

Vague Set Theoretic Approach to Fault Tree Analysis*

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Abstract: In conventional fault-tree analysis, the failure probabilities of component of a system are treated as exact values in estimating the failure probability of the top event. The fault detection and analysis of a system are the key measures to improve the reliability of the system. Due to the complexity in the structure and the variations in operating conditions, the occurrence of a fault inside a system is uncertain and random. Until now, the fault statistics is very limited due to the low failure rate. Present paper proposes a novel fault tree analysis method based on vague set theory.

Keywords: Vague sets, Reliability analysis, Fault tree analysis, Vague numbers, Vague Reliability.

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1. Introduction

Fuzzy set theory proposed by Zadeh¹ permits the replacement of the sharp boundaries in classical set theory by fuzzy boundaries. The concept of belongingness of an element in the context of classical sets changes to membership grade of the element to certain degree in fuzzy sets. The membership grade of an element x of the fuzzy set is given by a real number between zero and one. Due to fuzzy boundaries, this single value for the membership grade is the result of the combined effect of evidences in favour and against the inclusion of the element in the set². The utility of the application of fuzzy sets depends on the capability of the user to construct appropriate membership functions, which are often very precise. In many contexts it is difficult to assign a particular real number as a membership

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grade and in such cases it may be useful to identify meaningful lower and upper bounds for the membership grade. Such generalization of fuzzy sets is called vague sets.

The concept of fault tree analysis (FTA) was developed in 1962 at Bell telephone laboratories. FTA is now widely used in many fields, such as in nuclear reactor, chemical and aviation industries. Fault tree analysis (FTA) is a logical and diagrammatic method for evaluating system reliability. It is logical approach for systematically quantifying the possibility of abnormal system event. Starting from the top event the fault tree method employs Boolean algebra³ and logical modeling to represent the relations among various failure events at different levels of system decomposition. FTA can be a qualitative evaluation or quantitative analysis. However, current fault tree analysis still cannot be performed functionally without facing imprecise failure and improper modeling problems. FTA is now widely used in many fields such as in the nuclear reactor and chemical industries^{4,5}.

The reliability of a system is the probability that the system will perform a specified function satisfactorily during some interval of time under specified operating conditions⁶. Traditionally, the reliability of a system behaviour is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. However in real life systems, the information may be inaccurate or might have linguistic representation. In such cases the estimation of precise values of probability becomes very difficult. In order to handle this situation, fuzzy approach^{6,7,8} is used to evaluate the failure rate status. Fuzzy fault tree analysis has been used by several researchers^{3,4,5,9,10,11,12}. Singer¹³ proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. He used possibilistic AND, OR and NEG operators to construct possible fault tree.

Concept of vague sets given by Gau and Buehrer takes into account the favourable and unfavourable evidences separately providing a lower and an upper bound within which the membership grade may lie. Chen¹⁴ presented similarity measures between vague sets. Recently, Chen¹⁵ proposed fuzzy system reliability analysis based on vague set theory, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse $[0, 1]$. Chen's method has the advantages of modeling and analyzing the fuzzy system reliability in a more flexible and more intelligent manner. However, Chen's method limits its applicability to some special case of general vague set.

A more general vague fault- tree analysis is proposed in this paper. The work in this paper collects expert's knowledge and experience on the

problem domain and builds the possibility of failure of basic event so as to consider a source of obtaining system reliability of the top event. To work with the fault tree using vague sets, we modify fuzzy fault-tree analysis¹⁶ and integrate vague set arithmetic operations to implement fault-tree analysis on system fault diagnosis.

The paper is organized in six sections. Section 2 discusses definition of vague set and its operations. Section 3 proposes a new approach for fault-tree analysis using vague sets and represents an algorithm of vague fault-tree analysis. In section 4, a Power failure system is used to illustrate the algorithm of the fault tree analysis. Section 5, numerically verifies the results and compares them with others. The last section concludes the work.

