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Interval Valued Intuitionistic Fuzzy Sets in Medical Diagnosis*

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Abstract: Diagnostic decisions in medicine involve a variety of uncertainties related to the symptoms of patients and relation of symptoms and diseases. De et. al. [Fuzzy Sets and Systems vol.117 (2001) 209-213] have given intuitionistic fuzzy set theoretic approach for medical diagnosis. Interval estimation for membership functions may prove better than their point estimation approach in covering the uncertainties. Present paper defines composition rule of inference for interval valued intuitionistic fuzzy relations to extend the approach of De et.al to IVIFS. The work also introduces the concept of positive and negative ideal for IVIFS to reach the final decision on diagnosis.
Keywords: Medical diagnosis, Interval valued intuitionistic fuzzy sets, Interval valued intuitionistic fuzzy relations and Positive ideal interval valued intuitionistic fuzzy set.
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1. Introduction

Medical diagnosis is a process in which physicians are faced with a variety of uncertainties. Diagnosis of a patient depends on the past history of the patient, physical condition of the patient, and some investigative results (laboratory tests).The knowledge provided by each of these sources carries with it varying degrees of uncertainty. The past history offered by the patient may be subjective, exaggerated, underestimated, and incomplete. Errors may be made in the physical examination, and symptoms may be overlooked. The measurements provided by the investigative tests are often of limited precision and the exact borderline between normal and pathological is often unclear[1]. Fuzzy set introduced by Zadeh[2] in 1965 is a useful tool to deal with the uncertainty. These sets depend on the

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assumption of precise membership function which is a real valued function that takes real values in [0, 1]. Esogbue and Elder[3] gave a fuzzy set based mathematical model for medical diagnosis. Yao and Yao[4] worked on fuzzy decision making for medical diagnosis using fuzzy numbers. Zadeh also discussed sets whose membership functions were interval valued and took values in a closed interval [0, 1]. This class of fuzzy sets is called interval valued (i-v) fuzzy sets. Gorzalczany[5] studied the i-v fuzzy relations for approximate reasoning. Roy and Biswas[6] defined the compositions of interval valued fuzzy relations and discussed the medical diagnosis using interval valued fuzzy sets. Interval estimation covers the disadvantage of point estimation but it is still unable to give any information about non membership. Atanassov in 1986 extended the notion of fuzzy sets as intuitionistic fuzzy sets[7]. He considered the concept of membership function of favorable cases as well as the membership function for unfavorable cases. Intuitionistic fuzzy sets are more effective in handling the vagueness and are equipollent to interval valued fuzzy sets[8]. Zeshui Xu defined the concept of similarity measure for intuitionistic fuzzy sets and demonstrated its application in multiple attribute decision making[9]. De et al applied intuitionistic fuzzy relations for medical diagnosis by extending the Sanchez's approach[10].

Atanassov and Gargov further extended their concept of intuitionistic fuzzy sets by assuming the interval estimation for membership and nonmembership functions[8] and referred them as interval valued intuitionistic fuzzy sets .They also defined some operators for these sets[11].Zeshui and Yager worked on interval valued intuitionistic fuzzy preference relations and applied this concept to the evaluation of agreement within a group[12]. In the present paper, we define the composition rule of inference for interval valued intuitionistic fuzzy relations and extend the approach of De et.al. for medical diagnosis. We also define the concept of positive ideal and negative ideal for interval valued intuitionistic fuzzy sets that help in reaching the final diagnosis.

2. Basic Notions

Definition 2.1. An Interval Valued Intuitionistic Fuzzy Set (IVIFS) $\tilde{A}$ Over U (Universe of discourse) is an object having the form:

$$\tilde{A} = \left\{ \left\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{\upsilon}_{\tilde{A}}(x) \right\rangle | x \in U \right\},$$

where $\tilde{\mu}_{\tilde{A}}(x) = \left[\tilde{\mu}^L_{\tilde{A}}(x), \tilde{\mu}^U_{\tilde{A}}(x) \right]$, $\tilde{\upsilon}_{\tilde{A}}(x) = \left[\tilde{\upsilon}^L_{\tilde{A}}(x), \tilde{\upsilon}^U_{\tilde{A}}(x) \right]$

such that $\tilde{\mu}^U_{\tilde{A}}(x) + \tilde{\upsilon}^U_{\tilde{A}}(x) \le 1$

$$\tilde{\pi}_{\tilde{A}}(x)=\left[\tilde{\pi}^{L}_{\tilde{A}}(x),\tilde{\pi}^{U}_{\tilde{A}}(x)\right],\tilde{\pi}^{L}_{\tilde{A}}(x)=1-\tilde{\mu}^{U}_{\tilde{A}}(x)-\tilde{\upsilon}^{U}_{\tilde{A}}(x),\ \tilde{\pi}^{U}_{\tilde{A}}(x)=1-\tilde{\mu}^{L}_{\tilde{A}}(x)-\tilde{\upsilon}^{L}_{\tilde{A}}(x)$$

