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On a Projective Semi-Symmetric Connection

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Abstract: In this paper, we have extended the study of projective semi-symmetric connections on the para-contact manifold. We study the curvature conditions of semi-symmetric type on a SP-Sasakian manifold admitting a projective semi-symmetric non metric connection.
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1. Introduction

The study of semi-symmetric connections is a very attractive field for investigations in the past many decades. Semi-symmetric connection was introduced by A. Friedmann and J. A. Schouten[1] in 1924. In 1930, E. Bartolotti[2] extended a geometrical meaning to such a connection. Further, H. A. Hayden[3] studied a metric connection with torsion on Riemannian manifold. After a long gap, in 1970, the study of semi-symmetric connections was resumed by K. Yano[4]. In particular, he studied semi-symmetric metric connections. Afterwards several researchers have been carried out the study of semi-symmetric connections in a variety of directions such as[5-9].

In, 2001 P. Zhao and H. Song[10] defined and studied a type of semi-symmetric connection on Riemannian manifold which is projectively equivalent to the Levi-Civita connection ∇, i.e., has the same geodesic curves as ∇. This was termed as projective semi-symmetric connection. The studies on projective semi-symmetric connections have been further extended by P. Zhao[11], S.K. Pal et.al.[12] and others.

In continuation to the previous studies, we consider the projective semi-symmetric connection on a SP-Sasakian manifold. The paper is organised as follows: After preliminaries on SP-Sasakian manifold in section 2, we describe briefly the projective semi-symmetric connection in section 3. In section 4, we study a SP-Sasakian manifold admitting a projective semi-symmetric connection and show that a SP-Sasakian manifold satisfying the condition $\tilde{R}.\tilde{S}=0$ is an Einstein manifold. Further, in the section 5, we show that the condition $\tilde{R}.\tilde{R}=0$ implies that the SP-Sasakian manifold is an Einstein manifold.

2. Preliminaries

The notion of an (almost) para-contact manifold was introduced by I. Sato[13]. An n-dimensional differentiable manifold M is said to have almost para-contact structure (ϕ, ξ, η) where ϕ is a tensor field of type $(1,1)$, ξ is a vector field known as characteristic vector field and η is a 1-form satisfying the following relations

(2.1) $$\phi^2(X)=X-\eta(X)\xi \ ,$$

(2.2) $$\eta(\bar{X})=0,$$

(2.3) $$\phi(\xi)=0,$$

and

(2.4) $$\eta(\xi)=1.$$

A differentiable manifold with almost para-contact structure (ϕ, ξ, η) is called an almost para-contact manifold. Further, if the M has a Riemannian metric g satisfying

(2.5) $$\eta(X)=g(X,\xi),$$

and

(2.6) $$g(\phi X, \phi Y)=g(X,Y)-\eta(X)\eta(Y).$$

Then the set (ϕ,ξ,η,g) satisfying the conditions (2.1) to (2.6) is called an almost para-contact Riemannian structure and the manifold M with such a structure is called an almost para-contact Riemannian manifold[13,14].

Now, let (M, g) be an n-dimensional Riemannian manifold with a positive definite metric g admitting a $1-$form η which satisfies the conditions

(2.7) $$(\nabla_X\eta)Y-(\nabla_Y\eta)X=0,$$

and

(2.8) $$(\nabla_X\nabla_Y\eta)(Z)=-g(X,Z)\eta(Y)-g(X,Y)\eta(Z)+2\eta(X)\eta(Y)\eta(Z),$$

where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Moreover, If (M, g) admits a vector field ξ and a $(1, 1)$ tensor field ϕ such that

(2.9) $$g(X,\xi)=\eta(X),\quad \eta(\xi)=1 \quad \text{and} \quad \nabla_X\xi=\phi(X),$$

then it can be easily verified that the manifold under consideration becomes an almost paracontact Riemannian manifold. Such a manifold is called a para-Sasakian manifold or briefly a P-Sasakian manifold[15]. It is a special case of almost paracontact Riemannian manifold introduced by I. Sato. It is known[15] that on a P-Sasakian manifold the following relations hold

(2.10) $$\eta(R(X,Y,Z))=g(X,Z)\eta(Y)-g(Y,Z)\eta(X),$$

(2.11) $$R(\xi,X,Y)=\eta(Y)X-g(X,Y)\xi,$$

(2.12) $$R(\xi,X,\xi)=X-\eta(X)\xi,$$

(2.13) $$R(X,Y,\xi)=\eta(X)Y-\eta(Y)X,$$

(2.14) $$S(X,\xi)=-(n-1)\eta(X),$$

and

(2.15) $$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y),$$

where R is the curvature tensor and S is the Ricci tensor.