1. Basic Concepts of Vague Sets

Vague set: - A vague set \tilde{A} in the universe of discourse X is characterized by a membership function $\mu_{\tilde{A}} : X \rightarrow [0,1]$ and a non-membership function $\nu_{\tilde{A}} : X \rightarrow [0,1]$. The grade of membership for any element x in the vague set is bounded by a sub interval $[\mu_{\tilde{A}}(x), 1 - \nu_{\tilde{A}}(x)]$, where the grade $\mu_{\tilde{A}}(x)$ is called lower bound of membership grade of x derived from evidences for x and $\nu_{\tilde{A}}(x)$ is the lower bound of membership grade on the negation of x derived from the evidences against x and $\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. In the extreme case of equality $\mu_{\tilde{A}}(x) = 1 - \nu_{\tilde{A}}(x)$, the vague set reduces to the fuzzy set with interval value of the membership grade reducing to a single value $\mu_{\tilde{A}}(x)$. In general, however,

$$\mu_{\tilde{A}}(x) \leq \text{exact membership grade of } x \leq 1 - \nu_{\tilde{A}}(x).$$

Expressions (1) and (2) given below can be used to represent a vague set \tilde{A} for finite and infinite universe of discourse X respectively.

$$\tilde{A} = \sum_{k=1}^n [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k ,$$

$$\tilde{A} = \int_X [\mu_{\tilde{A}}(x_k), 1 - \nu_{\tilde{A}}(x_k)] / x_k .$$

A vague set is represented pictorially as

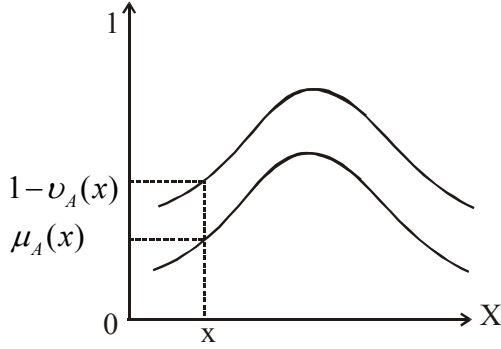


Fig-1

2.1 Convex vague set: Let \tilde{A} be a vague set of the universe of discourse X with $\mu_{\tilde{A}}$ and $\nu_{\tilde{A}}$ as its membership and non-membership functions respectively. The vague set is convex if and only if for every x_1, x_2 in X

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{Min}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)),$$

$$1 - \nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \text{Min}(1 - \nu_{\tilde{A}}(x_1), 1 - \nu_{\tilde{A}}(x_2)),$$

where $\lambda \in [0, 1]$.

2.2 Normal vague set: A vague set \tilde{A} in the universe of discourse X is called normal if

$$\exists x_i \in X, \text{ such that } 1 - \nu_{\tilde{A}}(x_i) = 1. \text{ That is } \nu_{\tilde{A}}(x_i) = 0.$$

2.3 Vague number: A vague number is a vague subset in the universe of the discourse X which is both convex and normal.

2.4. Triangular vague number: Chen³ defined triangular vague sets and arithmetic operations between them. On similar lines we introduce concept of a triangular vague number.

A triangular vague number \tilde{A} denoted by $[(a, b, c); k; 1]$ is characterized by a pair of membership functions: a lower membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{k(x-a)}{b-a}, & a \leq x \leq b \\ \frac{k(c-x)}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases},$$

and an upper membership function

$$\mu'_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases},$$

where $\mu'_A(x) = 1 - \nu_A(x)$ and $k \in [0,1]$. Figure 2 shows a triangular vague number.

