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# Dark Energy Cosmological Model in Brans-Dicke Theory

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**Abstract:** We have investigated Locally Rotationally Symmetric (LRS) Bianchi type-I dark energy cosmological model with time varying deceleration parameter (DP) in the framework of scalar-tensor Brans-Dicke theory of gravity. We presented the solution of modified Einstein's field equations by assuming bilinearly varying deceleration parameter (BVDP) along with relation $q(t)=\frac{\alpha(1-t)}{1+t}$, here $\alpha>0$ is an arbitrary constant. This study indicates that the Equation of State (EoS) parameter of dark energy is found to be time dependent and its calculated range is in good agreement with the recent observations. The nature of geometrical and physical parameters presented in graphical form.

**Keywords:** Bianchi type-1 Metric, Brans-Dicke Theory, Bilinearly Varying Deceleration Parameter, Dark Energy

**2010 AMS Classification Number:** 83F05.

## 1. Introduction

The Brans-Dicke theory[1] (BDT) was first proposed by Jordan and later refined by C. H. Brans and R. H. Dicke and it finally published by Brans and Dicke in 1961, which is originally based on 'Princeton' Ph.D. thesis. The seed of this theory came from Mach's principle, which states that local physical laws are determined by the large-scale structure of the universe[2]. It is a class of relativistic classical field theories of gravitation, called metric theory. According to BDT the universal gravitational constant $(G)$ remains not a constant, but it varies with space-time. The BDT also suggests that the space-time is equipped with a metric tensor and the gravitational field is represented by the Riemann curvature tensor. The key feature of BDT is the inclusion of scalar field $\phi$ in addition to space-time metric. The presence of scalar field $\phi$ is to describe gravitation together with the metric. The scalar field $\phi$ considers the source of all matter and it is the equivalent to reciprocal of Newtonian gravitational constant i.e. $\left(G \cong \frac{1}{\phi}\right)$. The scalar field $(\phi)$ could be considered as an extension of gravitational field from purely geometric to geometric plus scalar[3,4]. Since this theory contain a scalar field $\phi$ in addition to the metric tensor $g_{\mu\nu}$, so it is known as scalar tensor theory of gravitation. The scalar aspect is important because it cannot be gauged away as the metric and connection components may be local. It is the most promising theory among all existing alternative theories which may effectively address the problem of the early time inflation and the late time accelerating expansion of the universe[5]. In this context it can said that it is a modification of GTR. The Brans-Dicke field equations for combined scalar and tensor field may be written as

(1.1)
$$G_{\mu\nu} = -8\pi\phi^{-1}T_{\mu\nu} - \omega\phi^{-2}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\eta}\phi^{\eta}\right) - \phi^{-1}\left(\phi_{\mu;\nu} - g_{\mu\nu}\phi_{,\eta}^{;\eta}\right),$$

(1.2)
$$\phi_{,\eta}^{;\eta} = 8\pi\left(3+2W\right)^{-1}T,$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is Einstein tensor, $R$ is a scalar curvature, $\omega$ is a dimensionless coupling constant, $T_{\mu\nu}$ is stress-energy tensor, $T$ trace of energy momentum tensor, comma and semicolon denote partial and covariant differential.

Also, the law of conservation of energy may be written as

(1.3)
$$T_{,\nu}^{\mu\nu} = 0.$$

The field equations of this theory contain a coupling constant $w$, which is called Brans-Dicke coupling constant. During literature survey, it is also observed that several researchers have examined the convergence of scalar-tensor theory to GTR[6]. The BDT possesses a notable feature that it reduces to GTR when the Brans-Dicke parameter $\omega \to \infty$.

Many of our peer group researchers have discussed the various aspect of Brans-Dicke cosmology in their research work as cited in their corresponding papers[7-12]. During the study it is norticed that the recent research findings the discovery of the accelerated model of the expanding universe provided a very good platform for observational Cosmology. Recent studied also indicates the presence of unaccounted candidate for dark energy that opposes the self attraction of the matter and cause the expansion of the universe to accelerate[13,14]. This acceleration is due to negative pressure and positive energy density. The current observations of CMB (cosmic microwave background radiation) suggested that our physical universe is expanding, isotropic and homogeneous with a positive cosmological constant. Recently, several authors have been studied Bianchi type models in scalar-tensor theory and concluded that at present the universe has a phase transition from the early deceleration phase to current accelerating phase, which is also a good agreement with the recent observation.

