MHD Unsteady Slip Flow and Heat Transfer in a Channel With Slip at the Permeable Boundaries

P. R. Sharma and Ruchika Mehta
Department of Mathematics, University of Rajasthan, Jaipur-302055, India
E-mail: profprsharma@yahoo.com

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Abstract: Aim of the paper is to investigate the hydromagnetic unsteady flow of a viscous incompressible electrically conducting fluid in a channel with slip at the permeable non-conducting boundaries. The expressions for velocity and temperature distributions are obtained using regular perturbation technique, discussed numerically and shown through graphs. The expressions of skin-friction and Nusselt number at the boundaries are derived, discussed numerically and their numerical values for various values of physical parameters are shown through graphs.

Key words: Hydromagnetic, unsteady, permeable, channel, skin-friction, Nusselt number.

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1. Introduction

The study of flow of an electrically conducting fluid through a channel with permeable walls not only possesses a theoretical importance but also applicable to many biological and engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, blood flow problems etc. For almost a hundred years, Scientists and Engineers have applied the no-slip boundary condition to fluid flow over a solid surface. While the well accepted no-slip boundary condition has been validated experimentally for a number of macroscopic flows, it remains an assumption not based on physical principles.

Navier[1] proposed a more general boundary condition, which includes the possibility of fluid slip. An extensive theoretical work on hydromagnetic fluid flow in a channel under various situations has been presented by

Hartmann[2], Borkakati and Pop[3] etc. Ruckenstein and Rajora[4] investigated fluid slip in glass capillaries with surfaces made repellent to the flowing liquid. Pal et al.[5] investigated the effect of slip on longitudinal dispersion of tracer particles in a channel bounded by porous media. Makinde[6] studied the problem of laminar flow in channels of slowly varying width permeable boundaries. Barrat and Bocquet[7] discussed significant slip in nanoporous medium when the liquid is sufficiently non-wetting, which increases the effective permeability of the nanoporous medium. The closed form solution for steady periodic and transient velocity field under slip condition have been studied by Khaled and Vafai[8]. The effect of slip condition on MHD steady flow in a channel with peremeable boundaries has been discussed by Makinde and Osalusi[9].

Aim of the paper is to investigate the effect of magnetic field on slip velocity and heat transfer through unsteady flow of an electrically conducting fluid in a non-conducting channel of uniform width.

2. Formulation of the Problem

Consider the unsteady flow of an incompressible viscous electrically conducting fluid in a channel with slip at the peremeable boundaries under the influence of a transverse uniform magnetic field. It is assumed that the fluid has small electrical conductivity and the electro-magnetic force produced is also very small. Cartesian coordinate system is taken such that where x* lies along the centre of the channel and y* is the distance measured in the normal section such that y* = h is the channel's half width. Let u* and v* be the velocity components in of x*- and y*-directions, respectively. The governing equations of continuity, momentum and energy are given by

(2.1)
$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0,$$

(2.2)
$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}+\upsilon\nabla^2 u^*-\frac{\sigma B_0{}^2 u^*}{\rho},$$

(2.3)
$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\upsilon\nabla^2 v^*,$$

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*}+u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}\right)=\kappa\left(\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right)$$
$$+2\mu\left\{\left(\frac{\partial u^*}{\partial x^*}\right)^2+\left(\frac{\partial v^*}{\partial y^*}\right)^2+\frac{1}{2}\left(\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}\right)^2\right\}+\sigma u^{*2}B_0{}^2+Q^*. \tag{2.4}$$

where $\nabla^2=\frac{\partial^2}{\partial x^{*2}}+\frac{\partial^2}{\partial y^{*2}}$, p the fluid pressure , ρ the fluid density, $\upsilon(=\mu/\rho)$ the kinematic viscosity, σ the electrical conductivity, $B_0=\mu_e H_0$ the electromagnetic induction, μ_e the magnetic permeability, C_p specific heat at constant pressure, κ thermal conductivity and H_0 the intensity of magnetic field.

The corresponding boundary conditions are

$$y^*=0:\frac{\partial u^*}{\partial y^*}=0, v^*=0, \frac{\partial T^*}{\partial y^*}=0, \qquad y^*=h:\mu\frac{\partial u^*}{\partial y^*}=-\beta u^*,$$
$$v^*(t^*)=V\{1+\varepsilon\exp(i\omega^* t^*)\}, \qquad T^*=T_s\{1+\varepsilon\exp(i\omega^* t^*), \tag{2.5}$$

where μ is the dynamic viscosity coefficient, β the coefficient of sliding friction, V the mean suction velocity and T_s the static temperature.

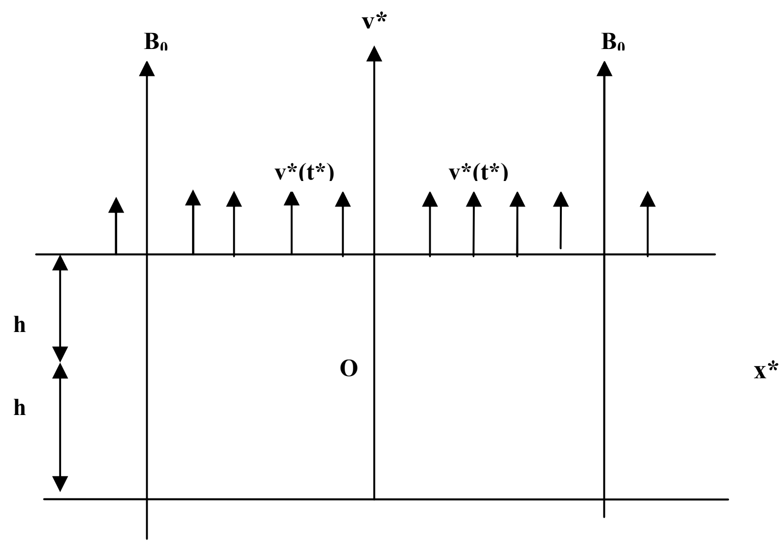

Fig. 1: Physical Model

3. Method of Solution

Intoducing the following dimensionless quantities

$$x=\frac{x^*}{h},\ y=\frac{y^*}{h},\ t=\frac{Vt^*}{h},\ \omega=\frac{h\omega^*}{V},\ u=\frac{u^*}{V},\ v=\frac{v^*}{V},\ p=\frac{p^*h}{\rho\nu V},$$

(3.1)

$$\mathrm{Re}=\frac{Vh}{\nu},\ M^2=\frac{h^2\sigma B_0{}^2}{\mu},\ L=\frac{\mu}{h\beta},\ Ec=\frac{V^2}{C_pT_0},\ \mathrm{Pr}=\frac{\mu C_p}{\kappa},$$

into the equations (2.1) to (2.4), we get

(3.2) $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

(3.3) $$\mathrm{Re}\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\mathrm{Re}\,M^2u,$$

(3.4) $$\mathrm{Re}\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2},$$

(3.5) $$\mathrm{Pr}\,\mathrm{Re}\left[\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}\right]=2\,\mathrm{Pr}\,Ec\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]$$

$$\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}++M^2\,\mathrm{Pr}\,Ecu^2+Q,$$

where Re is the cross-flow Reynolds number (with Re > 0 indicating suction and Re < 0 is for injection), M the Hartmann number, Pr the Prandtl number, Ec the Eckert number and L is the slip parameter.