Definition 2.2. Let $\tilde{A}$ and $\tilde{B}$ are two IVIFS of universe of discourse U. Then $\tilde{A}\subset\tilde{B}$ iff $\forall$ x$\in$U, $\tilde{\mu}^{L}_{\tilde{A}}(x)\le\tilde{\mu}^{L}_{\tilde{B}}(x)$ & $\tilde{\mu}^{U}_{\tilde{A}}(x)\le\tilde{\mu}^{U}_{\tilde{B}}(x)$ and

$\tilde{\upsilon}^{L}_{\tilde{A}}(x)\ge\tilde{\upsilon}^{L}_{\tilde{B}}(x)$ & $\tilde{\upsilon}^{U}_{\tilde{A}}(x)\ge\tilde{\upsilon}^{U}_{\tilde{B}}(x)$ and

$\tilde{A}=\tilde{B}$ iff $\forall x\in U$, $\tilde{\mu}^{L}_{\tilde{A}}(x)=\tilde{\mu}^{L}_{\tilde{B}}(x)$ & $\tilde{\mu}^{U}_{\tilde{A}}(x)=\tilde{\mu}^{U}_{\tilde{B}}(x)$,

$\tilde{\upsilon}^{L}_{\tilde{A}}(x)=\tilde{\upsilon}^{L}_{\tilde{B}}(x)$, & $\tilde{\upsilon}^{U}_{\tilde{A}}(x)=\tilde{\upsilon}^{U}_{\tilde{B}}(x)$,

$$\tilde{A}\cap\tilde{B}=\left\{\left\langle x,\left[\min(\tilde{\mu}^{L}_{\tilde{A}}(x),\tilde{\mu}^{L}_{\tilde{B}}(x)),\min\left(\tilde{\mu}^{U}_{\tilde{A}}(x),\tilde{\mu}^{U}_{\tilde{B}}(x)\right)\right],\left[\max(\tilde{\upsilon}^{L}_{\tilde{A}}(x),\tilde{\upsilon}^{L}_{\tilde{B}}(x))\right.\right.\right.$$
$$\left.\left.\left.\max(\tilde{\upsilon}^{U}_{\tilde{A}}(x),\tilde{\upsilon}^{U}_{\tilde{A}}(x))\right]\right\rangle \mid x\in U\right\}$$

$$\tilde{A}\cup\tilde{B}=\left\{\left\langle x,\left[\max(\tilde{\mu}^{L}_{\tilde{A}}(x),\tilde{\mu}^{L}_{\tilde{B}}(x)),\max\left(\tilde{\mu}^{U}_{\tilde{A}}(x),\tilde{\mu}^{U}_{\tilde{B}}(x)\right)\right],\left[\min(\tilde{\upsilon}^{L}_{\tilde{A}}(x),\tilde{\upsilon}^{L}_{\tilde{B}}(x))\right.\right.\right.$$
$$\left.\left.\left.\min(\tilde{\upsilon}^{U}_{\tilde{A}}(x),\tilde{\upsilon}^{U}_{\tilde{A}}(x))\right]\right\rangle \mid x\in U\right\}$$

Definition 2.3. Let U,V be two universe of discourses. An intuitionistic fuzzy relation $\tilde{R}$ form U to V is an intuitionistic fuzzy set, characterized by two membership functions called favorable $\tilde{\mu}_{\tilde{R}}:U\times V\to[0,1]$ and unfavorable $\tilde{\upsilon}_{\tilde{R}}:U\times V\to[0,1]$ and is denoted by

$$\tilde{R}=\left\{\left((x,y),\mu_{\tilde{R}}(x,y),\upsilon_{\tilde{R}}(x.y)\right)\mid(x.y)\in U\times V\right\}.$$

Definition 2.4. Let $\tilde{A}$ be a intuitionistic fuzzy set (IFS) of U, the max-min-max composition of the IFR $\tilde{R}$ on $U\times V$ with $\tilde{A}$ is an IFS $\tilde{B}$ of V denoted by $\tilde{B} = \tilde{R}o\tilde{A}$ is defined by following functions:
membership function

$$\mu_{\tilde{R}o\tilde{A}}(y)=\vee_{x}[\mu_{\tilde{A}}(x)\wedge\mu_{\tilde{R}}(x,y)]$$

non-membership function

$$\upsilon_{\tilde{R}o\tilde{A}}(y)=\wedge_{x}[\upsilon_{\tilde{A}}(x)\vee\upsilon_{\tilde{R}}(x,y)]$$

where $\vee$ and $\wedge$ represent max and min respectively.

