Stokes Flow Past a Swarm of Porous Spherical Particles with Stress Jump Condition

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Abstract: This paper concerns the problem of slow viscous flow through a swarm of porous spherical particles. As boundary conditions, continuity of velocity, continuity of normal stress and stress-jump condition at the porous and fluid interface are employed. On the hypothetical cell surface, uniform velocity and Happel boundary conditions are used. The drag force experienced by each porous spherical particle in a cell is evaluated. The earlier results reported for the drag force experienced by a solid sphere in a cell by Happel[1], and Qin and Kaloni[2] for a porous sphere in unbounded medium have been then deduced. Representative results are presented in graphical form by using mathematica software and discussed. The effect of stress jump coefficient β is observed in all the cases.

Key Words and Phrases: Cell model, Brinkman equation, Stream function, Permeability, Drag force.

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1. Introduction

Many process and phenomena in science, engineering and technology involve the motion of flow fluids past and through porous media. Flows through porous media occur commonly on geophysical and biochemical environment and also have engineering applications, like sedimentation, fluidization, petroleum industry, lubricant problems, etc.

Happel[1] proposed a cell model in which both particle and outer envelope are spherical. The cell model technique involves the concept that random assemblage of particles can be divided into a number of identical cells, one particle enveloped by each cell. Furthermore, the volume of fluid cell is so chosen that the solid volume fraction in the cell equals the solid volume fraction of the assemblage. Thus, the entire disturbances due to each particle are confined within the cell of fluid with which it is associated. Happel assumes that the inner surface is stationary and that fluid passes through a cell enveloping it. The following boundary conditions are imposed: (i) no-slip and impenetrability at the inner surface. (ii) zero tangential stress and uniform velocity conditions at the outer envelope.

When a Newtonian fluid flows between impermeable surfaces, the usual boundary condition is the no-slip condition on the boundary but when it flows over a permeable surface the no slip condition is no longer true. On that surface there will be a migration of fluids tangential to the boundary within the permeable surfaces i.e. there will be a net tangential drag due to the transfer of forward momentum across the permeable interface. Recently, Ochoa-Tapia and Whitaker[3,4] studied the momentum transfer at the boundary between a porous medium and a homogeneous fluid theoretically and experimentally. They suggested a boundary condition at the interface of a fluid and porous surface which is commonly known as stress-jump condition which can be expressed in the form

$$T_{nt}^{(2)} - T_{nt}^{(1)} = \frac{\beta\mu^{(1)}}{\sqrt{k}} v_t^{(1)}, \tag{1}$$

where $v_t^{(1)}$ being the tangential velocity in porous region, $T_{nt}^{(1)}$ and $T_{nt}^{(2)}$ are shear stresses in clear fluid and porous regions, respectively and k being the permeability of the porous region and β being the jump coefficient.

A Cartesian-tensor solution of the Brinkman equation is investigated by Qin and Kaloni[2] and they also evaluated the drag force on the porous sphere in unbounded medium. The problem of Stokes flow with slip and Kuwabara boundary conditions was studied by Datta and Deo[5]. The problem of flow past a porous sphere at small Reynolds number was discussed by Srivastava and Srivastava[6] and they also studies the effects of jump coefficient on the drag force. Bhattacharyya and Raja-Sekhar[7] have considered the problem of Stokes flow inside a porous spherical shell and solve it by applying the stress-jump condition at the porous interface. The stress jump coefficient at a fluid-porous dividing surface was also evaluated by Valdes-Parada *et al.*[8]. The motivations of these papers lead us to discuss the slow viscous flow through a swarm of porous spherical particles.

In this paper the solution of the problem of slow viscous flow through a swarm of porous spherical particles is investigated. As boundary conditions, continuity of velocity, continuity of normal stress and stress-jump condition at the porous and fluid interface are employed. On the hypothetical cell surface, uniform velocity and Happel boundary conditions are used. The drag force experienced by each porous spherical particle in a cell is evaluated. The earlier results reported for the drag force experienced by a solid sphere in a cell by Happel[1], and Qin and Kaloni[2] for a porous sphere in unbounded medium have been then deduced. Representative results are presented in graphical form by using mathematica software and discussed. The effect of stress jump coefficient β is observed in all the cases.