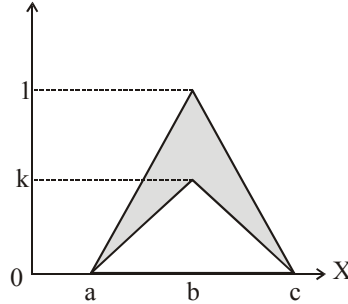


Fig-2

When $k = 1$, triangular vague number reduces to a triangular fuzzy number. In what follows now onwards, we shall use vague number for a triangular vague number.

2.5 Vague point: In a triangular vague number $\tilde{A} = [(a,b,c);k,1]$, if $a=c=b$, say, then

$$\tilde{A} = [(b,b,b);k,1] = b_k,$$

is said to be a vague point. A vague point b_k reduces to a fuzzy point b_1 for $k = 1$.

2.6 Arithmetic operations of triangular vague sets: A simple triangular vague set is represented as: $\langle [(a,b,c);\mu_1], [(a,b,c);\mu_2] \rangle$ or more concisely way as $\langle [(a,b,c);\mu_1;\mu_2] \rangle$, as shown in figure 2. From the definition of triangle vague set, we propose four arithmetic operations for triangular vague sets in the following:

Let A and B are two vague sets as shown in figure

If two vague sets $t_A \neq t_B$, and $1 - f_A \neq 1 - f_B$, then the arithmetic operations are defined as:

$$(2.6.1) \quad A = \langle [(a'_1, b_1, c'_1);\mu_1], [(a_1, b_1, c_1);\mu_2] \rangle,$$

$$(2.6.2) \quad B = \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle,$$

$$(2.6.3) \quad \begin{aligned} A(+)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle \\ &\quad + \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \left\langle [(a'_1 + a'_2, b_1 + b_2, c'_1 + c'_2); \min(\mu_1, \mu_3)], \right. \\ &\quad \left. [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \min(\mu_2, \mu_4)] \right\rangle, \end{aligned}$$

$$(2.6.4) \quad \begin{aligned} A(-)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle \\ &\quad - \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \left\langle [(a'_1 - a'_2, b_1 - b_2, c'_1 - c'_2); \min(\mu_1, \mu_3)], \right. \\ &\quad \left. [(a_1 - a_2, b_1 - b_2, c_1 - a_2); \min(\mu_2, \mu_4)] \right\rangle, \end{aligned}$$

$$(2.6.5) \quad \begin{aligned} A(\times)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle \\ &\quad + \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \left\langle [(a'_1 a'_2, b_1 b_2, c'_1 c'_2); \min(\mu_1, \mu_3)], \right. \\ &\quad \left. [(a_1 a_2, b_1 b_2, c_1 c_2); \min(\mu_2, \mu_4)] \right\rangle, \end{aligned}$$

$$(2.6.6) \quad \begin{aligned} A(/)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle \\ &\quad + \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \left\langle [(a'_1 / c'_2, b_1 / b_2, c'_1 / a'_2); \min(\mu_1, \mu_3)], \right. \\ &\quad \left. [(a_1 / c_2, b_1 / b_2, c_1 / a_2); \min(\mu_2, \mu_4)] \right\rangle, \end{aligned}$$

3. Proposed Algorithm for Fault tree Using Vague sets

Four arithmetic operations of triangular vague sets are used in fault tree analysis. In the present paper we have calculated the vague importance index of every basic event as follows:

Let P_{Ti} represent that P_T is not included in the i^{th} basic event of failure interval and $V(P_T, P_{Ti})$, called Vague Important Index (V.I.I.), denotes the difference between P_T and P_{Ti} . The larger values of $V(P_T, P_{Ti})$ represent that i th basic event has greater importance on P_T ; then

$$V(P_T, P_{T_i}) \equiv (a'_T - a'_{T_i}) + (a_T - a_{T_i}) + (b_T - b_{T_i}) + (c'_T - c'_{T_i}) + (c_T - c_{T_i}), \quad (3.1)$$

where $P_T = (a'_T, a_T, b_T, c_T, c'_T)$

and $P_{T_i} = (a'_{T_i}, a_{T_i}, b_{T_i}, c_{T_i}, c'_{T_i}),$

Following five steps are proposed in order to implement vague sets to fault tree analysis in power system fault diagnosis. These form the basis of the model for vague fault tree analysis.