Definition 2.5. Let $\tilde{R}$ and $\tilde{S}$ be two IFRs on $U\times V$ and $V\times W$ respectively. The max-min-max composition $\tilde{S}\,\mathrm{o}\,\tilde{R}$ is an IFR defined by membership function

$$\mu_{\tilde{S}o\tilde{R}}(x,z)=\vee_{y}[\mu_{\tilde{R}}(x,y)\wedge\mu_{\tilde{S}}(y,z)]$$

and non membership function

$$\upsilon_{\tilde{S}\circ\tilde{R}}(x,z) = \wedge_{y}[\upsilon_{\tilde{R}}(x,y) \vee \upsilon_{\tilde{S}}(y,z)] \ \ \forall\, y \in V, (x,y) \in U \times V, (y,z) \in V \times W .$$

3. Interval valued intuitionistic fuzzy relation

Definition 3.1. Let U and V denote universe of discourses. An IVIFR $\tilde{R}$ from $U \to V$ is an IVIFS in the space $U \times V$. It is characterized for each ordered pair (x, y) $\in U \times V$ by a membership interval valued function $\tilde{\mu}_{\tilde{R}}.: U \times V \to \varepsilon[0,1]$ and non membership interval valued function $\tilde{\upsilon}_{\tilde{R}} : U \times V \to \varepsilon[0,1]$, where $\tilde{\mu}_{\tilde{R}}(x,y) \subset [0,1]$ and $\tilde{\upsilon}_{\tilde{R}}(x,y) \subset [0,1]$ represent the interval range of association and non-association respectively. Therefore

$$\tilde{R} = \left\{ \left\langle (x,y), \tilde{\mu}_{\tilde{R}}(x,y), \tilde{\upsilon}_{\tilde{R}}(x,y) \right\rangle \mid (x,y) \in U \times V \right\} \text{ and } \tilde{\mu}^{U}_{\tilde{R}}(x,y) + \tilde{\upsilon}^{U}_{\tilde{R}}(x,y) \le 1$$

Definition 3.2. Let $\tilde{A}$ be an IVIFS of U and $\tilde{R}$ be an IVIFR from U to V then composition of $\tilde{R}$ with $\tilde{A}$, $\tilde{R} \circ \tilde{A}$, is an IVIFS of V defined by following functions:

(3.1) $$\tilde{\mu}_{\tilde{R}\circ\tilde{A}}(y) = \left[\wedge_{x\in U} \left(\tilde{\mu}^{L}_{\tilde{A}}(x) \wedge \tilde{\mu}^{L}_{\tilde{R}}(x,y) \right), \vee_{x\in U} \left(\tilde{\mu}^{U}_{\tilde{A}}(x) \wedge \tilde{\mu}^{U}_{\tilde{R}}(x,y) \right) \right],$$

(3.2) $$\tilde{\upsilon}_{\tilde{R}\circ\tilde{A}}(y) = \left[\wedge_{x\in U} \left(\tilde{\upsilon}^{L}_{\tilde{A}}(x) \wedge \tilde{\upsilon}^{L}_{\tilde{R}}(x,y) \right), \wedge_{x\in U} \left(\tilde{\upsilon}^{U}_{\tilde{A}}(x) \vee \tilde{\upsilon}^{U}_{\tilde{R}}(x,y) \right) \right].$$

Definition 3.3. Let $\tilde{R}$ and $\tilde{S}$ be two IVIFRs on $U \times V$ and $V \times W$ respectively. The composition relation $\tilde{S} \circ \tilde{R}$ is an IVIFS on $U \times W$ having following membership and non-membership interval valued functions.

(3.3) $$\tilde{\mu}_{\tilde{S}\circ\tilde{R}}(x,z) = \left[\wedge_{y\in V} \left(\tilde{\mu}^{L}_{\tilde{R}}(x,y) \wedge \tilde{\mu}^{L}_{\tilde{S}}(y,z) \right), \vee_{y\in V} \left(\tilde{\mu}^{U}_{\tilde{R}}(x,y) \wedge \tilde{\mu}^{U}_{\tilde{S}}(y,z) \right) \right],$$

(3.4) $$\tilde{\upsilon}_{\tilde{S}\circ\tilde{R}}(x,z) = \left[\wedge_{y\in V} \left(\tilde{\upsilon}^{L}_{\tilde{R}}(x,y) \wedge \tilde{\upsilon}^{L}_{\tilde{S}}(y,z) \right), \wedge_{y\in V} \left(\tilde{\upsilon}^{U}_{\tilde{R}}(x,y) \vee \tilde{\upsilon}^{U}_{\tilde{S}}(y,z) \right) \right].$$

Definition 3.4. An interval valued intuitionistic fuzzy relation $\tilde{R}$ on $U \times U$ is said to be

(i) Reflexive if and only if $\forall\ x \in U$, $\tilde{\mu}_{\tilde{R}}(x,x) = [1,1]$ and $\tilde{\upsilon}_{\tilde{R}}(x,x) = [0,0]$.