2. Mathematical formulations of the problem

Let us consider the creeping flow of a slow viscous fluid flow through a swarm of porous spherical particles. The governing equations written for two regions are as follows:

For the region (1), outside the porous sphere we assume the Stokes equation [Happel and Brenner[9]] as

(2) $$\mu \nabla^2 \mathbf{v}^{(1)} = \nabla p^{(1)}, \quad div\, \mathbf{v}^{(1)} = 0 .$$

For the region (2), inside the porous sphere we apply the Brinkman's equation [Zlatanovski[10]] as

(3) $$\mu_e \nabla^2 \mathbf{v}^{(2)} - \frac{\mu}{k} \mathbf{v}^{(2)} = grad\; p^{(2)}, \qquad div\, \mathbf{v}^{(2)} = 0 .$$

Here, μ_e is the effective viscosity for the Brinkman flow which is taken to be different from μ the coefficient of viscosity of clear fluid and k being the permeability of the porous medium. Also, $\mathbf{v}^{(i)}$ and $p^{(i)}, i = 1,2$ are the velocity vector and pressure at any point in the clear fluid and porous medium, respectively.

Using the following non-dimensional variables

(4) $$\psi^{(i)} = Ua^2 \tilde{\psi}^{(i)}, \quad p^{(i)} = \frac{\mu U}{a} \tilde{p}^{(i)}, \quad \mathbf{v}^{(i)} = U\, \tilde{\mathbf{v}}^{(i)}, \quad r^* = ar, \quad i = 1,2,$$

in the equations (2) and (3) and eliminating the pressures and dropping the tildes, we obtain the resulting equations in spherical polar coordinates (r, θ, ϕ) as

(5) $$\mathrm{E}^2\, (\mathrm{E}^2\; \psi^{(1)}) = 0,$$

(6) $$\mathrm{E}^2\, (\mathrm{E}^2\; - \alpha^2)\; \psi^{(2)} = 0,$$

where the dimensionless operator

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-\zeta^2)}{r^2}\frac{\partial^2}{\partial \zeta^2},$$

(7) $\alpha^2 = \sigma^2 / \gamma^2$, $\gamma^2 = \mu_e / \mu$, $\sigma^2 = a^2 / k$ and $\zeta = \cos\theta$.

Furthermore, the non-vanishing velocity components $(v_r^{(i)}, v_\theta^{(i)}, 0)$, shear and normal stresses respectively are given by

(8) $$v_r^{(i)} = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi^{(i)}}{\partial \theta}\ ; \qquad v_\theta^{(i)} = \frac{1}{r\sin\theta}\frac{\partial \psi^{(i)}}{\partial r}\ ;$$

(9) $$T_{r\zeta}^{(i)}(r,\theta) = \frac{\mu}{r\sin\theta}\left[\frac{\partial^2 \psi^{(i)}}{\partial r^2} - \frac{2}{r}\frac{\partial\ \psi^{(i)}}{\partial r} - \frac{(1-\zeta^2)}{r^2}\frac{\partial^2 \psi^{(i)}}{\partial \zeta^2}\right],$$

(10) $$T_{rr}^{(i)} = -p^{(i)} - \frac{2\mu}{r^2}\left[\frac{2}{r}\frac{\partial\ \psi^{(i)}}{\partial \zeta} - \frac{\partial^2 \psi^{(i)}}{\partial r \partial \zeta}\right], \qquad i = 1, 2 .$$

Also, the pressures may be obtained in both regions by integrating the following relations respectively:

(11) $$\frac{\partial p^{(1)}}{\partial r} = -\frac{\mu}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(E^2 \psi^{(1)}); \qquad \frac{\partial p^{(1)}}{\partial \theta} = \frac{\mu}{\sin\theta}\frac{\partial}{\partial r}(E^2 \psi^{(1)});$$

(12) $$\frac{\partial p^{(2)}}{\partial r} = -\frac{\mu_e}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\{(E^2 - (\tfrac{\sigma}{a})^2)\psi^{(2)}\},$$
$$\frac{\partial p^{(2)}}{\partial \theta} = \frac{\mu_e}{\sin\theta}\frac{\partial}{\partial r}\{(E^2 - (\tfrac{\sigma}{a})^2)\psi^{(2)}\}.$$

3. Statement and Solution of the problem

In the mathematical model, we assume that all the porous spherical particles are randomly and homogeneously distributed in the viscous fluid. The porous medium is assumed to be homogeneous and isotropic. Let us now consider that porous spherical particles are stationary and steady axisymmetric flow has been established around it by uniform velocity U directed in the positive z-axis (see figure-1).

