {a} is (1, 2)-s-closed but not (1, 2)-*gs-closed.

Example 3.18: Let X = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = {ϕ, {a, b}, X}, the subset {a, b} is (2, 1)-*gs-closed but not (2, 1)-s-closed.

Example 3.19: In example (3.7), the subset {a, b} is (1, 2)-*gs-closed but not (1, 2)-wg-closed.

Example 3.20: In example (3.18), the subset {a} is (1, 2)-wg-closed but not (1, 2)-*gs-closed.

Example 3.21: In example (3.18), the subset {a, b} is (1, 2)-*gs-closed but not (1, 2)-sg-closed.

Example 3.22: In example (3.6), the subset {a, b} is (2, 1)-sg-closed but not (2, 1)-*gs-closed.

Remark 3.23: The union of two (i, j)-*gs-closed sets need not be (i, j)-*gs-closed set as it can be seen from the following example.

Example 3.24: In example (3.6), {a} and {b} are (2, 1)-*gs-closed set but their union {a, b} is not (2, 1)-*gs-closed.

Remark 3.25: The intersection of two (i, j)-*gs-closed sets need not be (i, j)-*gs-closed set as it can be seen from the following example.

Example 3.26: Let X = {a, b, c}, τ_1 = {ϕ, {b}, X}, τ_2 = {ϕ, {b, c}, X}. Here {a, b} and {b, c} are (1, 2)-*gs-closed but their intersection {b} is not (1, 2)-*gs-closed.

The following diagram summarizes the above discussions.

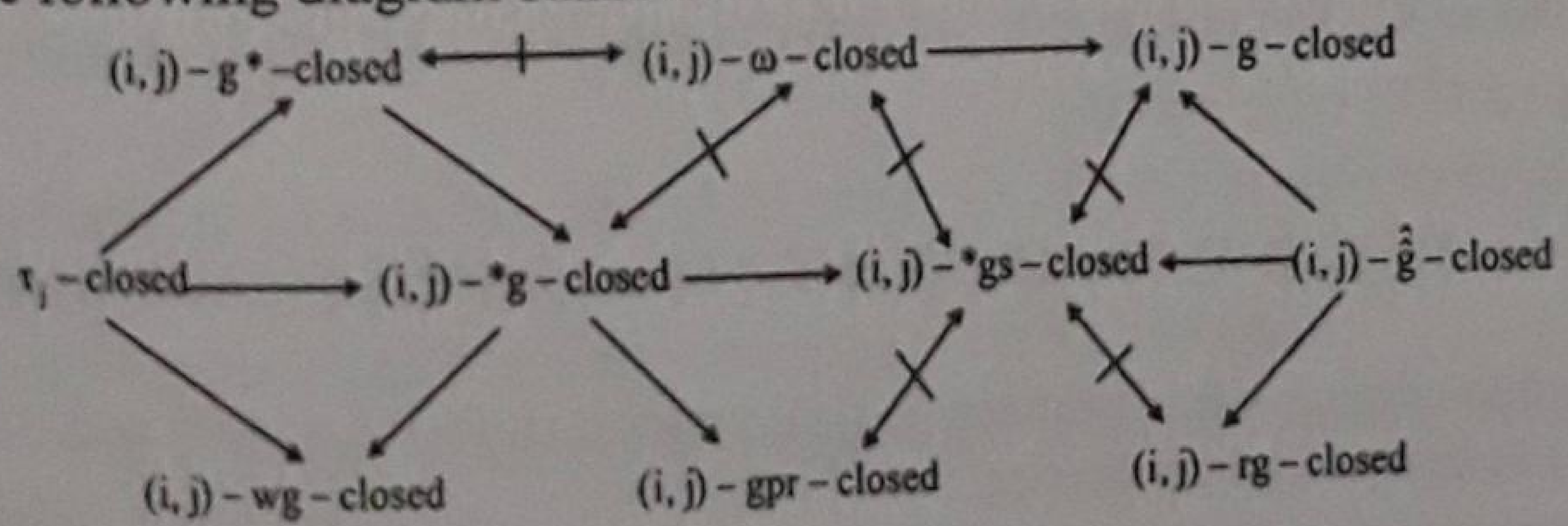

diagram (3.27)

Where A⟶B (resp. A⟷B) represents A implies B but converse is not necessary true (resp. A and B are independent).