(ii) Symmetric if and only if x, y$\in U$,

$$\tilde{\mu}_{\tilde{R}}(x,y) = \tilde{\mu}_{\tilde{R}}(y,x) \text{ and } \tilde{\upsilon}_{\tilde{R}}(x,y) = \tilde{\upsilon}_{\tilde{R}}(y,x).$$

(iii) Transitive if and only x, y, z $\in U$

$$\tilde{\mu}_{\tilde{R}}(x,z) \geq \left[\underset{y\in U}{\wedge} \left(\tilde{\mu}_{\tilde{R}}^{L}(x,y) \wedge \tilde{\mu}_{\tilde{R}}^{L}(y,z) \right), \underset{y\in U}{\vee} \left(\tilde{\mu}_{\tilde{R}}^{U}(x,y) \wedge \tilde{\mu}_{\tilde{R}}^{U}(y,z) \right) \right]$$

and

$$\tilde{\upsilon}_{\tilde{R}}(x,z) \leq \left[\underset{y\in U}{\wedge} \left(\tilde{\upsilon}_{\tilde{R}}^{L}(x,y) \wedge \tilde{\upsilon}_{\tilde{R}}^{L}(y,z) \right), \underset{y\in U}{\wedge} \left(\tilde{\upsilon}_{\tilde{R}}^{U}(x,y) \vee \tilde{\upsilon}_{\tilde{R}}^{U}(y,z) \right) \right].$$

Definition 3.5. If $\tilde{R}$ and $\tilde{S}$ are two IVIFR on $U \times V$ then $\tilde{R}$ is said to be a sub IVIFR of $\tilde{S}$ ($\tilde{R} \subset \tilde{S}$) if and only if for all (x, y)$\in U \times V$ we have

$$\tilde{\mu}_{\tilde{R}}^{L}(x,y) \leq \tilde{\mu}_{\tilde{S}}^{L}(x,y) \quad \& \quad \tilde{\mu}_{\tilde{R}}^{U}(x,y) \leq \tilde{\mu}_{\tilde{S}}^{U}(x,y)$$

and

$$\tilde{\upsilon}_{\tilde{R}}^{L}(x,y) \geq \tilde{\upsilon}_{\tilde{S}}^{L}(x,y) \quad \& \quad \tilde{\upsilon}_{\tilde{R}}^{U}(x,y) \geq \tilde{\upsilon}_{\tilde{S}}^{U}(x,y).$$

Definition 3.6. An IVIFR, $\tilde{R}$ on a single set U is called an IVIF tolerance relation on U, if R is reflexive and symmetric.

Definition 3.7. An IVIFR, $\tilde{R}$ on a single set U is called an IVIF equivalence relation on U, if $\tilde{R}$ is reflexive, symmetric and transitive.

Definition 3.8. If $\tilde{R}$ is an IVIFR on $U \times V$, its inverse relation $\tilde{R}^{-1}$ is an IVIFR on $V \times U$ such that $\forall$ (x, y)$\in U \times V$ and (y, x) $\in V \times U$ we have

$$\tilde{\mu}_{\tilde{R}^{-1}}(y,x) = \tilde{\mu}_{\tilde{R}}(x,y) \text{ and } \tilde{\upsilon}_{\tilde{R}^{-1}}(y,x) = \tilde{\upsilon}_{\tilde{R}}(x,y).$$

This, in turn, implies

$$\tilde{\mu}_{\tilde{R}^{-1}}^{L}(y,x) = \tilde{\mu}_{\tilde{R}}^{L}(x,y)\,;\, \tilde{\mu}_{\tilde{R}^{-1}}^{U}(y,x) = \tilde{\mu}_{\tilde{R}}^{U}(x,y),$$

$$\tilde{\upsilon}_{\tilde{R}^{-1}}^{L}(y,x) = \tilde{\upsilon}_{\tilde{R}}^{L}(x,y)\,; \text{ and } \tilde{\upsilon}_{\tilde{R}^{-1}}^{U}(y,x) = \tilde{\upsilon}_{\tilde{R}}^{U}(x,y).$$

Proposition 3.1. *If $\tilde{R}$ and $\tilde{S}$ are two interval valued intuitionistic fuzzy relations on $U \times V$ and $V \times W$ respectively then*