In this communication we have studied LRS Bianchi type-I cosmological models by considering BVDP in scalar-tensor Brans-Dicke theory of gravitation. The outline of the paper is as follows: In section 2, the metric and basic equations are described. Section 3 deals with the solutions of field equations. In Section 4, we discussed the physical and geometric properties of the models. Finally, results and discussion have been made in Section 5.

## 2. The Metric and Field Equations

We consider the LRS Bianchi-I metric as

(2.1) $$ds^2 = dt^2 - R_1^2(t)dx^2 - R_2^2(t)\left(dy^2 + dz^2\right),$$

where $R_1^2$ and $R_2^2$ are directional potential functions in the directions $x$-axes and $y$-axes, respectively and these are the function of cosmic time $t$ only. The energy momentum tensor for the configuration is expressed as

(2.2) $$T_\mu^\nu = \text{diag}\left[T_0^0, T_1^1, T_2^2, T_3^3\right],$$

It can be parameterized as

$$
\begin{aligned}
T_{\mu}^{\nu} &= \mathrm{diag}\left[\rho, -p_x, -p_y, -p_z\right] \\
&= \mathrm{diag}\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho \\
&= \mathrm{diag}\left[1, -\omega, -\left(\omega+\delta\right), -\left(\omega+\delta\right)\right]\rho ,
\end{aligned} \tag{2.3}
$$

here $p_x$, $p_y$, $p_z$ and $\omega_x$, $\omega_y$, $\omega_z$ are the directional pressure components and directional eqaution of state (EoS) parameter in the direction of $x$, $y$ and $z$, respectively. The EoS parameter $\omega = \frac{p}{\rho}$ is the deviation free parameter of the fluid and for isotropy we choose $\omega_x = \omega$. Also introducing the skewness parameter $\delta$ which is the deviation from $\omega$ along both $y$ and $z$ direction. The Brans-Dicke field eqs. (1.1)-(1.3) for the metric eq.(2.1) and with the help of eq.(2.3) can be expressed as

$$
\frac{2\ddot{R}_2}{R_2} + \frac{\dot{R}_2^{\,2}}{R_2^{\,2}} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{2\dot{R}_2\dot{\phi}}{R_2\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi\omega\rho}{\phi} , \tag{2.4}
$$

$$
\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2}\right)\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi\left(\omega+\delta\right)\rho}{\phi} , \tag{2.5}
$$

$$
2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2^{\,2}}{R_2^{\,2}} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \left(\frac{\dot{R}_1}{R_1} + \frac{2\dot{R}_2}{R_2}\right)\frac{\dot{\phi}}{\phi} = \frac{8\pi\rho}{\phi} , \tag{2.6}
$$

$$
\frac{\ddot{\phi}}{\phi} + \left(\frac{\dot{R}_1}{R_1} + \frac{2\dot{R}_2}{R_2}\right)\frac{\dot{\phi}}{\phi} = \frac{8\pi}{\left(3+2\omega\right)}\left(1-3\omega-2\delta\right)\rho . \tag{2.7}
$$

Now, using the law of conservation of energy momentum tensor $T_{\mu\nu}$ for perfect fluid, we have

$$
\dot{\rho} + \rho\left(1+\omega\right)\left(\frac{\dot{R}_1}{R_1} + \frac{2\dot{R}_2}{R_2}\right) + 2\rho\delta\frac{\dot{R}_2}{R_2} = 0 , \tag{2.8}
$$

here dot denotes the partial differential with respect to cosmic time $t$. Now, we define some cosmological parameters as:
The Hubble's parameter $(H)$ may be expressed as

(2.9) $$H = \frac{\dot{a}(t)}{a(t)} = \frac{1}{3}\left(H_1 + 2H_2\right) = \frac{1}{3}\left(\frac{\dot{R}_1}{R_1} + \frac{2\dot{R}_2}{R_2}\right),$$

here $H_1 = \frac{\dot{R}_1}{R_1}$, $H_2 = \frac{\dot{R}_2}{R_2} = H_3$ are directional Hubble parameter in the direction of $x$, $y$, $z$ respectively. The expansion scalar $(\theta)$, anisotropy parameter $(A_m)$ and shear scalar parameter $(\sigma)$ for metric eq. (2.1) defined as

(2.10) $$\theta = 3H = \frac{\dot{R}_1}{R_1} + \frac{2\dot{R}_2}{R_2},$$

(2.11) $$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3} H_i^{\,2} - \frac{1}{6}\theta^2,$$

(2.12) $$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2,$$

In next section we have obtain the solution of field equations and also calculate the values of above mention parameters by applying suitable assumptions.