Remark 3.28: *DS (1, 2) is generally not equal to *DS (2, 1) as it can be seen from the example (3.6).

Proposition 3.29: *If $\tau_i \subseteq \tau_j$ in (X, τ_i, τ_j) then *DS (j, i) $\subseteq$ *DS (i, j).*

The converse of the above proposition is not necessarily true as seen fr m following example.

Example 3.30: In example (3.18), *DS (1, 2) $\subseteq$ *DS (2, 1) but τ_2 is not contained in τ_1.

Proposition 3.31: *For each element x of (X, τ_1, τ_2), $\{x\}$ is τ_i- $\hat{g}$ -closed or $\{x\}^C$ is (i, j)-*gs-closed.*

Proposition 3.32: *If A is (i, j)-*gs-closed then τ_j-scl(A) – A contains no non-empty τ_i- $\hat{g}$ -closed set.*

Proposition 3.33: *If A is (i, j)-*gs-closed set in (X, τ_1, τ_2) then A is τ_j-sclosed if and only if τ_j-scl(A) – A is τ_i- $\hat{g}$ -closed.*

Proposition 3.34: *If A is an (i, j)-*gs-closed set of (X, τ_1, τ_2) such that A $\subseteq$ B $\subseteq$ τ_j-scl(A) then B is also an (i, j)-*gs-closed set of (X, τ_1, τ_2).*

Proposition 3.35: *Let A $\subseteq$ Y $\subseteq$ X and A is (i, j)-*gs-closed in X. Then A is (i, j)-*gs-closed relative to Y.*

Proof: Proof is same as proposition (3.34).

Theorem 3.36: *In a bitopological space (X, τ_1, τ_2), $\hat{G}O(X, \tau_i) \subseteq semiF_j$ (Family of all semiclosed sets in τ_j) if and only if every subset of X is an (i, j)-*gs-closed set.*

Proof: Suppose that $\hat{G}O(X, \tau_i) \subseteq$ semiF$_j$. Let A be a subset of X such that A $\subseteq$ U where U $\in$ $\hat{G}O(X, \tau_i)$ then τ_j-scl(A) $\subseteq$ τ_j-scl(U) = U and hence A is (i, j)-*gs-closed.

Conversely let every subset of X is (i, j)-*gs-closed. Let U $\in$ $\hat{G}O(X, \tau_i)$. Since U is (i, j)-*gs-closed, we have τ_j-scl(U) $\subseteq$ U. So U $\in$ semiF$_j$ and hence $\hat{G}O(X, \tau_i) \subseteq$ semiF$_j$.

3. (i, j)-T_s Spaces, (i, j)-T_s* Spaces and (i, j)-*T_s Spaces

In this section we introduce (i, j)-T_s spaces, (i, j)-T_s* spaces and (i, j)-*T_s spaces in bitopological spaces.

Definition 4.1: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-T_s space if every (i, j)-*gs-closed set is τ_j-closed.

Proposition 4.2: *If (X, τ_1, τ_2) is (i, j)-T_s space then it is (i) (i, j)-$T^*_{1/2}$ space (ii) (i, j)-$^*gT^*_{1/2}$ space (iii) (i, j)-$\hat{\hat{T}}_{1/2}$ space.*

The converse of the above proposition is not necessarily true as seen from following example.

Example 4.3: In example (3.6), (X, τ_1, τ_2) is $(2, 1)$-$T^*_{1/2}$ space, $(2, 1)$-$^*gT^*_{1/2}$ space and $(2, 1)$-$\hat{\hat{T}}_{1/2}$ space but not $(1, 2)$-T_s space.

Theorem 4.4: *A bitopological space (X, τ_1, τ_2) is an (i, j)-T_s space if $\{x\}$ is τ_j-open or τ_i-$\hat{g}$-closed for each $x \in X$.*

Proof: Suppose that $\{x\}$ is not τ_i-$\hat{g}$-closed then $\{x\}^C$ is (i, j)-*gs-closed by proposition (3.31). Since (X, τ_1, τ_2) is an (i, j)-T_s space, $\{x\}^C$ is τ_j-closed i.e. $\{x\}$ is τ_j-open.