(3.5) *(i)* $\left(\tilde{R}^{-1}\right)^{-1} = \tilde{R}$

(3.6) *(ii)* $\left(\tilde{S} \circ \tilde{R}\right)^{-1} = \tilde{R}^{-1} \circ \tilde{S}^{-1}$

Proof: (i) The inverse relation of a IVIFR $\tilde{R}$ is denoted by $\tilde{R}^{-1}$, defined by $\tilde{\mu}_{\tilde{R}^{-1}}(y,x) = \tilde{\mu}_{\tilde{R}}(x,y)$ and $\tilde{\upsilon}_{\tilde{R}^{-1}}(y,x) = \tilde{\upsilon}_{\tilde{R}}(x,y)$ $\forall$ x$\in$ U and $\forall$ y$\in$V i.e. the elements of inverse IVIF relation matrix is obtained by transposing the corresponding IVIF relation matrix. Now the inverse of an inverse IVIF relation is obtained by transposing the inverse IVIF relation matrix .It is therefore again the initial interval valued intuitionistic fuzzy relation. This completes the proof.

(ii) If $\tilde{R}: U \to V$ and $\tilde{S}: V \to W$ then $\tilde{R}^{-1}: V \to U$ and $\tilde{S}^{-1}: W \to V$

Also, $\tilde{S} \circ \tilde{R}: U \to W$ and $(\tilde{S} \circ \tilde{R})^{-1}: W \to U$, $R^{-1} \circ S^{-1}: W \to U$

Now, for membership,

$$\tilde{\mu}_{(\tilde{S}\circ\tilde{R})^{-1}}(z,x) = \tilde{\mu}_{(\tilde{S}\circ\tilde{R})}(x,z)$$

$$= \left[\underset{y\in V}{\wedge} \left(\tilde{\mu}_{\tilde{R}}^{L}(x,y) \wedge \tilde{\mu}_{\tilde{S}}^{L}(y,z) \right), \underset{y\in V}{\vee} \left(\tilde{\mu}_{\tilde{R}}^{U}(x,y) \wedge \tilde{\mu}_{\tilde{S}}^{U}(y,z) \right) \right]$$

$$= \left[\underset{y\in V}{\wedge} \left(\tilde{\mu}_{\tilde{S}^{-1}}^{L}(z,y) \wedge \tilde{\mu}_{\tilde{R}^{-1}}^{L}(y,x) \right), \underset{y\in V}{\vee} \left(\tilde{\mu}_{\tilde{S}^{-1}}^{U}(z,y) \wedge \tilde{\mu}_{\tilde{R}^{-1}}^{U}(y,x) \right) \right]$$

$$= \tilde{\mu}_{\tilde{R}^{-1}\circ\tilde{S}^{-1}}(z,x)$$

For non membership,

$$\tilde{\upsilon}_{(\tilde{S}\circ\tilde{R})^{-1}}(z,x) = \tilde{\upsilon}_{(\tilde{S}\circ\tilde{R})}(x,z)$$

$$= \left[\underset{y\in V}{\wedge} \left(\tilde{\upsilon}_{\tilde{R}}^{L}(x,y) \wedge \tilde{\upsilon}_{\tilde{S}}^{L}(y,z) \right), \underset{y\in V}{\wedge} \left(\tilde{\upsilon}_{\tilde{R}}^{U}(x,y) \vee \tilde{\upsilon}_{\tilde{S}}^{U}(y,z) \right) \right]$$

$$= \left[\underset{y\in V}{\wedge} \left(\tilde{\upsilon}_{\tilde{S}^{-1}}^{L}(z,y) \wedge \tilde{\upsilon}_{\tilde{R}^{-1}}^{L}(y,x) \right), \underset{y\in V}{\wedge} \left(\tilde{\upsilon}_{\tilde{S}^{-1}}^{U}(z,y) \vee \tilde{\upsilon}_{\tilde{R}^{-1}}^{U}(y,x) \right) \right]$$

$$= \tilde{\upsilon}_{R^{-1}\circ S^{-1}}(z,x).$$

Hence $\left(\tilde{S} \circ \tilde{R}\right)^{-1} = \tilde{R}^{-1} \circ \tilde{S}^{-1}.$

4. Sanchez's approach extended to IVIFR

Let us use following symbolic notations:

S: Set of symptoms; D: Set of diseases; P: Set of patients.

We define "Interval valued intuitionistic medical knowledge" from S to D in a manner similar to Sanchez's approach. It is assumed that the knowledge provided by a physician can be translated in interval valued degrees of association and non association between symptoms and diseases.

The complete methodology is divided into following three steps:

(i) Determination of the symptoms.

(ii) Formulation of medical knowledge based on interval valued intuitionistic fuzzy relations.