## 3. Solution of Field Equations

The field eqs. (2.4)-(2.7) are set of four independent equations along with six unknowns parameters *i.e.* $R_1$, $R_2$, $\rho$, $\delta$, $\omega$ and $\phi$. So, it is obvious that, few more constraints are required to find an explicit solution of this system of equations. To get an expclit solution we considered three assumptions as:
For exploring the dynamicity of the universe. We wish to study expansion nature of the universe by assuming DP $(q)$ as a bilinear function of cosmic time as suggested by Mishra and Avtar[8] and given by

(3.1) $$q(t) = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = \frac{\alpha(1-t)}{1+t},$$

here $\alpha > 0$ is an arbitary constant. On Integrating both the sides of the above equation and simplifying, we got expression for Hubble's parameter as:

(3.2) $$H(t) = \frac{1}{\int \{1+q(t)\}\,dt + k_1}.$$

On substituting the value of $q(t)$ from eq.(3.1) into eq.(3.2), we obtained expression for Hubble's parameter $(H)$ as

(3.3) $$H(t) = \frac{1}{(1-\alpha)t + 2\alpha\log(1+t) + k_1},$$

where $k_1$ is a constant of integration. For physical viabality we assumed $k_1 = 0$, because as $t \to 0$, $H \to \infty$ as at the early inflationary phase the universe expansion rate is very-very large.

(3.4) $$H(t) = \frac{1}{(1+\alpha)t} + \frac{\alpha}{(1+\alpha)^2} + \frac{-2\alpha+\alpha^2}{3(1+\alpha)^3}t + \frac{3\alpha - 2\alpha^2 + \alpha^3}{6(1+\alpha)^4}t^2 + O(t^3).$$

Now, on integrating eq.(3.4) w. r. t cosmic time $t$ and simplifying, we have expression for scale factor as

(3.5) $$a(t) = a_0 t^{\frac{1}{1+\alpha}}.e^{h(t)},$$

here $a_0$ is a constant of integration which is taken to be unity for convience and

$$h(t) = \frac{\alpha}{(1+\alpha)^2}t + \frac{-2\alpha+\alpha^2}{6(1+\alpha)^3}t^2 + \frac{3\alpha-2\alpha^2+\alpha^3}{18(1+\alpha)^4}t^3 + O(t^4).$$

Now, to avoid the complicacy of the problem we have assumed shear scalar parameter $(\sigma)$ is proportional to expansion parameter $(\theta)$ i.e. $\sigma \propto \theta$ as punbished by Throne[15] . This condition leads to following equations

(3.6) $$R_1 = \gamma R_2^m ,$$

here $\gamma$ is constant of proportionality. Hence for simplicity and without loss generality we took $\gamma = 1$, therefore,

(3.7) $$R_1 = R_2^m ,$$

where $m$ is the constant of proportionality. The observation of the velocity red-shift relation for extra galactic source indicates that the Hubble's expansion of the universe is isotropic at present within 30% as suggested by Kantowski et al., Kristian et al.[16,17].

(3.8) $$R_1(t) = \left[ t^{\frac{1}{1+\alpha}} .e^{h(t)} \right]^{\frac{3m}{(m+2)}} ,$$

(3.9) $$R_2(t) = \left[ t^{\frac{1}{1+\alpha}} .e^{h(t)} \right]^{\frac{3}{(m+2)}} .$$

Now considering a well-accepted power law relation between the scalar field $\phi$ and the scale factor $\alpha(t)$, as suggested by Pimentel and Johri & Kalyani[18,19] of the form

(3.10) $$\phi(t) \propto a(t)^{\beta} ,$$

(3.11) $$\phi(t) = \phi_0 a(t)^{\beta} = \left[ \phi_0{}^{\frac{1}{1+\alpha}} .e^{h(t)} \right]^{\beta} ,$$

where $\beta$ is an arbitary constant and $\phi_0$ is a constant of proportionality.