The corresponding boundary conditions in dimensionless form are

$$y=0:\frac{\partial u}{\partial y}=0, v=0, \frac{\partial \theta}{\partial y}=0,$$

(3.6)

$$y=1: u=-L\frac{\partial u}{\partial y}, v=1+\varepsilon\exp(i\omega t), \theta=1+\varepsilon\exp(i\omega t).$$

Equations (3.2) to (3.5) show that their solutions are not easily tractible. Therefore perturbation method is a global approach to find the solution of such differential equations.

Evidently, the parameter ε is assumed to be small such that $0 < \varepsilon < 1$, therefore assuming

(3.7) $$f(x,y,t) = f_0(x,y) + \varepsilon f_1(x,y)e^{i\omega t},$$

where f stands for u, v, p or θ. Using equation (3.7) into the equations (3.2) to (3.5) and equating the coefficients of O(ε), we get

Zeroth-order equations

(3.8) $$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0,$$

(3.9) $$\mathrm{Re}\left[u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right] = -\frac{\partial p_0}{\partial x} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} - \mathrm{Re}\,M^2 u_0,$$

(3.10) $$\mathrm{Re}\left[u_0\frac{\partial v_0}{\partial x} + v_0\frac{\partial v_0}{\partial y}\right] = -\frac{\partial p_0}{\partial y} + \frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2},$$

(3.11) $$\begin{aligned}\Pr\mathrm{Re}\left[u_0\frac{\partial \theta_0}{\partial x} + v_0\frac{\partial \theta_0}{\partial y}\right] &= \frac{\partial^2 \theta_0}{\partial x^2} + \frac{\partial^2 \theta_0}{\partial y^2} + +M^2\Pr Ec{u_0}^2 + Q \\ &+2\Pr Ec\left[\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial y}\right)^2 + \frac{1}{2}\left\{\left(\frac{\partial u_0}{\partial y}\right)^2 + \left(\frac{\partial v_0}{\partial x}\right)^2\right\} + \left(\frac{\partial u_0}{\partial y}\right)\left(\frac{\partial v_0}{\partial x}\right)\right]\end{aligned}$$

First-order equations

(3.12) $$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0,$$

(3.13) $$\begin{aligned}\mathrm{Re}\left[i\omega u_1 + u_1\frac{\partial u_0}{\partial x} + u_0\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_0}{\partial y} + v_0\frac{\partial u_1}{\partial y}\right] &= -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \\ &- \mathrm{Re}\,M^2 u_1,\end{aligned}$$

(3.14) $$\mathrm{Re}\left[i\omega v_1 + u_1\frac{\partial v_0}{\partial x} + u_0\frac{\partial v_1}{\partial x} + v_1\frac{\partial v_0}{\partial y} + v_0\frac{\partial v_1}{\partial y}\right] = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2},$$

(3.15) $$\mathrm{Re}\left[i\omega \theta_1 + u_1\frac{\partial \theta_0}{\partial x} + u_0\frac{\partial \theta_1}{\partial x} + v_1\frac{\partial \theta_0}{\partial y} + v_0\frac{\partial \theta_1}{\partial y}\right] = \frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2}$$

$$+4\Pr Ec\left[\left(\frac{\partial u_0}{\partial x}\right)\left(\frac{\partial u_1}{\partial x}\right)+\left(\frac{\partial v_0}{\partial y}\right)\left(\frac{\partial v_1}{\partial y}\right)\right]+2\Pr Ec\left[\left(\frac{\partial u_0}{\partial y}\frac{\partial u_1}{\partial y}\right)\right.$$

$$\left.+\left(\frac{\partial v_0}{\partial x}\frac{\partial v_1}{\partial x}\right)+\left(\frac{\partial v_0}{\partial x}\right)\left(\frac{\partial u_1}{\partial y}\right)+\left(\frac{\partial v_0}{\partial y}\right)\left(\frac{\partial v_1}{\partial x}\right)+M^2 u_0 u_1\right]$$

Now the corresponding boundary conditions are

(3.16)
$$y=0:\frac{\partial u_0}{\partial y}=0=\frac{\partial u_1}{\partial y}, v_0=0=v_1, \frac{\partial \theta_0}{\partial y}=0=\frac{\partial \theta_1}{\partial y},$$
$$y=1: u_0=-K\frac{\partial u_0}{\partial y}, u_1=-K\frac{\partial u_1}{\partial y}, v_0=1=v_1, \theta_0=1=\theta_1.$$

Eliminating p_0 and p_1 between the equations (3.9) and (3.10), and (3.13) and (3.14) respectively and introducing stream-function ψ and vorticity ω as given below

(3.17)
$$u_0=\frac{\partial \psi_0}{\partial y},\quad v_0=-\frac{\partial \psi_0}{\partial x},\quad \omega_0=-\left(\frac{\partial^2 \psi_0}{\partial x^2}+\frac{\partial^2 \psi_0}{\partial y^2}\right),$$

and

(3.18)
$$u_1=\frac{\partial \psi_1}{\partial y},\quad v_1=-\frac{\partial \psi_1}{\partial x},\quad \omega_1=-\left(\frac{\partial^2 \psi_1}{\partial x^2}+\frac{\partial^2 \psi_1}{\partial y^2}\right).$$

Using equations (3.17) and (3.18) into the equations (3.8) to (3.11) and (3.12) to (3.15) respectively; we get