(iii) To get the relationship between patients and diseases on the basis of the composition rules of IVIFR.

Let $\tilde{A}$ be an IVIFS representing state of a patient and $\tilde{R}$ be an IVIFR from S to D. Then $\tilde{B} = \tilde{R} \circ \tilde{A}$ gives the state of the patient in terms of diseases. $\tilde{B}$ is an IVIFS of the universe of discourse D. $\tilde{B}$ is computed on the basis of the following composition rule.

(4.1) $$\tilde{\mu}_{\tilde{B}}(d) = \left[\underset{s\in S}{\wedge}(\tilde{\mu}^L_{\tilde{A}}(s) \wedge \tilde{\mu}^L_{\tilde{R}}(s,d), \underset{s\in S}{\vee}(\tilde{\mu}^U_{\tilde{A}}(s) \wedge \tilde{\mu}^U_{\tilde{R}}(s,d) \right]$$

and

(4.2) $$\tilde{\upsilon}_{\tilde{B}}(d) = \left[\underset{s\in S}{\wedge}(\tilde{\upsilon}^L_{\tilde{A}}(s) \wedge \tilde{\upsilon}^L_{\tilde{R}}(s,d), \underset{s\in S}{\wedge}(\tilde{\upsilon}^U_{\tilde{A}}(s) \vee \tilde{\upsilon}^U_{\tilde{R}}(s,d) \right].$$

This is the case of one patient's diagnosis. This concept can be extended to a finite number of patients. Let there be n patients p_i; i=1, 2,…, n in a pathology. Let $\tilde{Q}$ be the interval valued intuitionistic fuzzy relation showing the relationship between patients and symptoms and $\tilde{R}$ be interval valued intuitionistic fuzzy relation showing the relationship between symptoms and diseases. Using the composition rule of inference defined in (5-6) we have

$$\tilde{T} = \tilde{R} \circ \tilde{Q}$$

Membership function between patients and diseases is given by

(4.3) $$\tilde{\mu}_{\tilde{T}}(d) = \left[\underset{s\in S}{\wedge}(\tilde{\mu}^L_{\tilde{Q}}(p,s) \wedge \tilde{\mu}^L_{\tilde{R}}(s,d), \underset{s\in S}{\vee}(\tilde{\mu}^U_{\tilde{Q}}(p,s) \wedge \tilde{\mu}^U_{\tilde{R}}(s,d) \right].$$

Non membership function between patients and diseases is given by

(4.4) $$\tilde{\upsilon}_{\tilde{T}}(d) = \left[\underset{s\in S}{\wedge}(\tilde{\upsilon}^L_{\tilde{Q}}(p,s) \wedge \tilde{\upsilon}^L_{\tilde{R}}(s,d), \underset{s\in S}{\wedge}(\tilde{\upsilon}^U_{\tilde{Q}}(p,s) \vee \tilde{\upsilon}^U_{\tilde{R}}(s,d) \right].$$

5. Diagnostic Decision

To get the diagnostic decision from the obtained IVIFR between patients and diseases, we define the concept of Positive ideal and Negative ideal for IVIFSs.

(5.1) Let $A^+ = \left\{ \left\langle c_j, \tilde{\mu}_{A^+}(c_j), \tilde{\upsilon}_{A^+}(c_j) \right\rangle | c_j \in C \right\}$ be the positive ideal IVIFS

and

(5.2) $A^- = \left\{ \left\langle c_j, \tilde{\mu}_{A^-}(c_j), \tilde{\upsilon}_{A^-}(c_j) \right\rangle | c_j \in C \right\}$ be the negative ideal IVIFS.

where $\tilde{\mu}_{A^+}(c_j) = \left[\tilde{\mu}^L_{A^+}(c_j), \tilde{\mu}^U_{A^+}(c_j) \right]$ and $\tilde{\upsilon}_{A^+}(c_j) = \left[\tilde{\upsilon}^L_{A^+}(c_j), \tilde{\upsilon}^U_{A^+}(c_j) \right]$