A P-Sasakian manifold satisfying

(2.16) $$(\nabla_X \eta)Y = -g(X, Y) + \eta(X)\eta(Y)$$

is called an special para-Sasakian manifold or briefly an SP-Sasakian manifold[15]. In an SP-Sasakian manifold, we also have

(2.17) $$`F(X, Y) = -g(X, Y) + \eta(X)\eta(Y),$$

where $`F(X, Y)$ refers to the fundamental 2-form of the manifold. On an SP-Sasakian manifold, we also have the following

(2.18) $$\phi X = -X + \eta(X)\xi .$$

The tensors $R \cdot R$ and $R \cdot S$ are defined by the followings[16, 17]

(2.19) $$(R(X, Y) \cdot R)(Z, W, U) = R(X, Y, R(Z, W, U)) - R(R(X, Y, Z), W, U) - R(Z, R(X, Y, W), U) - R(Z, W, R(X, Y, U)),$$

and

(2.20) $$(R(X, Y) \cdot S)(Z, W) = -S(R(X, Y, Z), W) - S(Z, R(X, Y, W)),$$

where $\cdot$ indicates that $R(X, Y)$ is acts as a derivation of the tensor algebra at each point of the manifold for tangent vectors X, Y. An n-dimensional manifold is said to be semi-symmetric manifold if the tensor $R(X, Y) \cdot R = 0$ and Ricci semi-symmetric if it satisfies $R \cdot S = 0$.

3. Projective Semi-symmetric Connection

In this section, we give a brief account of projective semi-symmetric connection and study it on a SP-Sasakian manifold.

A linear connection $\tilde{\nabla}$ on an n-dimensional Riemannian manifold (M, g) is called a semi-symmetric connection[4], if its torsion tensor T given by

$$T(X, Y) = \tilde{\nabla}_X Y - \tilde{\nabla}_Y X - [X, Y],$$

has the form

$$T(X, Y) = \pi(Y)X - \pi(X)Y, \tag{3.1}$$

where π is a 1-form associated with a vector field ρ, i.e.,

$$\pi(X) = g(X, \rho). \tag{3.2}$$

Further, a connection $\tilde{\nabla}$ is a metric connection if it satisfies

$$(\tilde{\nabla}_X g)(Y, z) = 0. \tag{3.3}$$

If the geodesic with respect to $\tilde{\nabla}$ are always consistent with those of the Levi-Civita connection ∇ on a Riemannian manifold, then $\tilde{\nabla}$ is called a connection projectively equivalent to ∇. If $\tilde{\nabla}$ is linear connection projective equivalent to ∇ as well as a semi-symmetric one, we call $\tilde{\nabla}$ is called projective semi-symmetric connection[10].

Now, we consider a projective semi-symmetric connection $\tilde{\nabla}$ introduced by P. Zhao and H. Song[10] given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \psi(Y)X + \psi(X)Y + \varphi(Y)X - \varphi(X)Y, \tag{3.4}$$

for arbitrary vector fields X and Y, where the 1-forms ψ and φ are given through the following relations

$$\psi(X) = \frac{n-1}{2(n+1)}\pi(X) \quad \text{and} \quad \varphi(X) = \frac{1}{2}\pi(X). \tag{3.5}$$

It is easy to see that the equations (3.4) and (3.5) give us

(3.6) $\left(\tilde{\nabla}_X g\right)(Y, Z) = \frac{1}{n+1}\left[2\pi(X)g(Y, Z) - n\pi(Y)g(X, Z) - n\pi(Z)g(X, Y)\right],$

which shows that the connection $\tilde{\nabla}$ given by (3.4) is a metric one.