Step I: Construct fault tree diagram using fault tree logical symbols and tracing back whole process from top to basic events.

Step II: Obtain possible failure intervals of basic events by using expert's knowledge and experience.

Step III: Calculate possible failure intervals of system using vague set arithmetic operations.

Step IV: Calculate the reliability interval of top events.

Step V: Find Vague Importance Index (V.I.I.) of basic events of system reliability by using $V(P_T, P_{T_i}), \forall i$ and to find the most important index power $\max_i V(P_T, P_{T_i})$ for the whole system.

4. Fault Tree Model of Power System

A power failure system is used to illustrate the above-mentioned approach. A power failure system includes electrical failure or hydraulic failure or failure due to manual mistake. The electrical failure is caused due to supply failure or due to supply out of range or due to the hydraulic pump power failure. The supply failure is resulted from supply out of range or due to sensor failure. The hydraulic failure is caused due to four reasons viz. oil pressure, oil level, oil temperature and clogged filters. As shown in fig. 3, the power failure is treated as a hazard that is, the top event in fault tree analysis. The power failure may be due to some basic events and these events may again occur due to some other events and so on.

The Boolean expression corresponding to this fault tree can be given as below

$$\begin{aligned} P &= X \cup Y \cup Z, \\ X &= A \cup B \cup C, \\ A &= A_1 \cap A_2 \\ B &= B_1 \cap B_2, \\ Y &= D \cup E \cup F \cup G, \end{aligned}$$

$D=D_1 \cap D_2$
 $E=E_1 \cap E_2,$
 $F=F_1 \cap F_2,$
 $G= G_1 \cup G_2 \cup G_3$