(3.19)
$$\nabla^2 \omega_0=\mathrm{Re}\left[\frac{\partial(\omega_0,\psi_0)}{\partial(x,y)}-M^2\frac{\partial^2 \psi_0}{\partial y^2}\right],\quad \omega_0=-\nabla^2\psi_0,$$

and

(3.20)
$$\nabla^2 \omega_1=\mathrm{Re}\left[\frac{\partial(\omega_1,\psi_1)}{\partial(x,y)}-M^2\frac{\partial^2 \psi_1}{\partial y^2}\right],\qquad \omega_1=-\nabla^2\psi_1.$$

The corresponding boundary conditions are

(3.21)
$$y=0: \frac{\partial^2\psi_0}{\partial y^2}=0,\ \frac{\partial\psi_0}{\partial x}=0,\ \frac{\partial^2\psi_1}{\partial y^2}=0,\ \frac{\partial\psi_1}{\partial x}=0,$$
$$y=1: \frac{\partial\psi_0}{\partial y}=-K\frac{\partial^2\psi_0}{\partial y^2}, \frac{\partial\psi_0}{\partial x}=-1\frac{\partial\psi_1}{\partial y}=-K\frac{\partial^2\psi_1}{\partial y^2}, \frac{\partial\psi_1}{\partial x}=-1.$$

Following Berman[10], substituting

(3.22) $$\psi_0 = xF_0(y), \omega_0 = xG_0(y), \quad \psi_1 = xF_1(y), \omega_1 = xG_1(y);$$

into the equations (3.19) and (3.20), and equating the coefficients of O(x), we get

(3.23) $$\frac{d^2G_0}{dy^2} = \mathrm{Re}\left[G_0\frac{dF_0}{dy} - F_0\frac{dG_0}{dy} + M^2G_0\right],$$

(3.24) $$G_0 = -\frac{d^2F_0}{dy^2},$$

(3.25) $$\frac{d^2G_1}{dy^2} = \mathrm{Re}\left[\mathrm{Re}\, i\omega G_1 + G_0\frac{dF_1}{dy} - F_1\frac{dG_0}{dy} + G_1\frac{dF_0}{dy} - F_0\frac{dG_1}{dy} + M^2G_1\right],$$

(3.26) $$G_1 = -\frac{d^2F_1}{dy^2}.$$

The corresponding boundary conditions are reduced to

(3.27)
$$y=0: \frac{d^2F_0}{dy^2}=0, F_0=0, \frac{d^2F_1}{dy^2}=0, F_1=0,$$
$$y=1: \frac{dF_0}{dy}=-K\frac{d^2F_0}{dy^2}, F_0=-1, \frac{dF_1}{dy}=-K\frac{d^2F_1}{dy^2}, F_1=-1.$$

Equations (3.23) to (3.26) are coupled non-linear differential equations with the boundary conditions (32). Since Re is very small, therefore F_0, F_1, G_0 and G_1 can be expanded in the power of Re, as given below

(3.28) $$H = H_0 + \mathrm{Re}\, H_1 + O(\mathrm{Re}^2),$$

where H stands for F_0, F_1, G_0 or G_1. Substituting (3.28) into the equations (3.23) to (3.26) and equating the coefficients of like powers of Re, we get

(3.29) $$\frac{d^2G_{00}}{dy^2}=0,\quad G_{00}=-\frac{d^2F_{00}}{dy^2},$$

(3.30) $$\frac{d^2G_{01}}{dy^2}=\left[G_{00}\frac{dF_{00}}{dy}-F_{00}\frac{dG_{00}}{dy}+M^2G_{00}\right],\quad G_{01}=-\frac{d^2F_{01}}{dy^2},$$

(3.31) $$\frac{d^2G_{01}}{dy^2}=0,\quad G_{10}=-\frac{d^2F_{10}}{dy^2},$$

(3.32) $$\frac{d^2G_{11}}{dy^2}=\left[i\omega G_{10}+G_{00}\frac{dF_{10}}{dy}-F_{10}\frac{dG_{00}}{dy}+G_{10}\frac{dF_{00}}{dy}-F_{00}\frac{dG_{10}}{dy}+M^2G_{10}\right],$$
$$G_{11}=-\frac{d^2F_{11}}{dy^2}.$$

The corresponding boundary conditions are

$$y=0:\ \frac{d^2F_{00}}{dy^2}=\frac{d^2F_{01}}{dy^2}=\frac{d^2F_{10}}{dy^2}=\frac{d^2F_{11}}{dy^2}=0,\quad F_{00}=F_{01}=F_{10}=F_{11}=0,$$

(3.33) $$y=1:\ \frac{dF_{00}}{dy}=-K\frac{d^2F_{00}}{dy^2},\quad \frac{dF_{01}}{dy}=-K\frac{d^2F_{01}}{dy^2},\quad \frac{dF_{10}}{dy}=-K\frac{d^2F_{10}}{dy^2},$$
$$\frac{dF_{11}}{dy}=-K\frac{d^2F_{11}}{dy^2},\quad F_{00}=F_{10}=-1,\quad F_{01}=F_{11}=0.$$

Equations (3.29) to (3.32) are ordinary second order differential equations and solved under the boundary conditions (3.33). Through straight forward calculations, the solutions of $G_{00}(y)$, $F_{00}(y)$, $G_{01}(y)$, $F_{01}(y)$, $G_{10}(y)$, $F_{10}(y)$, $G_{11}(y)$ and $F_{11}(y)$ are known and given by

(3.34) $$G_{00}(y)=a_1y=G_{10},$$

(3.35) $$F_{00}(y)=a_2y^3+a_3y=F_{10},$$

(3.36) $$G_{01}(y)=a_{10}y^5+a_{11}y^3+a_{12}y,$$

(3.37) $$F_{01}(y)=a_6y^7+a_7y^5+a_8y^3++a_9y,$$

(3.38) $$G_{11}(y)=\frac{a_1a_2}{5}y^5+\frac{M^2a_1}{6}y^3+C_9y+C_{10}+i\frac{\omega a_1}{6}y^3,$$

(3.39) $$F_{11}(y)=F_{11R}(y)+iF_{11I}(y),$$

where

(3.40) $$F_{11R}(y)=a_{23}y^7+a_{22}y^5+a_{24}y^3+a_{26}y,$$

and

(3.41) $$F_{11I}(y)=a_{21}y^5+a_{25}y^3+a_{27}y;$$

where a_1 to a_{27}, C_1 to C_{10} are constants and their expressions are given in Appendix.