We denote by $\tilde{R}$ and R the curvature tensors of the manifold relative to the projective semi-symmetric connection connections $\tilde{\nabla}$ and the Levi-Civita connection ∇. It is known that[10] that

(3.7) $$\tilde{R}(X, Y, Z) = R(X, Y, Z) + \alpha(X, Z)Y - \alpha(Y, Z)X + \beta(X, Y)Z,$$

where α and β are the tensors of type (0, 2) given by the following relations

(3.8) $$\alpha(X, Y) = \psi'(X, Y) + \varphi'(Y, X) - \psi(X)\varphi(Y) - \psi(Y)\varphi(X),$$

(3.9) $$\beta(X, Y) = \psi'(X, Y) - \psi'(Y, X) + \varphi'(Y, X) - \varphi'(X, Y).$$

The tensors ψ' and φ' of type (0, 2) are defined by the following two relations

(3.10) $$\psi'(X, Y) = (\nabla_X \psi)Y - \psi(X)\psi(Y),$$

and

(3.11) $$\varphi'(X, Y) = (\nabla_X \varphi)Y - \varphi(X)\varphi(Y).$$

Contraction of the vector field X in the equation (3.7) yields a relation between Ricci tensors of the manifold relative to the two connections $\tilde{\nabla}$ and ∇ which is given by

(3.12) $$\tilde{S}(Y, Z) = S(Y, Z) + \beta(Y, Z) - (n-1)\alpha(Y, Z).$$

Also, from the above equation, we get the following equation relating scalar curvatures $\tilde{r}$ and r of the manifold with respect to the two connections $\tilde{\nabla}$ and ∇

(3.13) $$\tilde{r} = r + b - (n-1)a,$$

where $b = \sum_{i=1}^{n} \beta(e_i, e_i)$ and $a = \sum_{i=1}^{n} \alpha(e_i, e_i)$.

In order to extend the studies of the projective semi-symmetric connection $\tilde{\nabla}$ on SP-Sasakian manifold, we identify the 1-form π of the connection $\tilde{\nabla}$ with the 1-form η of the P-Sasakian manifold. In view of this equality between π and η and the equations (3.5), we find that the expression (3.4) for the projective semi-symmetric connection $\tilde{\nabla}$ reduces to

$$\tilde{\nabla}_X Y = \nabla_X Y + \frac{1}{2}(c+1)\eta(Y)X + \frac{1}{2}(c-1)\eta(X)Y, \tag{3.14}$$

where the constant c is given by $c = \frac{n-1}{n+1}$. Now, it can be seen observed that

$$\left(\tilde{\nabla}_X \eta\right)(Y) = (\nabla_X \eta)(Y) - c\eta(X)\eta(Y). \tag{3.15}$$

On a SP-Sasakian manifold, we have $(\nabla_X \eta)(Y) = (\nabla_Y \eta)(X)$. Therefore, the above equation yields

$$\left(\tilde{\nabla}_X \eta\right)(Y) = \left(\tilde{\nabla}_Y \eta\right)(X).$$

Thus, the connection $\tilde{\nabla}$ given by the equation (3.4) becomes special projective semi-symmetric connection studied by S.K. Pal et. al.[12]. It can also be verified very easily that for such a projective semi-symmetric connection the tensor β vanishes and the tensor α is symmetric, i.e.,

$$\beta(X, Y) = 0 \quad \text{and} \quad \alpha(X, Y) = \alpha(Y, X). \tag{3.16}$$

As a consequence of these, the expressions for curvature tensor, the tensor α, Ricci tensors and scalar curvatures given by (3.7), (3.8), (3.12) and (3.13) takes the following simpler forms

$$\tilde{R}(X, Y, Z) = R(X, Y, Z) + \alpha(X, Z)Y - \alpha(Y, Z)X, \tag{3.17}$$

$$\alpha(X, Y) = \mu(\nabla_X \eta)(Y) - \mu^2 \eta(X)\eta(Y), \tag{3.18}$$

(3.19) $$\tilde{S}(Y, Z) = S(Y, Z) - (n-1)\alpha(Y, Z)$$

and

(3.20) $$\tilde{r} = r - (n-1)a ,$$

where $\mu = \frac{1}{2}(c+1)$.

It may also be notes that the Ricci tensor $\tilde{S}(Y, Z)$ of the special projective semi-symmetric connection is symmetric.

Now, we derive some of the results concerning the tensor α which we shall need in subsequent sections.
Replacing X by ξ in the equation (3.18) and using (2.4), we get

(3.21) $$\alpha(\xi, Y) = \mu(\nabla_{\xi}\eta)(Y) - \mu^2\eta(Y) .$$

Next, covariant differentiation of the equation (2.4) with respect to the Levi-Civita connection ∇ gives

$$(\nabla_X \eta)(\xi) + \eta(\nabla_X \xi) = 0 ,$$

which, in view of the equations (2.2), (2.7) and (2.9), yields

$$(\nabla_{\xi}\eta)(X) = (\nabla_X \eta)(\xi) = 0 .$$

Using the above equation and the fact that the tensor α is symmetric in the above equation (3.21), we get

(3.22) $$\alpha(\xi, Y) = \alpha(Y, \xi) = \lambda\eta(Y) ,$$

where we have put $\lambda = -\mu^2$.