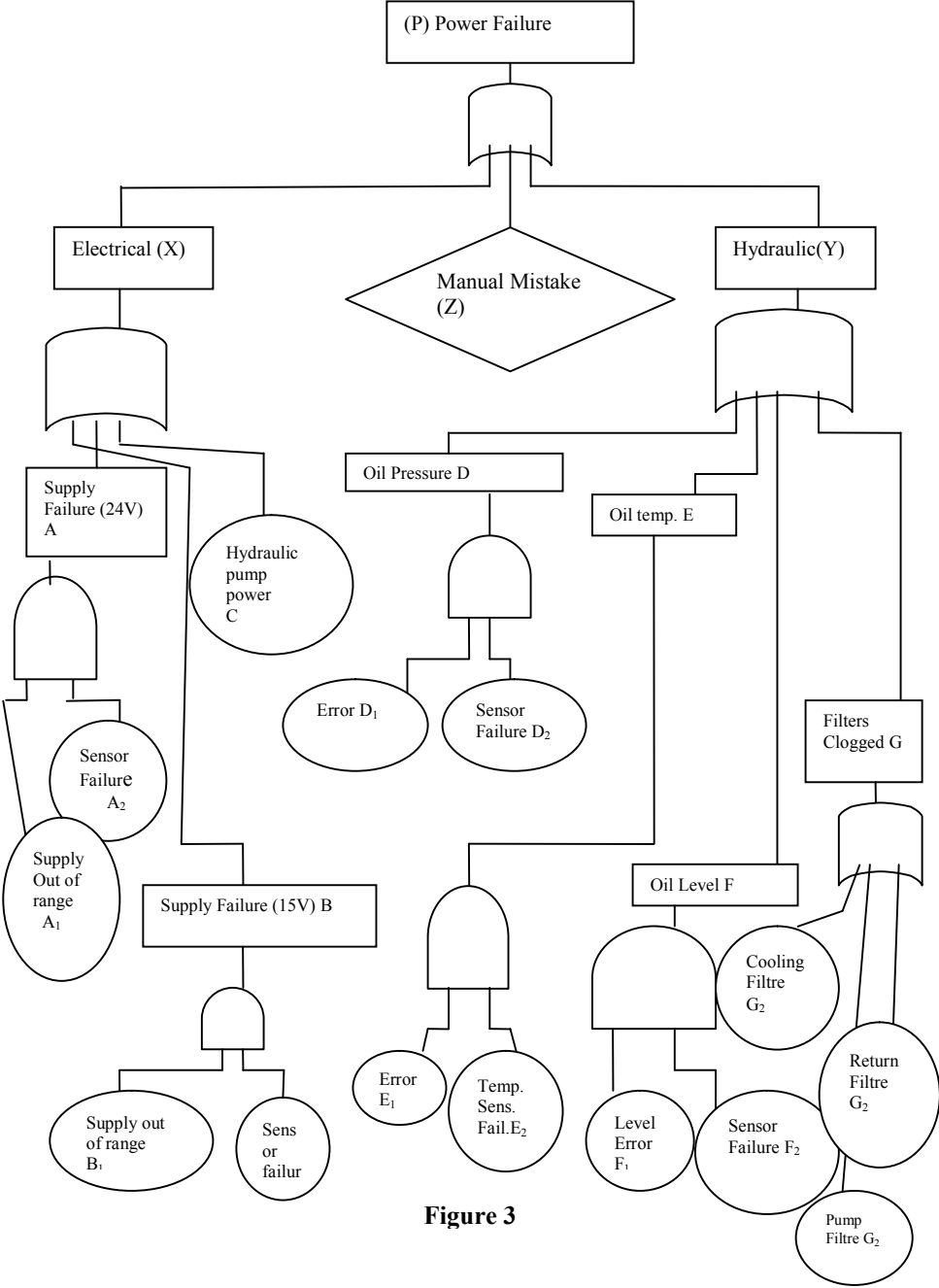


Figure 3

where P denotes power failure, X = electrical failure due to failure of hydraulic pump power, Z = manual mistake, Y = Hydraulic failure due to oil pressure, oil level, oil temperature or filters clogged, A = Supply failure due to supply out of range and sensor failure, A_1 = Hydraulic pump power failure, A_2 = Supply out of range, G_1 = Sensor failure, C = Oil pressure, E = Oil level, D = Oil temperature, G = Filters clogged, F_1 = Temperature error, B_1 = Temperature sensor failure, B_2 = Pressure error, E_1 = Pressure sensor failure, E_2 = Level error, F_1 = Level sensor failure, F_2 = Cooling filter, D_1 = Pump filter, D_2 = Return filter.

4.1 Solution of the Model:

$$P = X \cup Y \cup Z$$

$$\begin{aligned} P &= (A \cup B \cup C) \cup Z \cup (D \cup E \cup F \cup G) \\ &= ((A_1 \cap A_2) \cup (B_1 \cap B_2) \cup C) \cup Z \cup \\ &\quad ((D_1 \cap D_2) \cup (E_1 \cap E_2) \cup (F_1 \cap F_2) \cup (G_1 \cup G_2 \cup G_3)), \\ (4.1.1) \quad P &= ((A_1 \cap A_2) \cup (B_1 \cap B_2) \cup C) \cup Z \cup \\ &\quad ((D_1 \cap D_2) \cup (E_1 \cap E_2) \cup (F_1 \cap F_2) \cup (G_1 \cup G_2 \cup G_3)), \end{aligned}$$

where \cap and \cup mean relations of parallel and series respectively.