Proposition 4.5: *(i, j)-T_s space and (i, j)-$T_{1/2}$ space are independent.*

Definition 4.6: A bitopological space (X, τ_1, τ_2) is said to be strongly pairwise T_s-space if it is both $(1, 2)$-T_s space and $(2, 1)$-T_s space.

Proposition 4.7: *If (X, τ_1, τ_2) is strongly pairwise T_s-space then it is strongly pairwise $T^*_{1/2}$-space (resp. strongly pairwise $^*gT^*_{1/2}$-space).*

Definition 4.8: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-T^*_s space if every (i, j)-*gs-closed set is τ_j-semiclosed.

Proposition 4.9: *Every (i, j)-T_s space is (i, j)-T^*_s space.*

The converse of the above proposition is not necessarily true as seen from following example.

Example 4.10: In example (3.16), (X, τ_1, τ_2) is $(2, 1)$-T^*_s space but not $(2, 1)$-T_s space.

Remark 4.11: (i, j)-T^*_s space is not necessarily (i, j)-$T_{1/2}$ space as it can be seen from the following example.

Example 4.12: In example (3.7), (X, τ_1, τ_2) is $(1, 2)$-T^*_s space but not $(1, 2)$-$T_{1/2}$ space.

Definition 4.13: A bitopological space (X, τ_1, τ_2) is said to be an (i, j)-$^*T_s$ space if every (i, j)-*gs-closed set is (i, j)-*g-closed.

Proposition 4.14: *Every (i, j)-T_s space is (i, j)-$^*T_s$ space.*

The converse of the above proposition is not necessarily true as seen from following example.

Example 4.15: In example (3.6), (X, τ_1, τ_2) is $(2, 1)$-$^*T_s$ space but not $(2, 1)$-T_s space.

Remark 4.16: (X, τ_1, τ_2) is not generally (1, 2)-T_s space even if both (X, τ_1) and (X, τ_2) are T_s-space as it can be seen from the following example.

Example 4.17: Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. Here (X, τ_1) and (X, τ_2) both are T_s-space but (X, τ_1, τ_2) is not (1, 2)-T_s space.

Theorem 4.18: *A bitopological space (X, τ_1, τ_2) is an (i, j)-T_s space if and only if it is both (i, j)-$^*T_s$ space and (i, j)-$^*gT^*_{1/2}$ space.*

Theorem 4.19: *A bitopological space (X, τ_1, τ_2) is (i, j)-T_s space if $\{x\}$ is τ_j-open or τ_i-$\hat{g}$-closed.*

Proof: Suppose that $\{x\}$ is not τ_i-$\hat{g}$-closed then $\{x\}^C$ is (i, j)-*gs-closed by proposition (3.31). Since (X, τ_1, τ_2) is (i, j)-T_s space, $\{x\}^C$ is τ_j-closed i. e. $\{x\}$ is τ_j-open.

The following diagram summarizes the above discussions.