Following Verma and Bansal[11], θ_0 and θ_1 can be expanded in the powers of x, as given below

(3.42) $$\theta_0=\phi_0+x^2\zeta_0 \quad \text{and} \quad \theta_1=\phi_0+x^2\zeta_1.$$

Substituting (3.42) into the equations (3.11) and (3.15), and equating the coefficients of like powers of x, we get

(3.43) $$\zeta_0''=\Pr\mathrm{Re}(2F_0'\zeta_0-F_0\zeta_0')-\Pr Ec\{(F_0'')^2+M^2(F_0')^2\},$$

(3.44) $$\phi_0''+2\zeta_0=-\Pr\mathrm{Re}\,F_0\phi_0'-4\Pr Ec(F_0')^2-Q,$$

(3.45) $$\begin{aligned}\zeta_1''&=\Pr\mathrm{Re}(i\omega\zeta_1+2F_1'\zeta_0+2F_0'\zeta_1-F_0\zeta_1'-F_1\zeta_0')\\&\quad-2\Pr Ec(F_0''F_1''+M^2F_0'F_1'),\end{aligned}$$

(3.46) $$\phi_1''+2\zeta_1=\Pr\mathrm{Re}(i\omega\phi_1-F_0\phi_1'-F_1\phi_0')-8\Pr EcF_0'F_1'.$$

The corresponding boundary conditions are reduced to

(3.47) $$\begin{aligned}&y=0:\frac{d\phi_0}{dy}=0=\frac{d\phi_1}{dy},\frac{d\zeta_0}{dy}=0=\frac{d\zeta_1}{dy};\\&y=1:\phi_0=1=\phi_1,\zeta_0=0=\zeta_1.\end{aligned}$$

Since Re is small, therefore ϕ_0, ϕ_1, ζ_0 and ζ_1 can be expanded in the powers of Re as given below

(3.48) $$\phi_0 = \sum_{i=0}^{\infty} \mathrm{Re}^i \, \phi_{0i}, \qquad \zeta_0 = \sum_{i=0}^{\infty} \mathrm{Re}^i \, \zeta_{0i};$$

(3.49) $$\phi_1 = \sum_{i=0}^{\infty} \mathrm{Re}^i \, \phi_{1i}, \qquad \zeta_1 = \sum_{i=0}^{\infty} \mathrm{Re}^i \, \zeta_{1i}.$$

Substituting (3.48) and (3.49) into the equations (3.43) to (3.46) and equating the coefficients of like powers of Re, we get

(3.50) $$\zeta_{00}'' = -\Pr Ec[(F_{00}'')^2 + M^2(F_{00}')^2,$$

(3.51) $$\phi_{00}'' + 2\zeta_{00} = -4\Pr Ec(F_{00}')^2 - Q,$$

(3.52) $$\zeta_{01}'' = \Pr[2F_{00}'\zeta_{00} - F_{00}\zeta_{00}'] - 2\Pr Ec[F_{00}''F_{01}'' + M^2 F_{00}'F_{01}'],$$

(3.53) $$\phi_{01}'' + 2\zeta_{01} = -\Pr F_{00}\phi_{00}' - 8\Pr Ec F_{00}'F_{01}',$$

(3.54) $$\zeta_{10}'' = -2\Pr Ec[F_{00}''F_{10}'' + M^2 F_{00}'F_{10}'],$$

(3.55) $$\phi_{10}'' + 2\zeta_{10} = -8\Pr Ec F_{00}'F_{10}',$$

(3.56) $$\begin{aligned}\zeta_{11}'' &= \Pr[i\omega\zeta_{10} + 2(F_{10}'\zeta_{00} + F_{00}'\zeta_{10}) - (F_{00}\zeta_{10}' + F_{10}\zeta_{00}') \\ &-2\Pr Ec[F_{00}''F_{11}'' + F_{01}''F_{10}'' + M^2(F_{01}'F_{10}' + F_{00}'F_{11}')],\end{aligned}$$

(3.57) $$\phi_{11}'' + 2\zeta_{11} = \Pr[i\omega\phi_{10} - (F_{00}\phi_{10}' + F_{10}\phi_{00}')] - 8\Pr Ec[F_{01}'F_{10}' + F_{00}'F_{11}'].$$

The corresponding boundary conditions are reduced to

(3.58) $$\begin{aligned}&y = 0: \zeta_{00}' = 0 = \phi_{00}', \zeta_{01}' = 0 = \phi_{01}', \zeta_{10}' = 0 = \phi_{10}', \zeta_{11}' = 0 = \phi_{11}', \\ &y = 1: \zeta_{00} = 0, \phi_{00} = 1, \zeta_{01} = 0 = \phi_{01}, \zeta_{10} = 0, \phi_{10} = 1, \zeta_{11} = 0 = \phi_{11.}\end{aligned}$$

The equations (3.50) to (3.57) are ordinary coupled differential equations and solved under the boundary conditions (3.58). Through straight forward calculations, the solutions of $\zeta_{00}(y)$, $\phi_{00}(y)$, $\zeta_{01}(y)$, $\phi_{01}(y)$, $\zeta_{10}(y)$, $\phi_{10}(y)$, $\zeta_{11}(y)$ and $\phi_{11}(y)$ are known and given by

(3.59) $$\zeta_{00}(y) = b_6 y^6 + b_7 y^4 + b_8 y^2 + b_5,$$

(3.60) $$\phi_{00}(y) = b_{27}y^8 + b_{28}y^6 + b_{29}y^4 + b_{30}y^2 + b_{26},$$

(3.61) $$\zeta_{01}(y) = b_{16}y^{10} + b_{17}y^8 + b_{18}y^6 + b_{19}y^4 + b_{20}y^2 + b_{15},$$

(3.62) $$\phi_{01}(y) = b_{39}y^{12} + b_{40}y^{10} + b_{41}y^8 + b_{42}y^6 + b_{43}y^4 + b_{44}y^2 + b_{38},$$

(3.63) $$\zeta_{10}(y) = b_{50}y^6 + b_{51}y^4 + b_{52}y^2 + b_{49},$$

(3.64) $$\phi_{10}(y) = b_{59}y^8 + b_{60}y^6 + b_{61}y^4 + b_{62}y^2 + b_{58},$$

(3.65) $$\zeta_{11}(y) = \zeta_{11R}(y) + i\zeta_{11I}(y),$$

(3.66) $$\phi_{11}(y) = \phi_{11R}(y) + i\phi_{11I}(y),$$

where

(3.67) $$\zeta_{11R}(y) = b_{78}y^{10} + b_{79}y^8 + b_{80}y^6 + b_{81}y^4 + b_{82}y^2 + b_{83},$$