Let P_i represents the failure possibility of basic event I , then the possibility of failure of X can be expressed as

$$(4.1.2) \quad P_X = 1 - (1 - P_A)(1 - P_B)(1 - P_C).$$

The failure possibility of Z is

$$(4.1.3) \quad P_Z = 1 - (1 - P_D)(1 - P_E)(1 - P_F)(1 - P_G).$$

The failure possibility of A is

$$(4.1.4) \quad P_A = P_{A_1} \cdot P_{A_2}.$$

The failure possibility of B is

$$(4.1.5) \quad P_B = P_{B_1} \cdot P_{B_2}.$$

The failure possibility of D is

$$(4.1.6) \quad P_D = P_{D_1} \cdot P_{D_2}.$$

The failure possibility of E is

$$(4.1.7) \quad P_E = P_{E_1} . P_{E_2} .$$

The failure possibility of F is

$$(4.1.8) \quad P_F = P_{F_1} . P_{F_2} .$$

The failure possibility of G is

$$(4.1.9) \quad P_G = 1 - (1 - P_{G_1})(1 - P_{G_2})(1 - P_{G_3}) .$$

Then the top event possibility of power failure system can be described as:

$$(4.1.10) \quad P_T = \{1 - (1 - P_X)(1 - P_Y)(1 - P_Z)\} \\ \times \{1 - \{1 - (1 - \{1 - (1 - P_A)(1 - P_B)(1 - P_C)\} \times (1 - P_Z)) \\ \times \{1 - \{1 - (1 - P_D)(1 - P_E)(1 - P_F)(1 - P_G)\}\}\},$$

$$(4.1.11) \quad P_T = [\{1 - (1 - P_{A_1} . P_{A_2})(1 - P_{B_1} . P_{B_2})(1 - P_C)\} \times \{1 - P_Z\} \\ \times \{(1 - P_{D_1} . P_{D_2})(1 - P_{E_1} . P_{E_2})(1 - P_{F_1} . P_{F_2})\} \\ \times \{(1 - P_{G_1})(1 - P_{G_2})(1 - P_{G_3})\},$$

4.2 Possible Failure Range of Basic Events:

Failure poss.	a _i	a _i [']	b _i	c _i [']	c _i	μ _{1-FA(U)}	μ _{tA(U)}
P_{T_C}	0.0001	0.0005	0.0001	0.0002	0.0003	0.9	0.8
$P_{T_{A_1}}$	0.0	0.002	0.005	0.008	0.01	0.9	0.7
$P_{T_{A_2}}$	0.003	0.005	0.006	0.008	0.02	0.8	0.8
$P_{T_{B_1}}$	0.007	0.007	0.007	0.009	0.009	1.0	1.0
$P_{T_{B_2}}$	0.003	0.004	0.005	0.006	0.007	0.9	0.8
$P_{T_{D_2}}$	0.003	0.006	0.007	0.008	0.009	0.9	0.8
$P_{T_{D_2}}$	0.006	0.007	0.008	0.009	0.01	0.8	0.7
$P_{T_{E_1}}$	0.003	0.006	0.006	0.007	0.009	0.7	0.6
$P_{T_{E_2}}$	0.002	0.003	0.004	0.007	0.008	0.9	0.8

$P_{T_{F_1}}$	0.001	0.002	0.003	0.004	0.005	1.0	0.8
$P_{T_{F_2}}$	0.004	0.004	0.005	0.006	0.006	0.8	0.7
$P_{T_{G_1}}$	0.003	0.003	0.005	0.006	0.007	0.8	1.0
$P_{T_{G_2}}$	0.008	0.008	0.009	0.009	0.01	0.9	0.6
$P_{T_{G_3}}$	0.005	0.004	0.004	0.005	0.02	1.0	0.7