(3.12) $$\frac{\sigma}{H} = \sqrt{3}.\frac{(m-1)}{(m+2)} .$$

The observatiional results published by Collins et al.[20] suggested that the limit $\frac{\sigma}{H} \leq 0.3$ and the ratio of shear scalar $\sigma$ to $H$ in the neighbourhood of spatially homogeneous space is found to be constant,

(3.13) $$\frac{\sigma}{\theta} = \text{constant},$$

which also satisfies the condition of normal congruence to the homogeneous expansion.

## 4. Physical and Geometric Properties of the Model

The expressions for physical parameters such as spatial volume parameter $(V)$, directional Hubble's parameters $(H_1, H_2)$, expansion scalar $(\theta)$, shear scalar $(\sigma)$ and anisotropic parameter $(A_m)$ may be expressed as

(4.1) $$V(t) = t^{\frac{3}{1+\alpha}}.e^{3h(t)},$$

(4.2) $$H_1(t) = \frac{3m}{(m+2)}\left\{\frac{1}{(1+\alpha)t} + g(t)\right\},$$

(4.3) $$H_2(t) = H_3(t) = \frac{3}{(m+2)}\left\{\frac{1}{(1+\alpha)t} + g(t)\right\},$$

(4.4) $$\theta(t) = \frac{3}{(1-\alpha)t + 2\alpha\log(1+t)},$$

(4.5) $$\sigma^2(t) = \frac{3(m-1)^2}{(m+2)^2\{(1-\alpha)t + 2\alpha\log(1+t)\}},$$

(4.6) $$A_m = 2\left(\frac{m-1}{m+2}\right)^2.$$

The metric eq.(2.1) afrter putting the values of $R_1(t)$ and $R_2(t)$ now can be expressed as

(4.7) $$ds^2 = dt^2 - \left[t^{\frac{1}{1+\alpha}}.e^{h(t)}\right]^{\frac{6m}{(m+2)}} dx^2 - \left[t^{\frac{1}{1+\alpha}}.e^{h(t)}\right]^{\frac{6m}{(m+2)}} \left(dy^2 + dz^2\right).$$

With the help of above mentioned equations we obtained the expressions for energy density $(\rho)$, EoS parameter $(\omega)$ and deviation parameter $\delta$ as

(4.8) $$\rho(t) = \frac{\phi_0}{16\pi}\left\{\frac{18(2m+1) - b(\omega b - 6)(m+2)^2}{n^2(m+2)^2}\right\} \times \left\{t^{\frac{\beta}{1+\alpha}}.e^{h(t)}\right\}\left(t^k e^t\right)^{\frac{\beta}{n}},$$

(4.9) $$\omega(t) = \frac{-2\left\{\beta^2(m+2)^2(\omega+2) - 6\beta(m+2) + 54\right\}}{18(2m+1) - b(\omega\beta - 6)(m+2)^2}$$
$$\times \frac{1}{(1-\alpha)t + 2\alpha\log(1+t)} - \frac{kn\left\{(m+2)\left(\beta(m+2)+6\right)\right\}}{18(2m+1) - b(\omega b - 6)(m+2)^2} \times \frac{1}{t^2},$$

(4.10) $$\delta(t) = \frac{1}{2} - \frac{(m+2)\left\{\beta^2(\omega+3)(m+2) - 3\beta^2(2\omega+9)\right\}}{18(2m+1) - b(\omega\beta - 6)(m+2)^2}$$
$$\times \left\{\frac{1}{(1-\alpha)t + 2\alpha\log(1+t)}\right\}^2 - \frac{\left\{\left(\beta^2+\beta\right)(m+2)+6\right\}}{18(2m+1) - \beta(\omega\beta - 6)(m+2)^2} \times \frac{1}{t^2}.$$

## 5. Results, Discussion and Conclusions

On the basis of above investigations following conclusions are made:

(i) It is observed from eq. (4.1) that spatial volume $V$ is zero at intial stage i.e. $t = 0$ and after that it is increasing at an expontial rate with cosmic time. It indicates that universe originated from Big-Bang singularity.

(ii) From eq. (3.1) we observed that $q>0$ for $0<t<1$, $q=0$ at $t=1$ and $q<0$ for $t>1$. From this we reached at the concolusion that for $t<1$ universe expansion rate was decreasing during this period or in other words we can say that universe expansion rate was decerasing at early time. Also $q=0$ when $t=1$, i.e. constant expasion rate at $t=1$ and for $t>1$ universe is expanding at an accelerating rate.