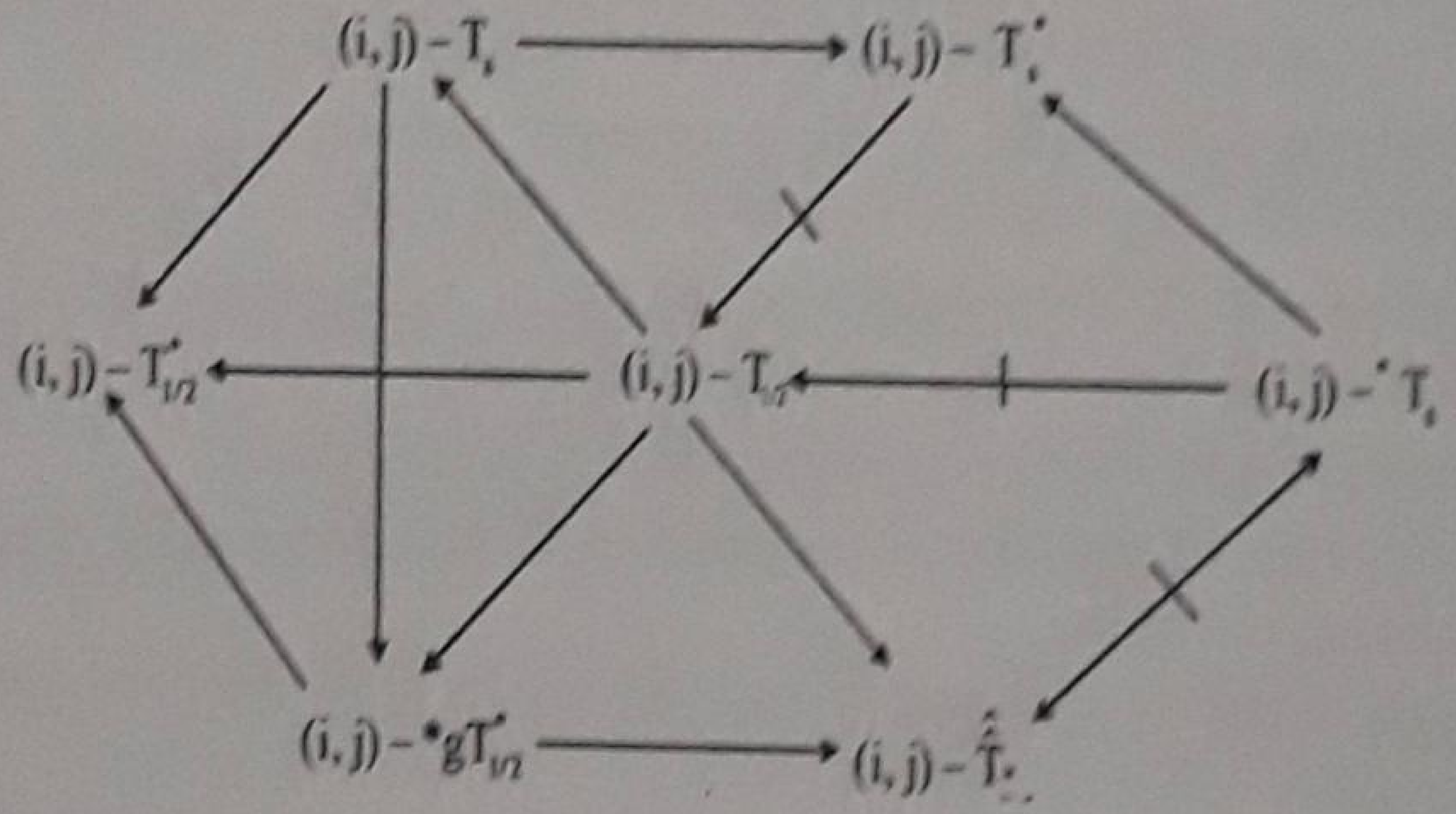

Diagram (4.20)

4. *gs-Continuous maps in Bitopological Spaces

In this section we introduce *gs-continuous maps and then discuss its properties.

Definition 5.1: A map. $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called *DS(i, j)-σ_k-continuous if the inverse image of every σ_k-closed set is an (i, j)-*gs-closed set.

Proposition 5.2: *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i) τ_j-σ_k-continuous (ii) $D^*(i, j)$-σ_k-continuous (iii) $^*D(i, j)$-σ_k-continuous (iv) $\hat{\hat{D}}(i, j)$-σ_k-continous then it is $^*DS(i, j)$-σ_k-continuous.*

However the reverse implications of the above proposition are not true in general as seen from the following examples.

Example 5.3: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X}, τ_2 = {ϕ, {a, b}, X }, σ_1 = {ϕ, {a}, Y}, σ_2 = {ϕ, {a}, {a, b}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping then f is *DS(1, 2)-σ_2-continuous but not τ_1-σ_2-continuous.

Example 5.4: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, X}, τ_2 = (ϕ, {a}, {b, c}, X }, σ_1 = {ϕ, {b}, Y}, σ_2 = { ϕ, {c}, {b, c}, {a, c}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping then f is *DS (1, 2)-σ_2-continuous but not D*(i, j)-σ_2-continuous.

Example 5.5: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = (ϕ, {a}, {b, c}, X }, σ_1 = {ϕ, {b}, Y}, σ_2 = { ϕ, {a}, {a, b}, {a, c}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping then f is *DS(2, 1)-σ_2-continuous but not *D(2, 1)-σ_2-continuous.