$\tilde{\mu}^L_{A^+}(c_j) = \max_i \tilde{\mu}^L_{A_i}(c_j)$, $\tilde{\mu}^U_{A^+}(c_j) = \max_i \tilde{\mu}^U_{A_i}(c_j)$ and

$$\tilde{\upsilon}^L_{A^+}(c_j) = \min_i \tilde{\upsilon}^L_{A_i}(c_j), \tilde{\upsilon}^U_{A^+}(c_j) = \min_i \tilde{\upsilon}^U_{A_i}(c_j)$$

$$\tilde{\mu}_{A^-}(c_j) = \left[\tilde{\mu}^L_{A^-}(c_j), \tilde{\mu}^U_{A^-}(c_j)\right] \text{ and } \tilde{\upsilon}_{A^-}(c_j) = \left[\tilde{\upsilon}^L_{A^-}(c_j), \tilde{\upsilon}^U_{A^-}(c_j)\right]$$

$$\tilde{\mu}^L_{A^-}(c_j) = \min_i \tilde{\mu}^L_{A_i}(c_j) \; , \; \tilde{\mu}^U_{A^-}(c_j) = \min_i \tilde{\mu}^U_{A_i}(c_j) \text{ and}$$

$$\tilde{\upsilon}^L_{A^-}(c_j) = \max_i \tilde{\upsilon}^L_{A_i}(c_j), \quad \tilde{\upsilon}^U_{A^-}(c_j) = \max_i \tilde{\upsilon}^U_{A_i}(c_j).$$

Here C is the set of criteria and A_i's are alternatives. Again define the degree of similarity of the positive ideal and negative ideal from the i^{th} alternative as:

(5.3)

$$S(A^+, A_i) = 1 - \left[\frac{1}{4n}\sum_{j=1}^{n}\left(\left|\mu^L_{A^+}(x_j) - \mu^L_{A_i}(x_j)\right| + \left|\mu^U_{A^+}(x_j) - \mu^U_{A_i}(x_j)\right| + \left|\upsilon^L_{A^+}(x_j) - \upsilon^L_{A_i}(x_j)\right| \right.\right.$$
$$\left.\left. + \left|\upsilon^U_{A^+}(x_j) - \upsilon^U_{A_i}(x_j)\right| + \left|\pi^L_{A^+}(x_j) - \pi^L_{A_i}(x_j)\right| + \left|\pi^U_{A^+}(x_j) - \pi^U_{A_i}(x_j)\right|\right)\right]$$

(5.4)

$$S(A^-, A_i) = 1 - \left[\frac{1}{4n}\sum_{j=1}^{n}\left(\left|\mu^L_{A^-}(x_j) - \mu^L_{A_i}(x_j)\right| + \left|\mu^U_{A^-}(x_j) - \mu^U_{A_i}(x_j)\right| + \left|\upsilon^L_{A^-}(x_j) - \upsilon^L_{A_i}(x_j)\right| \right.\right.$$
$$\left.\left. + \left|\upsilon^U_{A^-}(x_j) - \upsilon^U_{A_i}(x_j)\right| + \left|\pi^L_{A^-}(x_j) - \pi^L_{A_i}(x_j)\right| + \left|\pi^U_{A^-}(x_j) - \pi^U_{A_i}(x_j)\right|\right)\right]$$

The relative degree of similarity measure can be defined as follows:

(5.5) $$d(A_i) = \frac{S(A^+, A_i)}{S(A^+, A_i) + S(A^-, A_i)}.$$

In an IVIFR between patients and diseases, we assume diseases as the alternatives while the combination of membership and nonmembership intervals for a patient is considered to be the criterion.

The diagnosis for a patient *P* is said to favour the disease *k* if

(5.6) $$d(A_k) = \max_i d(A_i).$$

6. Case Study

We illustrate our results through the case discussed by De et.al.
Let P= {Paul, Jadu, Kundu, and Rohit} be the set of patients.
S = {Temperature, Headache, Stomach pain, Cough, Chest pain} be the set of symptoms.
D = {Viral fever, Malaria, Typhoid, Stomach Problem, Chest Problem}

be the set of diseases possibly associated to these symptoms. The interval valued intuitionistic fuzzy relation ($\tilde{Q}$) from P to S and the interval valued intuitionistic fuzzy relation ($\tilde{R}$) from S to D are given below in Tables-1 and 2 respectively.

Table-1

$\tilde{Q}$	Temperature	Headache	Stomach pain	Cough	Chest pain
Paul	[0.7,0.8] [0.1,0.2]	[0.5,0.7] [0.1,0.2]	[0.1,0.2] [0.7,0.8]	[0.5,0.7] [0.1,0.2]	[0.1,0.2] [0.5,0.7]
Jadu	[0.0,0.1] [0.7,0.85]	[0.3,0.5] [0.3,0.45]	[0.5,0.7] [0.1,0.2]	[0.05,0.15] [0.6,0.8]	[0.05,0.15] [0.7,0.85]
Kundu	[0.7,0.85] [0.05,0.15]	[0.7,0.85] [0.05,0.15]	[0.0,0.1] [0.5,0.7]	[0.1,0.25] [0.6,0.75]	[0.0,0.1] [0.4,0.6]
Rohit	[0.5,0.7] [0.05,0.2]	[0.4,0.6] [0.3,0.4]	[0.2,0.4] [0.3,0.5]	[0.65,0.75] [0.15,0.25]	[0.2,0.4] [0.3,0.5]