(iii) Fig. 1 indicates the variation of DP versus cosmic time $t$ for selected values of $\alpha$. It is clear from Fig. 1 that universe shows transitional phase of expansion i.e. ealry time deceleration phase to current time accelerating phase. For $\alpha=1$ the DP lies between $-1<q<1$, such result are having close resemblance with SNeIa supernova data recorded by Riess et al., Permuttter et al.)[12,13].

(iv) In Fig. 2, the nature of H with cosmic time $t$ has been presented. The Hubble's parameters are decreasing with time since Big-Bang. At early time $H$ was decreasing rapidly as compare to present epoch.

(v) Fig. 3, depicts the varition of energy density of the fluid $(\rho)$ versus time $t$. In Fig. 3, we observed that $\rho$ is decreasing function of time and it converges to a small positive value as $t\to\infty$.

(vi) It is easy to see from Fig. 4 that the EoS parameter $\omega$ is an increasing function of time and it is also noticed from Fig.4 that the values of the EoS parameter is converges to $-1$ i.e. $\omega=-1$, which strengthen the idea among researcher that phantom energy is a hypothetical form of dark energy. Even some times it is more importent than the cosmological constant $\Lambda$. It also indicates the cause of the accelerating expansion and generate Big-Rip scenario and also validate the results published by Caldwell[21].

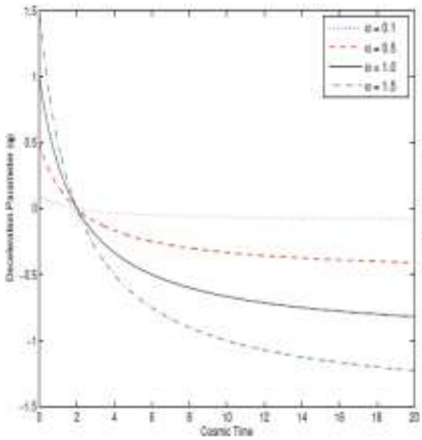

**Figure 1.** Plot of deceleration parameter q versus cosmic time t

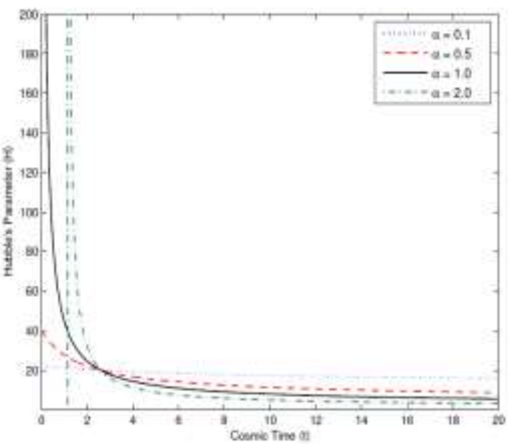


**Figure 2.** Plot of Hubble's parameter H versus cosmic time t

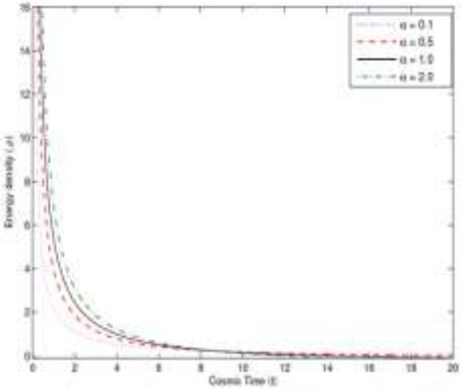


**Figure 3.** Plot of energy density ρ versus cosmic time t

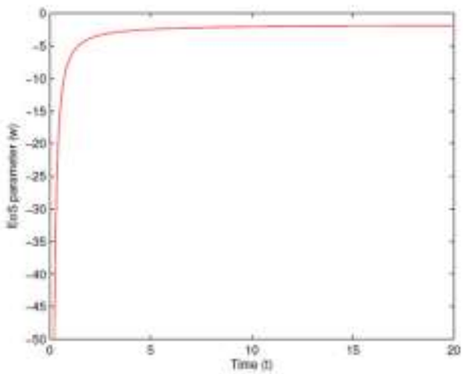


**Figure 4.** Plot of EoS parameter $\omega$ versus cosmic time t.

Note: For Fig. 3 and Fig. 4, we have taken following values of constants: $k = 1$, $m = 2$, $\phi_0 = 1$, $\beta = 0.5$, $\omega = 0.01$

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