Now, putting ξ for each of the vector fields X, Y and Z in the equation (3.17) and using the equations (2.11), (2.13) and (3.22), we obtain the followings

(3.23) $$\tilde{R}(\xi, Y, Z) = \lambda'\eta(Z)Y - \theta(Y, Z)\xi ,$$

(3.24) $$\tilde{R}(X,\xi,Z)=\theta(X,Z)\xi-\lambda'\eta(Z)X$$

and

(3.25) $$\tilde{R}(X,Y,\xi)=\lambda'\eta(X)Y-\lambda'\eta(Y)X\ ,$$

where $\lambda'=(1+\lambda)$ and the tensor θ is a symmetric tensor given by

(3.26) $$\theta(Y,Z)=g(Y,Z)+\alpha(Y,Z).$$

Again, taking $Z=\xi$ in the above equation and using the equation (3.22), we have

(3.27) $$\theta(Y,\xi)=\lambda'\eta(Y).$$

4. Ricci Semi-symmetric SP-Sasakian Manifold

In this section, we consider a SP-Sasakian manifold which admits a projective semi-symmetric connection $\tilde{\nabla}$ and is Ricci Semi-symmetric with respect the connection $\tilde{\nabla}$, i.e., satisfies the condition of the type $\tilde{R}\cdot\tilde{S}=0$.

Theorem 4.1: *If a* SP-*Sasakian manifold admitting a projective semi-symmetric connection* $\tilde{\nabla}$ *is Ricci semi-symmetric with respect to the connection* $\tilde{\nabla}$ *then the manifold is an Einstein manifold.*

Proof: Let the projective semi-symmetric connection $\tilde{\nabla}$ on SP-Sasakian manifold satisfies

(4.1) $$\left(\tilde{R}(X,Y)\cdot\tilde{S}\right)(Z,W)=0,$$

where $\tilde{R}$ and $\tilde{S}$ are the curvature tensor and the Ricci tensor of the manifold relative to the connection $\tilde{\nabla}$. The equation (2.20) gives

$$-\tilde{S}\left(\tilde{R}(X,Y,Z),W\right)-\tilde{S}\left(Z,\tilde{R}(X,Y,W)\right)=0.$$

Taking $X=\xi$ in the above equation, it follows that

$$\tilde{S}\left(\tilde{R}(\xi, Y, Z), W\right)+\tilde{S}\left(Z, \tilde{R}(\xi, Y, W)\right)=0,$$

which, in view of the equation (3.23), gives

(4.2) $$\lambda'\eta(Z)\tilde{S}(Y, W)-\theta(Y, Z)\tilde{S}(\xi, W)+\lambda'\eta(W)\tilde{S}(Z, Y)$$
$$-\theta(Y, W)\tilde{S}(Z, \xi)=0.$$

Replacing $Z=\xi$ in the equation (3.19) and using the equation (2.14) and (3.22), we obtain

(4.3) $$\tilde{S}(Y, \xi)=d\eta(Y),$$

where $d=-(n-1)\lambda'$.

Now, putting $W=\xi$ in the equation (4.2) and using the equation (4.3), we get

$$\lambda'\tilde{S}(Y, Z)=d\theta(Y, Z).$$

Now, in view of the equations (3.19) and (3.26), we obtain

(4.4) $$S(Y, Z)=-(n-1)g(Y, Z).$$

Thus, the manifold is Einstein manifold. This proves the theorem.

5. Semi-Symmetric SP-Sasakian Manifold

In this section, we consider a SP-Sasakian manifold which admits a projective semi-symmetric connection $\tilde{\nabla}$ and is Semi-symmetric with respect the connection $\tilde{\nabla}$, i.e., satisfies the condition of the type $\tilde{R}\cdot\tilde{R}=0$.