Table-2

$\tilde{R}$	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	[0.3,0.5] [0.0,0.1]	[0.6,0.8] [0.0,0.1]	[0.2,0.4] [0.2,0.4]	[0.05,0.15] [0.6,0.8]	[0.05,0.15] [0.75,0.85]
Headache	[0.2,0.4] [0.45,0.55]	[0.1,0.3] [0.5.0.7]	[0.5,0.7] [0.05,0.15]	[0.1,0.3] [0.3,0.5]	[0.0,0.1] [0.7,0.9]
Stomach pain	[0.05,0.15] [0.6,0.8]	[0.0,0.05] [0.85,0.95]	[0.1,0.3] [0.6,0.7]	[0.7,0.85] [0.0,0.1]	[0.1,0.2] [0.7,0.8]
Cough	[0.3,0.5] [0.2,0.4]	[0.6,0.8] [0.0,0.1]	[0.1,0.3] [0.5,0.7]	[0.1,0.3] [0.6,0.7]	[0.1,0.2] [0.7,0.8]
Chest pain	[0.05,0.15] [0.6,0.8]	[0.05,0.15] [0.7,0.85]	[0.0,0.1] [0.8,0.9]	[0.15,0.25] [0.65,0.75]	[0.75,0.85] [0.1,0.15]

Using the composition rule defined in (3.3) and (3.4), we get the interval valued intuitionistic fuzzy relation between patients and diseases as given in Table-3.

Table-3

$\tilde{T}$	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Paul	[0.05,0.5] [0.0,0.2]	[0.0,0.8] [0.0,0.2]	[0.0,0.7] [0.05,0.2]	[0.05,0.3] [0.0,0.5]	[0.0,0.2] [0.1,0.7]
Jadu	[0.0,0.4] [0.0,0.55]	[0.0,0.3] [0.0,0.7]	[0.0,0.5] [0.05,0.45]	[0.0,0.7] [0.0,0.2]	[0.0,0.2] [0.1,0.8]
Kundu	[0.0,0.5] [0.0,0.15]	[0.0,0.8] [0.0,0.15]	[0.0,0.7] [0.05,0.15]	[0.0,0.3] [0.0,0.5]	[0.0,0.2] [0.05,0.6]
Rohit	[0.05,0.5] [0.0,0.2]	[0.0,0.75] [0.0,0.2]	[0.0,0.6] [0.05,0.4]	[0.05,0.4] [0.0,0.5]	[0.0,0.4] [0.05,0.5]

Using (5.1)-(5.4), positive and negative ideal IVIFS for patient Paul and the degree of similarity of the positive and negative ideals from the ith alternative are respectively given below:

$$A^+ = \{\langle Paul, [0.05, 0.8], [0.0, 0.2]\rangle\}, A^- = \{\langle Paul, [0, 0.2], [0.1, 0.7]\rangle\}$$

$$S(A^+, A_1) = 0.85, S(A^+, A_2) = 0.975, S(A^+, A_3) = 0.925,$$

$$S(A^+, A_4) = 0.8, S(A^+, A_5) = 0.65,$$

$$S(A^-, A_1) = 0.7, S(A^-, A_2) = 0.65, S(A^-, A_3) = 0.725,$$

$$S(A^-, A_4) = 0.85, S(A^-, A_5) = 1,$$

Using (5.5) we get the following relative distances for each alternative:

$$d_{A_1} = 0.5483, d_{A_2} = 0.6, d_{A_3} = 0.5606, d_{A_4} = 0.4848, d_{A_5} = 0.3939$$

Following results for other patients can also be obtained in a similar manner. These results are given in Table-4 with bold entries indicating the diagnosed disease.

Table-4

$\tilde{T}$	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Paul	0.5483	0.6	0.5606	0.4848	0.3939
Jadu	0.5	0.45	0.5151	0.6060	0.3939
Kundu	0.5312	0.5970	0.5522	0.4477	0.4029
Rohit	0.5147	0.5492	0.4929	0.4583	0.4444

7. Conclusion

Medical knowledge concerning the symptom-disease relationship constitutes a major source of imprecision and uncertainty in the diagnostic process. Linguistically described state of the patients is yet another source. A good decision model must take into account the favorable and unfavorable evidences separately to cover up the vagueness and incompleteness. Present work prescribes a model based on interval valued intuitionistic fuzzy sets for medical diagnosis. It defines composition rule of inference for interval valued intuitionistic fuzzy relations to extend the approach of De et.al to interval valued intuitionistic fuzzy sets. The work also introduces the concept of positive and negative ideal for IVIFS to reach to the final decision on the diagnosis.

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