In the case of axisymmetric incompressible creeping flow the regular solution of the stokes equation outside the porous sphere can be expressed as

(13) $$\psi^{(1)}(r,\zeta)=[A_1\frac{1}{r}+B_1r^2+C_1r+D_1r^4]G_2(\zeta),$$

where $G_2(\zeta)=\frac{1}{2}(1-\zeta^2)$.

The regular solution of the Brinkman equation inside the porous sphere in which origin lies may be taken as

(14) $$\psi^{(2)}(r,\zeta)=[A_2r^2+D_2y_2(\alpha r)]G_2(\zeta),$$

where $y_2(\alpha r)=\alpha\cosh(\alpha r)-\dfrac{\sinh(\alpha r)}{r}$.

4. Matching conditions

At the interface of the porous surface and fluid $r^*=a$, we assume that the velocity components and normal stress are continuous and jump condition in the shearing stress is employed. Therefore, the following matching condition at the interface is used.

On the interface of porous sphere ($r^*=a$):

The continuity of velocity components across the porous sphere and normal stress implies that we may take

(15) $$v_r^{(1)}=v_r^{(2)}; \qquad v_\theta^{(1)}=v_\theta^{(2)}$$

(16) $$T_{rr}^{(1)}(r,\zeta)=T_{rr}^{(2)}(r,\zeta) \quad \text{on} \quad r=1.$$

Applying the stress jump condition on the porous and fluid interface as suggested by Ochoa-Tapia and Whittaker[3,4]

(17) $$T_{r\theta}^{(2)}-T_{r\theta}^{(1)}=\frac{\beta\mu}{\sqrt{k}}v_\theta^{(1)} \quad \text{on} \quad r=1,$$

where β being the jump coefficient at the interface and the sign of β may be either positive or negative. In particular, if $\beta=0$ then we get the continuity of shearing stress.

On the cell surface ($r^*=b$):

The continuity of normal component of velocity provides

(18) $$v_r^{(2)}=U\cos\theta \quad \text{on} \quad r=\ell=b/a.$$

The vanishing of shear stress i.e., Happel condition implies that we take

(19) $$\frac{\partial^2\psi^{(2)}}{\partial r^2}-\frac{2}{r}\frac{\partial\psi^{(2)}}{\partial r}-\frac{(1-\zeta^2)}{r^2}\frac{\partial^2\psi^{(2)}}{\partial\zeta^2}=0 \quad \text{on} \quad r=\ell .$$

5. Determination of arbitrary constants

Applying these above boundary conditions given by equations (15)-(19), we obtain

(20) $$A_2+D_2y_2(\alpha)=A_1+B_1+C_1+D_1,$$

(21) $$2A_2+\{\alpha^2y_1(\alpha)-y_2(\alpha)\}D_2=-A_1+2B_1+C_1+4D_1,$$

(22) $$\gamma^2[\alpha^2A_2+2\{\alpha^2y_1(\alpha)-3y_2(\alpha)\}D_2]=-\left[6A_1+3C_1+6D_1\right],$$

(23) $$\gamma^2\{(\alpha^2+6)y_2(\alpha)-2\alpha^2y_1(\alpha)\}D_2-6(A_1+D_1)$$
$$=\beta\sigma\{-A_1+2B_1+C_1+4D_1\},$$

(24) $$A_1+B_1\ell^3+C_1\ell^2+D_1\ell^5=-\ell^3,$$

(25) $$A_1+D_1\ell^5=0.$$

Solving these above equations (20)-(25), we find the following values of the unknown constants

(26) $$A_1=\frac{1}{\Delta}\Big[\ell^6\sigma[-y_1(\alpha)\{2\sigma^2(-2\beta+\sigma)+\alpha^2\beta(6+\sigma^2)\}$$
$$+y_2(\alpha)\{6\gamma^2\sigma+\sigma^3+\beta(18-12\gamma^2+\sigma^2)\}]\Big],$$