5. Numerical Computations

5.1 Traditional reliability: Traditionally, probability method is the method for dealing with the heterogeneous problems, and probability can only show the randomness of success or failure events. This method is constrained to its usage on the condition of great amount of data sample and all event outcomes are assumed to be under certainty. In the traditional approach we calculate failure possibility of power system as the following:

$$\begin{aligned}
 P_T &= [\{1 - (1 - P_{A_1} \cdot P_{A_2}) (1 - P_{B_1} \cdot P_{B_2}) (1 - P_C)\} \times \{1 - P_Z\} \\
 &\quad \times \{(1 - P_{D_1} \cdot P_{D_2}) (1 - P_{E_1} \cdot P_{E_2}) (1 - P_{F_1} \cdot P_{F_2})\} \\
 &\quad \times \{(1 - P_{G_1}) (1 - P_{G_2}) (1 - P_{G_3})\}, \\
 &= \{1 - (1 - 0.005 \times 0.007) \times (1 - 0.006 \times 0.004) \times (1 - 0.003) \\
 &\quad \times (1 - 0.008) \times (1 - 0.005 \times 0.005) \times (1 - 0.009) (1 - 0.002) \\
 &\quad \times (1 - 0.0001) (1 - 0.006) \times (1 - 0.007) (1 - 0.005)\} \\
 &\quad \times \{1 - 0.0012\} (1 - 0.0025) (1 - 0.0012) \\
 &= 1 - 0.96250 \\
 &= 0.03750
 \end{aligned}$$

Thus we find the failure probability of the power failure system as 0.03750 and the reliability of the system as 0.96250.

5.2 Proposed method: According to arithmetic operations of triangle vague set (2.6.1) to (2.6.6), the failure range of Power failure system" can be described as:

$$\begin{aligned}
 P_T &= [\{1 - (1 - P_{A_1} \cdot P_{A_2}) (1 - P_{B_1} \cdot P_{B_2}) (1 - P_C)\} \times \{1 - P_Z\} \\
 &\quad \times \{(1 - P_{D_1} \cdot P_{D_2}) (1 - P_{E_1} \cdot P_{E_2}) (1 - P_{F_1} \cdot P_{F_2})\} \\
 &\quad \times \{(1 - P_{G_1}) (1 - P_{G_2}) (1 - P_{G_3})\}, \\
 &= \{1(-) [(0.992, 0.995, 0.998); 0.8], [(0.99, 0.995, 1.0); 0.9] \\
 &\quad (\times) [(0.992, 0.993, 0.994); 0.8], [(0.991, 0.993, 0.997); 0.9]
 \end{aligned}$$

$$\begin{aligned}
& (\times) [(1, 1, 1); 0.6], [(1, 1, 1); 0.7] \\
& (\times) [(0.991, 0.992, 0.993); 0.7], [(0.99, 0.992, 0.994); 0.8] \\
& (\times) [(1, 1, 1); 0.6], [(1, 1, 1); 0.8] \\
& (\times) [(0.99974, 0.9999, 0.99995); 0.8], [(0.9997, 0.9999, 0.99999); 0.9] \\
& (\times) [(0.992, 0.994, 0.995); 0.8], [(0.99, 0.994, 0.997); 0.8] \\
& (\times) [(0.991, 0.993, 0.993); 1.0], [(0.991, 0.993, 0.993); 1.0] \\
& (\times) [(0.994, 0.995, 0.996); 0.8], [(0.993, 0.995, 0.997); 0.9] \\
& = \{1(-) [(0.95270, 0.96250, 0.96934); 0.6], [(0.94596, 0.96250, 0.97818); 0.7]\} \\
& = [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7]
\end{aligned}$$

On the basis of above calculations, we find that the failure interval of "Power failure system" as following

$$= [(0.0466, 0.03750, 0.04030); 0.6], [(0.03182, 0.03750, 0.04404); 0.7]$$

Thus the reliability interval of "Power failure system" can be described as the following vague number.

$$(5.2.1) [(0.9540, 0.96250, 0.96934); 0.6], [(0.9596, 0.96250, 0.97818); 0.7]$$

Expression (5.2.1) interprets the reliability to lie in the interval (0.9540, 0.96934) with truth value 0.6 and in the interval (0.9596, 0.97818) with truth value 0.7. It can be observed that the crisp value of traditional reliability lies within the obtained intervals.