(3.68) $$\zeta_{11I}(y) = b_{74}y^8 + b_{75}y^6 + b_{76}y^4 + b_{77}y^2 + b_{84},$$

(3.69) $$\phi_{11R}(y) = b_{103}y^{12} + b_{104}y^{10} + b_{105}y^8 + b_{106}y^6 + b_{107}y^4 + b_{108}y^2 + b_{109},$$

and

(3.70) $$\phi_{11I}(y) = b_{98}y^{10} + b_{99}y^8 + b_{100}y^6 + b_{101}y^4 + b_{102}y^2 + b_{110}.$$

where b_1 to b_{109} are constants and their expressions are given in Appendix. Hence ϕ and ζ are obtained as given below

(3.71) $$\phi = \phi_{00} + \mathrm{Re}\,\phi_{01} + \varepsilon\phi_{10}\cos\omega t + \varepsilon\,\mathrm{Re}(\phi_{11R}\cos\omega t - \phi_{11I}\sin\omega t),$$

(3.72) $$\zeta = \zeta_{00} + \mathrm{Re}\,\zeta_{01} + \varepsilon\zeta_{10}\cos\omega t + \varepsilon\,\mathrm{Re}(\zeta_{11R}\cos\omega t - \zeta_{11I}\sin\omega t).$$

Finally, the expressions of $u(x, y, t)$, $v(x, y, t)$ and $\theta(x, y, t)$ are known and given by

(3.73) $$\begin{aligned} u(x, y, t) &= x(F_{00}' + \mathrm{Re}\,F_{01}') \\ &\quad + \varepsilon x\left[F_{10}'\cos\omega t + \mathrm{Re}(F_{11R}'\cos\omega t - F_{11I}'\sin\omega t)\right], \end{aligned}$$

$$v(x,y,t) = -(F_{00} + \mathrm{Re}\, F_{01}) - \varepsilon\left[F_{10}\cos\omega t + \mathrm{Re}(F_{11R}\cos\omega t - F_{11I}\sin\omega t)\right], \tag{3.74}$$

$$\theta(x,y,t) = \phi(y,t) + x^2\zeta(y,t). \tag{3.75}$$

4. Skin-friction Coefficient

Skin-friction coefficient at upper plate is given by

$$(C_f)_1 = \frac{\tau h}{\mu V} = \left(\frac{\partial u}{\partial y}\right)_{y=1}, \tag{4.1}$$

where $\tau = \mu\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)_{y^*=h}$.

Hence the expression of skin-friction coefficient at upper plate is given by

$$(C_f)_1 = x(F_{00}'' + \mathrm{Re}\, F_{01}'') + \varepsilon x\left[F_{10}''\cos\omega t + \mathrm{Re}(F_{11R}''\cos\omega t - F_{11I}''\sin\omega t)\right]. \tag{4.2}$$

5. Nusselt number

The rate of heat transfer in terms of Nusselt number at upper plate is given by

$$(Nu)_1 = \frac{qh}{\kappa T_w} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=1}, \tag{5.1}$$

where $q = -\kappa\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=h}$.

Hence, the expression of the Nusselt number at upper plate is given by

$$(Nu)_1 = -\left(\frac{d\phi}{dy} + x^2\frac{d\zeta}{dy}\right)_{y=1}. \tag{5.2}$$

6. Results and discussion

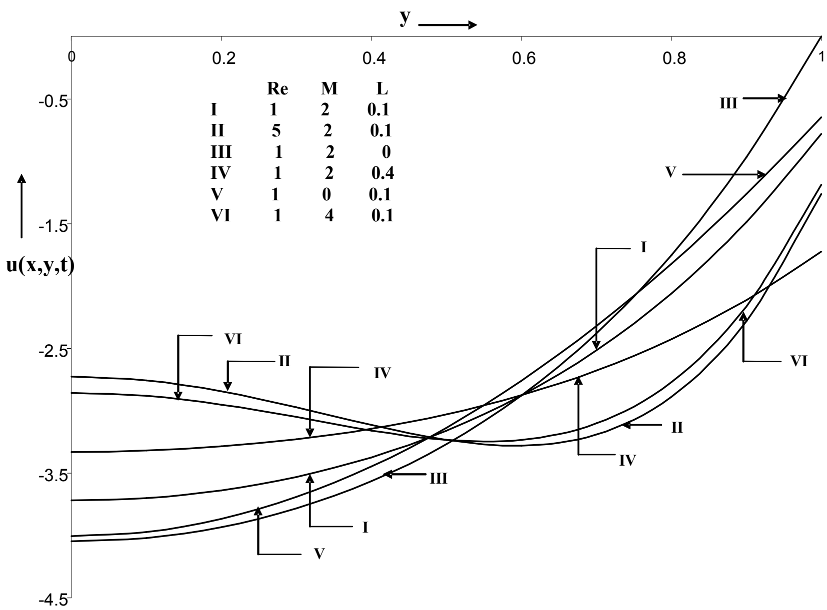

Fig. 2 Variation of axial velocity versus y when ω = 5, ωt = π/6, ε = 0.5

It is observed from Fig. 2 that axial velocity of fluid increases near the lower plate due to increase in the cross-flow Reynolds number, slip parameter or magnetic field intensity, while different behaviour is noted at the upper plate.

It is noted from Fig. 3 that transverse velocity of fluid decreases due to increase in the cross-flow Reynolds number, slip parameter or magnetic field intensity.

Fig. 4 depicts that fluid temperature increases near the lower plate due to increase in the Prandtl number, the cross-flow Reynolds number, the Eckert number, magnetic field intensity or volumetric rate of heat generation parameter, while it decreases due to increase in slip parameter.

It is noted from Fig. 5 that skin-friction coefficient at the upper plate increases due to increase in the magnetic field intensity or cross-flow Reynolds number, while it decreases due to increase in slip parameter; but it decreases due to increase in the cross-flow Reynolds number in the absence of magnetic field intensity.