(27) $$B_1=\frac{1}{\Delta}\Big[\ell[y_2(\alpha)\{3\{18+8\sigma^2+\sigma^4+\beta\sigma(6+\sigma^2)+6\gamma^2(2-2\beta\sigma+\sigma^2)\}$$
$$+\ell^5\{-54-9\sigma^2+2\sigma^4-6\gamma^2(-9+4\beta\sigma-2\sigma^2)+\beta\sigma(27+2\sigma^2)\}\}$$
$$-y_1(\alpha)[2\sigma^2\{6-6\beta\sigma+3\sigma^2+\ell^5(9-4\beta\sigma+2\sigma^2)\}$$
$$+\alpha^2\{3\{\beta\sigma(2+\sigma^2)+2(3+\sigma^2)\}+\ell^5\{-6(3+\sigma^2)+\beta\sigma(9+2\sigma^2)\}\}]]\Big],$$

$$
\begin{aligned}
C_1 = \frac{1}{\Delta}\Big[& -\ell\sigma[y_2(\alpha)\{\sigma\{3\ell^5(-6+6\gamma^2+\sigma^2)+2(9+6\gamma^2+\sigma^2)\} \\
& +\beta\{-36-24\gamma^2+2\sigma^2+\ell^5(36-36\gamma^2+3\sigma^2)\}\} \\
& -y_1(\alpha)[-2(2+3\ell^5)(2\beta-\sigma)\sigma^2+\alpha^2\{-6(-1+\ell^5)\sigma \\
& +\beta\{2(-6+\sigma^2)+3\ell^5(4+\sigma^2)\}\}]]\Big],
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
D_1 = \frac{1}{\Delta}\Big[& -\ell\sigma[-y_1(\alpha)\{2\sigma^2(-2\beta+\sigma)+\alpha^2\beta(6+\sigma^2)\} \\
& +y_2(\alpha)\{6\gamma^2\sigma+\sigma^3+\beta(18-12\gamma^2+\sigma^2)\}]\Big],
\end{aligned} \tag{29}
$$

$$
\begin{aligned}
A_2 = \frac{1}{\Delta}\Big[& 3\ell[y_2(\alpha)\{\ell^5(-18+18\gamma^2+\beta\sigma+\sigma^2)+2(9+6\gamma^2+2\beta\sigma+2\sigma^2)\} \\
& -y_1(\alpha)\{2(2+3\ell^5)\sigma^2+\alpha^2(6+4\beta\sigma+\ell^5(-6+\beta\sigma))\}]\Big],
\end{aligned} \tag{30}
$$

$$
D_2 = \frac{1}{\Delta}6\ell\sigma[(4+\ell^5)\beta+(-1+\ell^5)\sigma], \tag{31}
$$

where

$$
\begin{aligned}
\Delta = {} & y_2(\alpha)[3\ell^5\sigma\{\beta(12-12\gamma^2+\sigma^2)+\sigma(-6+6\gamma^2+\sigma^2)\} \\
& +2\sigma\{\beta(-18-12\gamma^2+\sigma^2)+\sigma(9+6\gamma^2+\sigma^2)\} \\
& -3\ell\{18+8\sigma^2+\sigma^4+\beta\sigma(6+\sigma^2)+6\gamma^2(2-2\beta\sigma+\sigma^2)\} \\
& +\ell^6\{54+9\sigma^2-2\sigma^4+6\gamma^2(-9+4\beta\sigma-2\sigma^2)-\beta\sigma(27+2\sigma^2)\}] \\
& +y_1(\alpha)[3\ell\{2\sigma^2(2-2\beta\sigma+\sigma^2)+\alpha^2(6+2\beta\sigma+2\sigma^2+\beta\sigma^3)\} \\
& -2\sigma\{2\sigma^2(-2\beta+\sigma)+\alpha^2\{3\sigma+\beta(-6+\sigma^2)\}\} \\
& -3\ell^5\sigma\{2\sigma^2(-2\beta+\sigma)+\alpha^2\{-2\sigma+\beta(4+\sigma^2)\}\} \\
& +\ell^6\{2\sigma^2(9-4\beta\sigma+2\sigma^2)+\alpha^2\{-6(3+\sigma^2)+\beta\sigma(9+2\sigma^2)\}\}],
\end{aligned} \tag{32}
$$