In order to find the vague importance index, we calculate P_T as the followings:

$$\begin{aligned}
P_{T_C} &= [(0.03061, 0.03741, 0.04705); 0.6], [(0.02181, 0.03741, 0.05376); 0.7], \\
P_{T_{A_1}} &= [(0.02872, 0.03267, 0.03962); 0.6], [(0.02182, 0.03267, 0.04449); 0.7], \\
P_{T_{A_2}} &= [(0.02579, 0.03169, 0.03962); 0.6], [(0.01888, 0.03169, 0.04449); 0.7], \\
P_{T_{B_1}} &= [(0.02383, 0.03072, 0.03865); 0.6], [(0.01493, 0.03072, 0.04545); 0.7], \\
P_{T_{B_2}} &= [(0.02677, 0.03267, 0.04155); 0.6], [(0.01888, 0.03267, 0.04737); 0.7], \\
P_{T_{D_1}} &= [(0.02481, 0.03072, 0.03962); 0.6], [(0.01888, 0.03072, 0.04545); 0.7], \\
P_{T_{D_2}} &= [(0.02383, 0.02974, 0.03865); 0.6], [(0.01592, 0.02974, 0.04449); 0.7], \\
P_{T_{E_1}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8], \\
P_{T_{E_2}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8], \\
P_{T_{F_1}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.8],
\end{aligned}$$

$$\begin{aligned}
P_{T_{F_2}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7], \\
P_{T_{G_1}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7], \\
P_{T_{G_2}} &= [(0.03066, 0.03750, 0.04730); 0.6], [(0.02182, 0.03750, 0.05404); 0.7], \\
P_{T_{G_3}} &= [(0.02066, 0.02750, 0.03730); 0.6], [(0.01182, 0.02750, 0.04404); 0.7],
\end{aligned}$$

In view of equation (7), the V.I.I. of all basic events are calculated as following:

$$\begin{aligned}
V(P_T, P_{TA1}) &= 0.00068, & V(P_T, P_{TA2}) &= 0.024, & V(P_T, P_{TC}) &= 0.03085, & V(P_T, P_{TZ}) &= 0 \\
V(P_T, P_{TB1}) &= 0.03774, & V(P_T, P_{TB2}) &= 0.02408, & V(P_T, P_{TD1}) &= 0.03184, & V(P_T, P_{TD2}) &= 0.03869, \\
V(P_T, P_{TE1}) &= 0, & V(P_T, P_{TE2}) &= 0, & V(P_T, P_{TF1}) &= 0, & V(P_T, P_{TF2}) &= 0, \\
V(P_T, P_{TG1}) &= 0, & V(P_T, P_{TG2}) &= 0.00034, & V(P_T, P_{TG3}) &= 0.02184.
\end{aligned}$$

Basic events having vague importance index zero or a very small number indicate that those events play either no role or very negligible role in the top event. These events can therefore be ignored while calculating the crisp reliability using traditional method. In the present example, events A1, E1, E2, F1, F2, G1 and G2 can be ignored in the fault tree given in Figure 3. Repeating the calculations of Section 5.1 after ignoring above events, one gets the crisp reliability estimate of the ‘Power Failure System’ to be 0.9899. Thus our approach of vague importance index could be useful in avoiding the underestimation of the reliability of the system.

6. Conclusion

A new vague fault tree analysis model is proposed in this paper that modifies the vague set arithmetic operations for implementing fault tree analysis. Proposed method leads to two interval estimates of reliability with different truth values. The reliability estimate obtained by traditional approach lies inside the intervals. This work also introduces the concept of Vague Importance Index that helps in discarding unimportant events from the classical fault tree analysis to avoid under/ over estimation of reliability. Results of vague fault tree analysis are more flexible than the fuzzy fault tree analysis because the later method cannot¹⁶ describe the uncertainty of confidence level.

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