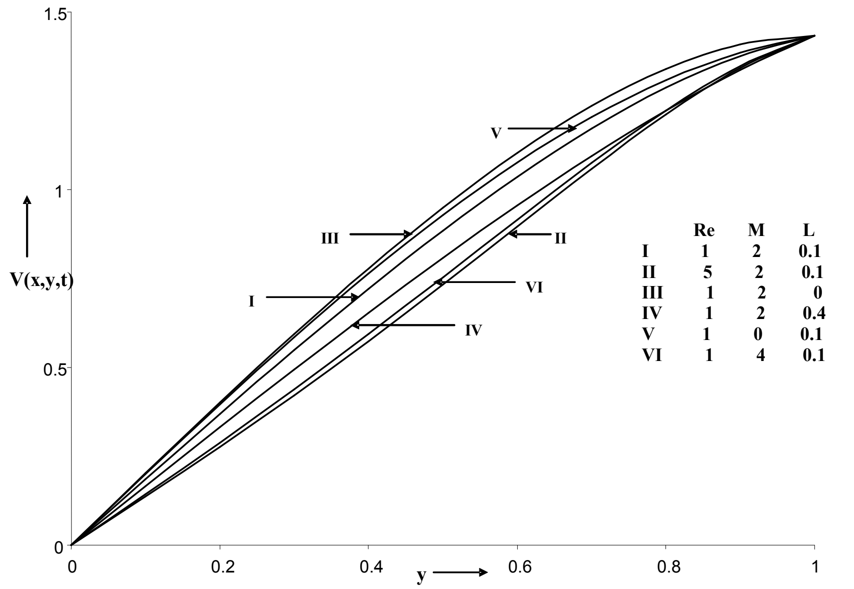

Fig. 3 Variation of transverse velocity v versus y when ω = 5,ωt = π/6, ε = 0.5

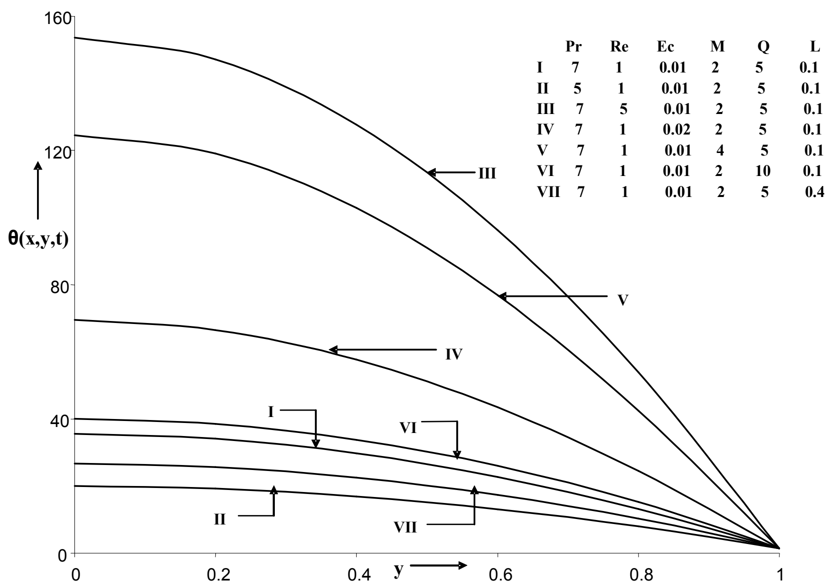

Fig. 4 Temperature distribution versus y when ε = 0.5, ω = 5, ωt = π/3

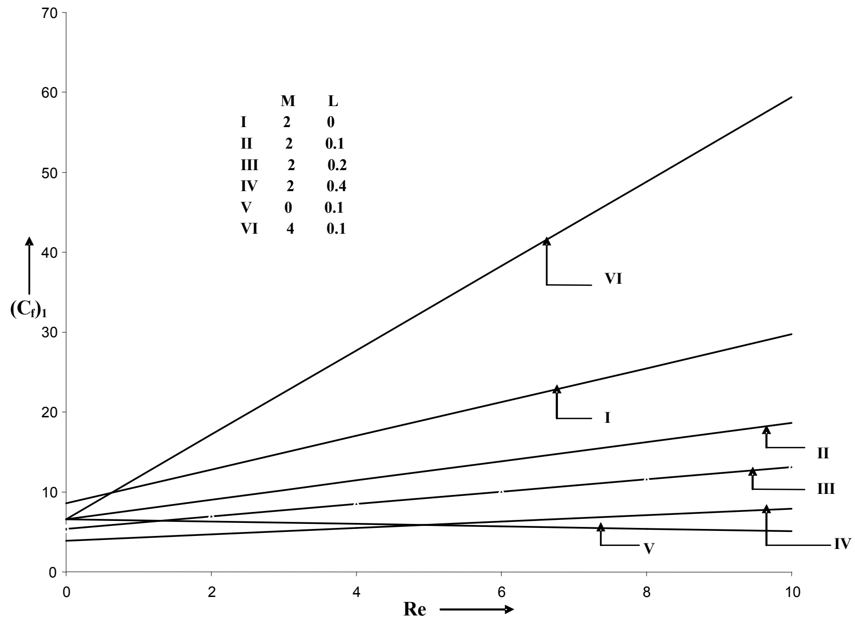

Fig. 5 Variation of skin-friction at upper plate versus Re when ε = 0.5, ω = 5, ωt = π/3

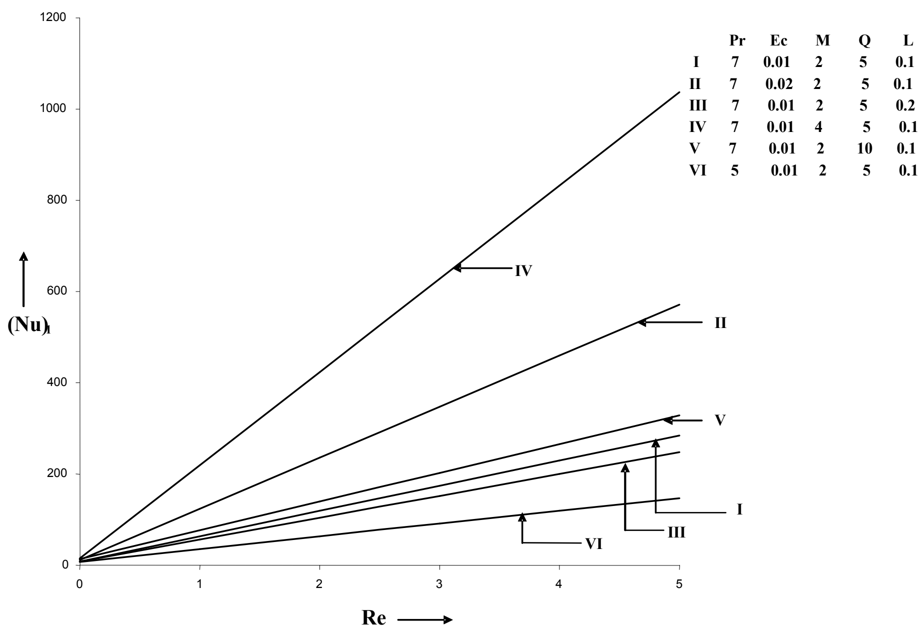

Fig. 6 Nusslet number at upper plate versus Re when ε = 0.5, ω = 5, ωt = π/3

It is noted from Fig. 6 that the Nusselt number at the upper plate decreases due to increase in slip parameter, while it increases due to increase in the Eckert number, the Prandtl number,the magnetic field intensity, cross-flow Reynolds number or volumetric rate of heat generation parameter .