$y_1(\alpha) = \sinh\alpha \quad and \quad y_2(\alpha) = \alpha\cosh\alpha - \sinh\alpha.$

6. Evaluation of the drag force

The drag force experienced by a porous sphere in a cell can be evaluated by using the simple elegant formula [Happel and Brenner[9]] as

(33) $$F = \pi\,\mu U\,a\int_0^{\pi} \varpi^3\,\frac{\partial}{\partial r}\left(\frac{E^2\psi^{(1)}}{\varpi^2}\right) r\,d\theta.$$

Here since $\varpi = r\sin\theta$ and $E^2\psi^{(1)} = [-\frac{2}{r}C_1 + 10\,r^2 D_1]\,G_2(\zeta)$, inserting these above values in (33) and integrating, we find that

(34)
$$\begin{aligned} F &= 4\pi\,\mu\,a\,UC_1 \\ &= -4\pi\,\mu\,a\,U\,\ell\sigma[y_2(\alpha)\{\sigma\{3\ell^5(-6+6\gamma^2+\sigma^2)+2(9+6\gamma^2+\sigma^2)\} \\ &\quad +\beta\{-36-24\gamma^2+2\sigma^2+\ell^5(36-36\gamma^2+3\sigma^2)\} \\ &\quad -y_1(\alpha)[-2(2+3\ell^5)(2\beta-\sigma)\sigma^2+\alpha^2\{-6(-1+\ell^5)\sigma+\beta\{2(-6+\sigma^2)+3\ell^5(4+\sigma^2)\}\}]/\Delta, \end{aligned}$$

where Δ is given by equation (32). Also, the drag coefficient C_D can be defined as

(35) $$C_D = \frac{F}{(1/2)\,\rho U^2\pi\,a^2} = \frac{16\,C_1}{\mathrm{Re}},$$

where $\mathrm{Re} = \frac{2\,aU}{\nu}$ and $\nu = \frac{\mu}{\rho}$ being the Reynolds number and kinematic viscosity of fluid, respectively.

The following special cases can be deduced:

I. Porous sphere with jump condition in unbounded medium $(\ell \to \infty)$ **:**

In this case, the value of drag force F experienced by a porous sphere comes out as

(36) $$\begin{aligned} F = 12\,\pi\,\mu\,a U\alpha\,\gamma[&\alpha^2\{2\alpha\gamma(-1+\gamma^2)+\beta(4+(-4+\alpha^2)\gamma^2)\}y_1(\alpha) \\ &-\{\beta(12+(-12+\alpha^2)\gamma^2)+\alpha\gamma(-6+(6+\alpha^2)\gamma^2)\}y_2(\alpha)]/\Delta_1 \end{aligned}$$

where

$$(37)\qquad \begin{aligned}\Delta_1 &= \alpha^2[2\alpha^3\beta\gamma^3+\alpha\beta\gamma(9-8\gamma^2)+18(-1+\gamma^2)\\ &+\alpha^2(-6\gamma^2+4\gamma^4)]y_1(\alpha)-[2\alpha^3\beta\gamma^3+2\alpha^4\gamma^4\\ &+54(-1+\gamma^2)+3\alpha^2\gamma^2(-3+4\gamma^2)-3\alpha\beta\gamma(-9+8\gamma^2)]y_2(\alpha)\end{aligned}$$

II. Porous sphere in a cell without jump $(\beta=0)$ **:**

In this case, the value of drag force F experienced by a porous sphere for the case of $\gamma=1$ comes out as

$$(38)\qquad F=\frac{4\pi\,\mu\,aU\alpha^2\ell[-10\alpha^2 y_1(\alpha)+\{30+\alpha^2(2+3\ell^5)\}y_2(\alpha)]}{\Delta_2},$$

where

$$(39)\qquad \begin{aligned}\Delta_2 &= [2\alpha^2\{(5-6\ell+\ell^6)\alpha^2-15\ell\}y_1(\alpha)+\{\alpha^4(2\ell^6-3\ell^5+3\ell-2)\\ &+3(\alpha^2(\ell^6+14\ell-10)+30\ell)\}y_2(\alpha)].\end{aligned}$$

III. Porous sphere in an unbounded medium without jump $(\beta=0)$ **:**

In this case, the value of drag force F experienced by a porous sphere of radius a turns out as

$$(40)\qquad F=6\pi\,\mu\,aU[1+\lambda_1+\frac{3}{2\alpha^2}]^{-1},\qquad \lambda_1=y_1(\alpha)/y_2(\alpha).$$

A well- known result was reported earlier by Qin and Kaloni[2] for the drag force experienced by a porous sphere in an unbounded fluid.