Example 5.6: In example (5.5), f is *DS (2, 1)-σ_2-continuous but not $\hat{D}(2, 1)$-σ_2-continuous.

Remark 5.7: *DS(i, j)-σ_k-continuous map is independent from (i) D(i, j)-σ_k-continuous (ii) C(i, j)-σ_k-continuous (iii) D_r(i, j)-σ_k-continuous (iv) ζ(i, j)-σ_k-continuous (v) W(i, j)-σ_k-continous as it can be seen from the following examples.

Example 5.8: Let X = Y = {a, b, c}, τ_1 = {ϕ,{a}, {c}, {a, c}, X}, τ_2 = (ϕ, {a}, X }, σ_1 = {ϕ, {a}, {c}, {a, c}, Y}, σ_2 = { ϕ, {a, b}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping then f is *DS(1, 2)-σ_2-continuous but not D(1, 2)-σ_2-continuous.

Example 5.9: Let X = Y = {a, b, c}, τ_1 = {ϕ,{a}, {b, c}, X}, τ_2 = (ϕ, {a}, {a, b}, X }, σ_1 = {ϕ, {b}, {c}, {b, c}, Y}, σ_2 = { ϕ, {a}, {b}, {a, b}, Y}. Define f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping then f is D(1, 2)-σ_2-continuous but not *DS(1, 2)-σ_2-continuous.

Example 5.10: In example (5.8), f is *DS (1, 2)-σ_2-continuous but not C (2, 1)-σ_2-continuous.

Example 5.11: In example (5.9), f is C (1, 2)-σ_2-continuous but not *DS(1, 2)-σ_2-continuous.

Example 5.12: In example (5.8), f is *DS (1, 2)-σ_2-continuous but not D_r(2, 1)-σ_2-continuous.

Example 5.13: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = (ϕ, {a}, {b, c}, X }, σ_1 = {ϕ, {b}, {b, c}, Y}, σ_2 = { ϕ, {c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is D_r(1, 2)-σ_2-continuous but not *DS(1, 2)-σ_2-continuous.

Example 5.14: Let X = Y = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = (ϕ, {a, b}, X }, σ_1 = {ϕ, {b}, {a, b}, Y}, σ_2 = { ϕ, {a}, {c}, {a, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is ζ(1, 2)-σ_2-continuous but not *DS(1, 2)-σ_2-continuous.

Example 5.15: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, {b}, {a, b}, X}, τ_2 = {ϕ, {a}, {b, c}, X }, σ_1 = {ϕ, {b, c}, Y}, σ_2 = { ϕ, {a}, {b}, {a, b}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is *DS(2, 1)-σ_1-continuous but not ζ(2, 1)-σ_1-continuous.

Example 5.16: Let X = Y = {a, b, c}, τ_1 = {ϕ, {b}, {c}, {b, c}, {a, c}, X}, τ_2 = {ϕ, {a, b}, X }, σ_1 = {ϕ, {b, c}, Y}, σ_2 = { ϕ, {a}, {b}, {a, b}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is W(1, 2)-σ_1-continuous but not *DS(1, 2)-σ_1-continuous.

Example 5.17: Let X = Y = {a, b, c}, τ_1 = {ϕ, {a}, {c}, {a, c}, X}, τ_2 = {ϕ, {a}, X }, σ_1 = {ϕ, {a}, Y}, σ_2 = { ϕ, {b}, {c}, {b, c}, Y}. Define f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) by identity mapping then f is *DS(2, 1)-σ_2-continuous but not W(2, 1)-σ_2-continuous.

*(i) If (X, τ_1, τ_2) is an (i, j)-T_s space then f is τ_j-σ_k-continuous if and only if it is *DS(i, j)-σ_k-continuous.*

(ii) If (X, τ_1, τ_2) is an (i, j)-T_s space then f is (a) D(i, j)-σ_k-continuous (b) *D(i, j)-σ_k continuous (c) $\hat{\hat{D}}$(i, j)-σ_k-continuous if and only if it is *DS(i, j)-σ_k-continuous.*

Definition 5.20: A map. f : (X, τ_1, τ_2) $\rightarrow$ (Y, σ_1, σ_2) is called *DS(i, j)-σ_k-irresolute if the inverse image of every σ_k-*gs-closed set in Y is (i, j)-*gs-closed set in X.