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Appendix

$$C_1 = \frac{-3}{3k+1}, \quad C_3 = -\frac{(6k+3)}{2(3k+1)}, \quad a_1 = C_1, \quad a_2 = -\frac{C_1}{6}, \quad a_3 = C_3,$$

$$a_4 = -\frac{a_1 a_2}{420} - \frac{H^2 a_1}{120}, \quad a_5 = -\frac{a_1 a_2}{60}(6k+1) - \frac{H^2 a_1}{24}(4k+1),$$

$$C_5 = \frac{3}{3k+1}(a_5 - a_4), \quad C_7 = \frac{C_5}{6} - a_4, \quad a_6 = -\frac{a_1 a_2}{420}, \quad a_7 = -\frac{H^2 a_1}{120},$$

$$a_8 = -\frac{C_5}{6}, \quad a_9 = C_7, \quad a_{10} = \frac{a_1 a_2}{10}, \quad a_7 = \frac{H^2 a_1}{6}, \quad a_{12} = C_5,$$

$$a_{13} = -\frac{a_1 a_2}{210} - \frac{H^2 a_1}{120}, \; a_{14} = -\frac{\omega a_1}{120}, \; a_{15} = -\frac{a_1 a_2}{30}(6k+1) - \frac{H^2 a_1}{24}(4k+1),$$

$$a_{16} = -\frac{\omega a_1}{24}(4k+1), \; a_{17} = \frac{3(a_{15} - a_{13})}{3k+1}, \; a_{18} = \frac{3(a_{16} - a_{14})}{3k+1}, \; a_{19} = \frac{a_{17}}{6} - a_{13},$$

$$a_{20} = \frac{a_{18}}{6} - a_{14}, \quad a_{21} = -\frac{\omega a_1}{120}, \; a_{22} = \frac{H^2 a_1}{120}, \; a_{23} = -\frac{a_1 a_2}{210}, \; a_{24} = -\frac{a_{17}}{6},$$

$$a_{25} = -\frac{a_{18}}{6}, \quad a_{26} = a_{19}, \quad a_{27} = a_{20}, \quad b_1 = -9 \Pr EcH^2 a_2{}^2,$$

$$b_2 = -\Pr Ec(6H^2 a_2 a_3 + 36 a_2{}^2), \quad b_3 = -\Pr EcH^2 a_3{}^2, \quad b_5 = -(b_6 + b_7 + b_8),$$

$$b_6 = \frac{b_1}{30}, \quad b_7 = \frac{b_2}{12}, \quad b_8 = \frac{b_3}{2}, \quad b_9 = -42 \Pr Eca_2 a_6 H^2,$$

$$b_{10} = \Pr(2a_2 b_7 - 4a_3 b_6) - 504 \Pr Eca_2 a_6 - 30 \Pr Eca_2 a_7 H^2 - 14 \Pr Eca_3 a_6 H^2,$$

$$b_{11} = \Pr(4a_2 b_8 - 2a_3 b_7) - 240 \Pr Eca_2 a_7 - 18 \Pr Eca_2 a_8 H^2 - 10 \Pr Eca_3 a_7 H^2,$$

$$b_{12} = 6 \Pr a_2 b_5 - 72 \Pr Eca_2 a_8 - 6 \Pr Eca_2 a_9 H^2 - 6 \Pr Eca_3 a_8 H^2,$$

$$b_{13} = 2 \Pr a_3 b_5 - 2 \Pr Eca_3 a_9 H^2, \quad b_{15} = -(b_{16} + b_{17} + b_{18} + b_{19} + b_{20}),$$

$$b_{16} = \frac{b_9}{90}, \; b_{17} = \frac{b_{10}}{56}, \; b_{18} = \frac{b_{11}}{30}, \; b_{19} = \frac{b_{12}}{12}, \; b_{20} = \frac{b_{13}}{2}, \; b_{21} = -2b_6,$$

$$b_{22} = -2b_7 - 36 \Pr Eca_2{}^2, \; b_{23} = -2b_8 - 24 \Pr Eca_2 a_3,$$

$$b_{24} = -2b_5 - 4 \Pr Eca_3{}^2 - Q, \quad b_{26} = 1 - (b_{27} + b_{28} + b_{29} + b_{30}), \quad b_{27} = \frac{b_{21}}{56},$$

$$b_{28}=\frac{b_{22}}{30},\qquad b_{29}=\frac{b_{23}}{12},\qquad b_{30}=\frac{b_{24}}{2},\qquad b_{31}=-2b_{16}-8\Pr a_2b_{27},$$

$$b_{32}=-2b_{17}-6\Pr a_2b_{28}-8\Pr a_3b_{27}-168\Pr Eca_2a_6,$$

$$b_{33}=-2b_{18}-4\Pr a_2b_{29}-6\Pr a_3b_{28}-120\Pr Eca_2a_7-56\Pr Eca_3a_6,$$

$$b_{34}=-2b_{19}-2\Pr a_2b_{30}-4\Pr a_3b_{29}-72\Pr Eca_2a_8-40\Pr Eca_3a_7,$$

$$b_{35}=-2b_{25}-2\Pr a_3b_{30}-24\Pr Eca_2a_9-24\Pr Eca_3a_8,$$

$$b_{36}=-2b_{15}-8\Pr Eca_3a_8,\quad b_{38}=-(b_{39}+b_{40}+b_{41}+b_{42}+b_{43}+b_{44}),$$