It is easy to see that in view of the equation (2.16) in (3.18), the tensor α takes the following form

(5.1) $$\alpha(X, Y)=-\mu g(X, Y)+\nu\eta(X)\eta(Y),$$

where we have put $\nu=\mu-\mu^2$. This, in view of the equations (2.4) and (2.5) and symmetry of tensor α, produces easily that

$$\alpha(\xi, Y) = \alpha(Y, \xi) = \lambda\eta(Y), \tag{5.2}$$

where by λ we mean $-\mu^2$.

Also, by taking inner product with ξ, we get the following from the equation (3.17)

$$\eta(\tilde{R}(X, Y, Z)) = \eta(R(X, Y, Z)) + \alpha(X, Z)\eta(Y) - \alpha(Y, Z)\eta(X), \tag{5.3}$$

which, due to the equations (2.10) and (5.1), gives

$$\eta(\tilde{R}(X, Y, Z)) = -(\mu - 1)[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)]. \tag{5.4}$$

Now, we state the following theorem.

Theorem 5.1: *If a* SP-*Sasakian manifold admitting a projective semi-symmetric connection* $\tilde{\nabla}$ *is semi-symmetric with respect to* $\tilde{\nabla}$ *then the manifold is an Einstein manifold.*

Proof: Let the projective semi-symmetric connection $\tilde{\nabla}$ on SP-Sasakian manifold satisfies

$$(\tilde{R}(X, Y) \cdot \tilde{R})(Z, W, U) = 0, \tag{5.5}$$

for all vector fields X, Y, Z, W and U. Then, in particular for $X = \xi$, we have

$$(\tilde{R}(\xi, X) \cdot \tilde{R})(Y, Z, W) = 0,$$

which, in view of the equation (2.19), implies that

$$\tilde{R}(\xi, X, \tilde{R}(Y, Z, W)) - \tilde{R}(\tilde{R}(\xi, X, Y), Z, W)$$
$$-\tilde{R}(Y, \tilde{R}(\xi, X, Z), W) - \tilde{R}(Y, Z, \tilde{R}(\xi, X, W)) = 0.$$

Now, using (2.10), (3.23), (3.26) and (5.4) in the above equation, we find

That

$$-(\mu-1)\lambda'\left[g(Y,W)\eta(Z)-g(Z,W)\eta(Y)\right]X$$
$$-\left[g\left(X,\tilde{R}(Y,Z,W)\right)+\alpha\left(X,\tilde{R}(Y,Z,W)\right)\right]\xi$$
$$-\lambda'\eta(Y)\tilde{R}(X,Z,W)+\theta(X,Y)\tilde{R}(\xi,Z,W)$$
$$-\lambda'\eta(Z)\tilde{R}(Y,X,W)+\theta(X,Z)\tilde{R}(Y,\xi,W)$$
$$-\lambda'\eta(W)\tilde{R}(Y,Z,X)+\theta(X,W)\tilde{R}(Y,Z,\xi)=0.$$

Again, using the equations (3.23), (3.24), (3.25) and (5.2), in the above equation and taking inner product with ξ, we get

$$`\tilde{R}(Y,Z,W,X)(1-\mu)=-(\mu-1)\lambda' g(Y,W)\eta(Z)\eta(X)$$
$$+(\mu-1)\lambda' g(Z,W)\eta(Y)\eta(X)-\nu\eta(X)\eta\left(\tilde{R}(Y,Z,W)\right)$$
$$-\lambda'\eta(Y)\eta(X)\eta\left(\tilde{R}(X,Z,W)\right)+\lambda'\eta(Z)\eta(W)\theta(X,Y)$$
$$-\theta(X,Y)\theta(Z,W)-\lambda'\eta(Z)\eta\left(\tilde{R}(Y,X,W)\right)-\lambda'\eta(Y)$$
$$\times\eta(W)\theta(X,Z)+\theta(X,Z)\theta(Y,W)-\lambda'\eta(W)\eta\left(\tilde{R}(Y,Z,X)\right)$$
$$+(1+\lambda)\eta(Y)\eta(Z)\theta(X,W)-(1+\lambda)\eta(Z)\eta(Y)\theta(X,W),$$

where $`\tilde{R}(X,Y,Z,W)=g\left(\tilde{R}(X,Y,Z),W\right)$ is the curvature tensor of type (0,4) relative to the connection $\tilde{\nabla}$.