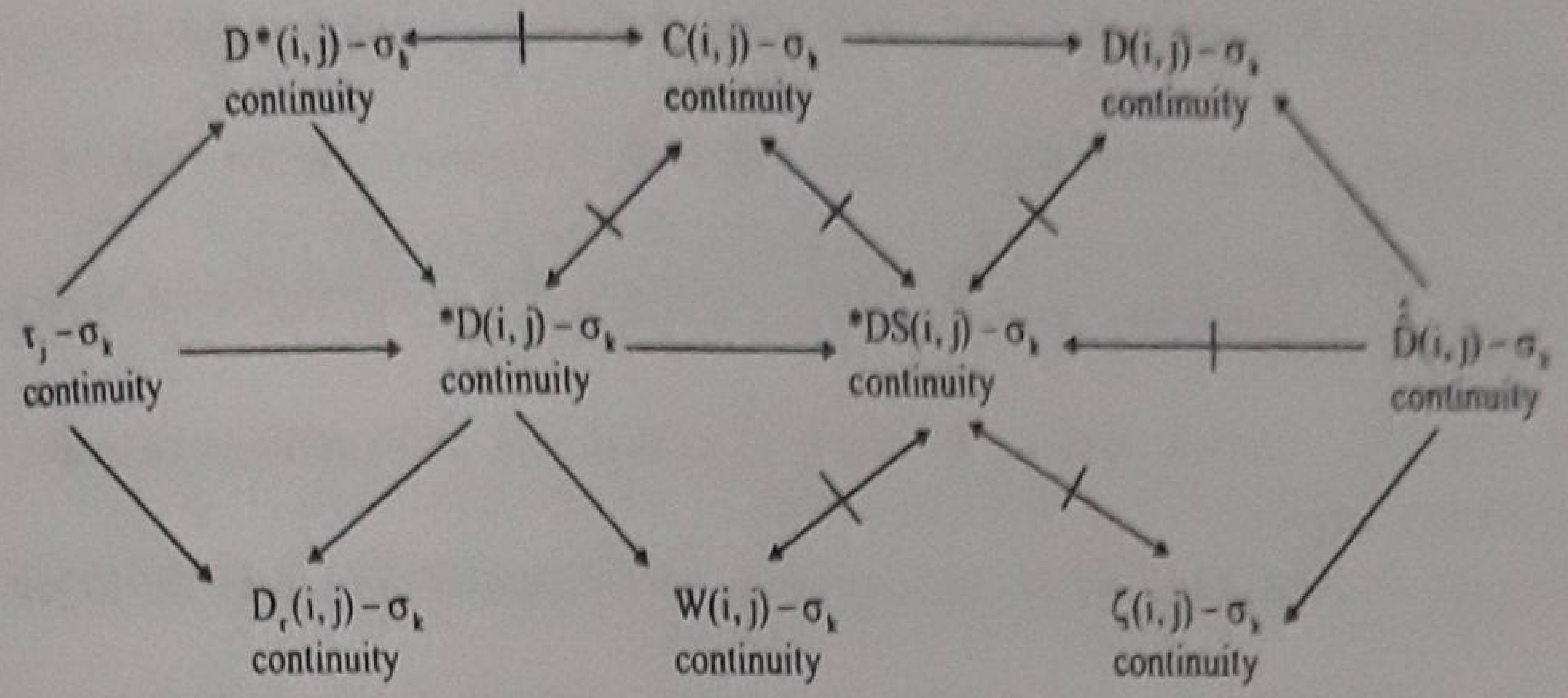

Diagram (5.18)

Proposition 5.21: *If f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is *DS (i, j)-σ_k-irresolute then it is *DS (i, j)-σ_k-continuous.*

However the reverse implication of the above proposition is not true in general as seen from the following example.

Example 5.22: Let X = Y = {a, b, c}, $\tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$, $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma_1 = \{\phi, \{a\}, \{a, b\}, Y\}$, $\sigma_2 = \{\phi, \{a\}, Y\}$. Define f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ by identity mapping, then f is *DS (1, 2)-σ_2-continuous but not *DS (1, 2)-σ_2-irresolute.

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