$$b_{39}=\frac{b_{31}}{132},\quad b_{40}=\frac{b_{32}}{90},\quad b_{41}=\frac{b_{33}}{56},\quad b_{42}=\frac{b_{34}}{30},\quad b_{43}=\frac{b_{35}}{12},\quad b_{44}=\frac{b_{36}}{2},$$

$$b_{45}=-18\Pr EcH^2a_2^{\,2},\qquad b_{46}=-2\Pr Ec(36a_2^{\,2}+6H^2a_2a_3),$$

$$b_{47}=-2\Pr EcH^2a_3^{\,2},\qquad b_{49}=-b_{50}-b_{51}-b_{52},\qquad b_{50}=\frac{b_{45}}{30},\qquad b_{51}=\frac{b_{46}}{12},$$

$$b_{52}=\frac{b_{47}}{2},\qquad b_{53}=-2b_{50},\qquad b_{54}=-2b_{51}-72\Pr Eca_2^{\,2},$$

$$b_{55}=-2b_{52}-48\Pr Eca_2a_3,\qquad b_{56}=-2b_{49}-8\Pr Eca_3^{\,2},$$

$$b_{58}=1-b_{59}-b_{60}-b_{61}-b_{62},\quad b_{59}=\frac{b_{53}}{56},\quad b_{60}=\frac{b_{54}}{30},\quad b_{61}=\frac{b_{55}}{12},\quad b_{62}=\frac{b_{56}}{2},$$

$$b_{63}=\Pr\omega b_{50}-30\Pr EcH^2a_2a_{21},$$

$$b_{64}=\Pr\omega b_{51}-240\Pr EcH^2a_{21}-18\Pr EcH^2a_2a_{25}-10\Pr EcH^2a_3a_{21},$$

$$b_{65}=\Pr\omega b_{52}-72\Pr Eca_2a_{25}-6\Pr EcH^2a_2a_{27}-6\Pr EcH^2a_3a_{25},$$

$$b_{66}=\Pr\omega b_{49}-2\Pr EcH^2a_3a_{27},\quad b_{67}=-42\Pr EcH^2a_2(a_6+a_{23}),$$

$$b_{68} = 2\Pr a_2(b_7 + b_{51}) - 4\Pr a_3(b_6 + b_{50}) - 12.42\Pr Eca_2(a_6 + a_{23})$$
$$-30\Pr EcH^2 a_2(a_7 + a_{22}) - 14\Pr EcH^2 a_3(a_6 + a_{23}),$$

$$b_{69} = 4\Pr a_2(b_8 + b_{52}) - 2\Pr a_3(b_7 + b_{51}) - 240\Pr Eca_2(a_7 + a_{22})$$
$$-18\Pr EcH^2(a_8 + a_{24}) - 10\Pr EcH^2 a_3(a_7 + a_{22}),$$

$$b_{70} = 6\Pr a_2(b_5 + b_{49}) - 72\Pr Eca_2(a_8 + a_{24}) - 6\Pr EcH^2 a_2(a_9 + a_{26})$$
$$-6\Pr EcH^2 a_3(a_8 + a_{24}),$$

$$b_{71} = 2\Pr a_3(b_5 + b_{49}) - 6\Pr EcH^2 a_3(a_9 + a_{26}), \quad b_{74} = \frac{b_{63}}{56}, \quad b_{75} = \frac{b_{64}}{30},$$

$$b_{76} = \frac{b_{65}}{12}, \quad b_{77} = \frac{b_{66}}{2}, \quad b_{78} = \frac{b_{67}}{90}, \quad b_{79} = \frac{b_{68}}{56}, \quad b_{80} = \frac{b_{69}}{30}, \quad b_{81} = \frac{b_{70}}{12},$$

$$b_{82} = \frac{b_{71}}{2}, \quad b_{83} = -(b_{78} + b_{79} + b_{80} + b_{81} + b_{82}), \quad b_{84} = -(b_{74} + b_{75} + b_{76} + b_{77}),$$

$$b_{73} = b_{83} + ib_{84}, \; b_{85} = -2b_{74} - \Pr \omega b_{59},$$

$$b_{86} = -2b_{75} - \Pr \omega b_{60} - 120\Pr Eca_2 a_{21},$$

$$b_{87} = -2b_{76} - \Pr \omega b_{61} - 72\Pr Eca_2 a_{25} - 40\Pr Eca_3 a_{21},$$

$$b_{88} = -2b_{77} - \Pr \omega b_{62} - 24\Pr Eca_{27} - 24\Pr Eca_3 a_{25},$$

$$b_{89} = -2b_{84} - \Pr \omega b_{58} - 8\Pr Eca_3 a_{27}, \quad b_{90} = -2b_{78} + 8\Pr a_2(b_{27} + b_{59}),$$

$$b_{92} = -2b_{80} + 4\Pr a_2(b_{29} + b_{61}) + 6\Pr a_3(b_{28} + b_{60})$$
$$-120\Pr Eca_2(a_7 + a_{22}) - 56\Pr Eca_3(a_6 + a_{23}),$$

$$b_{93} = -2b_{81} + 2\Pr a_2(b_{30} + b_{62}) + 4\Pr a_3(b_{29} + b_{61})$$
$$-72\Pr Eca_2(a_8 + a_{24}) - 40\Pr Eca_3(a_7 + a_{22}),$$

$$b_{94} = -2b_{82} + 2\Pr a_3(b_{30} + b_{62}) - 24\Pr Eca_2(a_9 + a_{26}) - 24\Pr Eca_3(a_8 + a_{24}),$$

$$b_{95} = -2b_{83} - 8\Pr Eca_3(a_9 + a_{26}), \quad b_{98} = \frac{b_{85}}{90}, \quad b_{99} = \frac{b_{86}}{56}, \quad b_{100} = \frac{b_{87}}{30},$$

$$b_{101} = \frac{b_{88}}{12}, \quad b_{102} = \frac{b_{89}}{2}, \quad b_{103} = \frac{b_{90}}{132}, \quad b_{104} = \frac{b_{91}}{90}, \quad b_{105} = \frac{b_{92}}{56}, \quad b_{106} = \frac{b_{93}}{30},$$

$$b_{107} = \frac{b_{94}}{12}, \qquad b_{108} = \frac{b_{95}}{2}, \qquad b_{109} = -(b_{103} + b_{104} + b_{105} + b_{106} + b_{107} + b_{108}),$$

$$b_{110} = -(b_{98} + b_{99} + b_{100} + b_{101} + b_{102}), \qquad b_{97} = b_{109} + ib_{110},$$