Using (3.26) and (5.4) in the above equation, it follows that

$$`\tilde{R}(Y,Z,W,X)(1-\mu)=\nu(\mu-1)\eta(X)g(Y,W)\eta(Z)$$
$$+\lambda'\mu\eta(Z)\eta(W)g(X,Y)-\lambda'\mu\eta(Y)\eta(W)g(Z,X)$$
$$+\lambda'\eta(Z)\eta(W)\alpha(X,Y)-\lambda'\eta(Y)\eta(W)\alpha(X,Z)$$
$$-\left[g(X,Y)+\alpha(X,Y)\right]\left[g(Z,W)+\alpha(Z,W)\right]$$
$$+\left[g(X,Z)+\alpha(X,Z)\right]\left[g(W,Y)+\alpha(W,Y)\right].$$

Using the equations (5.1) in the above equation, we find

$$`\tilde{R}(Y, Z, W, X)(1-\mu) = (2\mu - \mu^2 - 1) g(X, Y) g(Z, W)$$

$$+(\mu - 1)\nu\eta(Z)\eta(W) g(X,Y) + (\mu^2 - 2\mu + 1) g(X,Z) g(Y,W)$$

$$-(\mu - 1)\nu g(X, Z)\eta(Y)\eta(W).$$

Now, contracting the above equation with respect to X and Y, we obtain

$$\tilde{S}(Z, W)(1-\mu) = (1-n)(\mu^2 - 2\mu + 1) g(Z, W)$$

$$+(n-1)(\mu - 1)\nu\eta(W)\eta(Z),$$

which, on using the equation (3.19) and (5.1), gives

$$S(Z, W) = (1-n) g(Z, W).$$

This proves the theorem.

References

1. A. Friedmann and J. A. Schouten, Uber Die Geometric Der Halbsymmetrischen Ubertragung, *Math. Zeitschr.*, **21**(1924), 211-223.

2. E. Bartolotti, Sulla Geometria Della Variata A Connection Affine, *Ann. di Mat.*, **4(8)** (1930), 53-101.

3. H. A. Hayden, Subspaces of a Space with Torsion, *Proc. London Math. Soc.*, **34** (1932), 27-50.

4. K. Yano, On Semi-Symmetric Metric Connections, *Rev. Roumaine Math. Pures Appl.*, **15** (1970), 1579-1586.

5. S. K. Chaubey and R. H. Ojha, On Semi-Symmetric Non-Metric Connection, *Filomat*, **26(2)** (2012), 269-275.

6. T. Imai, Notes on Semi-Symmetric Metric Connections, *Tensor, N.S.*, **24** (1972), 293-296.

7. R. S. Mishra and S. N. Pandey, Semi-Symmetric Metric Connections in an Almost Contact Manifold, *Indian J. Pure Appl. Math.*, **9(6)** (1978), 570-580.

8. R. N. Singh, Shravan K. Pandey and Giteshwari Pandey, On Semi-Symmetric Non-Metric Connections in a Co-Symplectic Manifold, *Journal of International Academy of Physical Sciences*, **16(1)** (2012), 1-16.

9. R. N. Singh, M. K. Pandey and D. Gautam, On Nearly Quasi Einstein Manifold, *Int. Journal of Math. Analysis*, **5(36)** (2011), 1767-1773.

10. P. Zhao and H. Song, An Invariant of the Projective Semi Symmetric Connection, *Chinese Quarterly J. of Math.*, **16(4)** (2001), 49-54.

11. P. Zhao, Some Properties of Projective Semi-Symmetric Connections, *Int. Math. Forum,* **3(7)** (2008), 341-347.

12. S. K. Pal, M. K. Pandey and R. N. Singh, On a Type Of Projective Semi-Symmetric Connection, *Int. J. of Anal. and Appl., (N.S.)*, **7(2)** (2015), 153-161.

13. I. Sato, On a Structure Similar to the Almost Contact Structure, *Tensor, (N.S.),* **30** (1976), 219-224.

14. T. Adati and T. Miyazava, On Para-Contact Riemannian Manifolds, *Tru Math.*, **13 (2) (**1977), 27-39.

15. T. Adati and T. Miyazava, Some Properties of P-Sasakian Manifolds, *Tru Math.*, **13** (1977), 33-42.

16. P. Majhi and U. C. De, Classification of $N(k)$-Contact Metric Manifolds Satisfying Certain Curvature Conditions, *Acta Math. Univ. Comenianae,* **84** (2015), 167-178.

17. Z. I. Szabo, Structure Theorem on Riemannian Space Satisfying $R(X,Y)\cdot R=0$. I. the Local Version, *J, Diff.Geom.*, **17** (1982), 531-582.