IV. A solid sphere in a cell:

In this case, the value of drag force from the equation (38) by taking limit as $\alpha\to\infty$, comes

$$(41)\qquad F=6\pi\,\mu U\,a[1+\tfrac{2}{3}\lambda^{5/3}][1-\tfrac{3}{2}\lambda^{1/3}+\tfrac{3}{2}\lambda^{5/3}-\lambda^2]^{-1},$$

where $\lambda=(a/b)^3=\ell^{-3}$ being the solid volume fraction. This result for the drag force agrees with the earlier result reported by Happel[1].

V. A solid sphere in unbounded medium:

In this case, the value of drag force F experienced by the solid sphere comes out as

$$F = 6\pi \mu a U \,. \tag{42}$$

A well-known result for the drag reported earlier by Stokes for flow past a solid sphere in an unbounded medium.

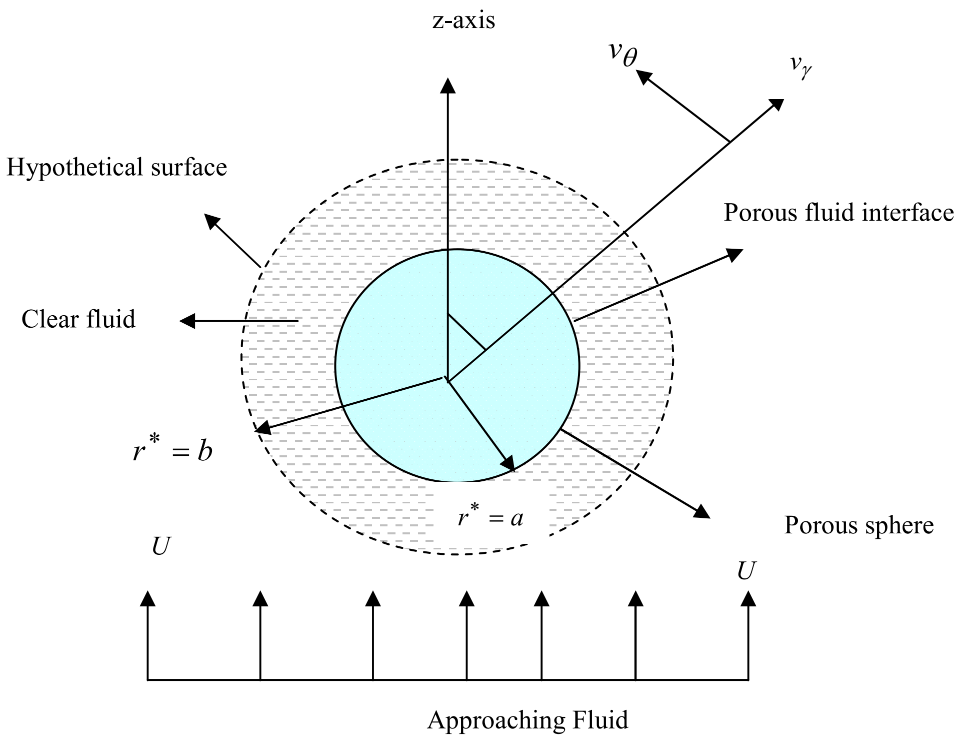


Fig.-1: The physical situation and the description of coordinate system for the model

7. Conclusions

The variation of the drag coefficient C_D against permeability parameter σ for the porous sphere for various values of stress jump coefficient $\beta = -0.5,\ 0,\ 0.5$ has been shown in figures-2, 3 and 4 for different cell sizes ℓ and viscosity ratios γ. These figures show that the drag coefficient C_D increases with increase of σ, i.e. with decrease of permeability but it decreases with increase of jump coefficient β. Here, it may be mentioned that the values of jump coefficient β varies in the range –1 to 1 as experimentally found [Ochoa-Tapia and Whitaker[3,4]].

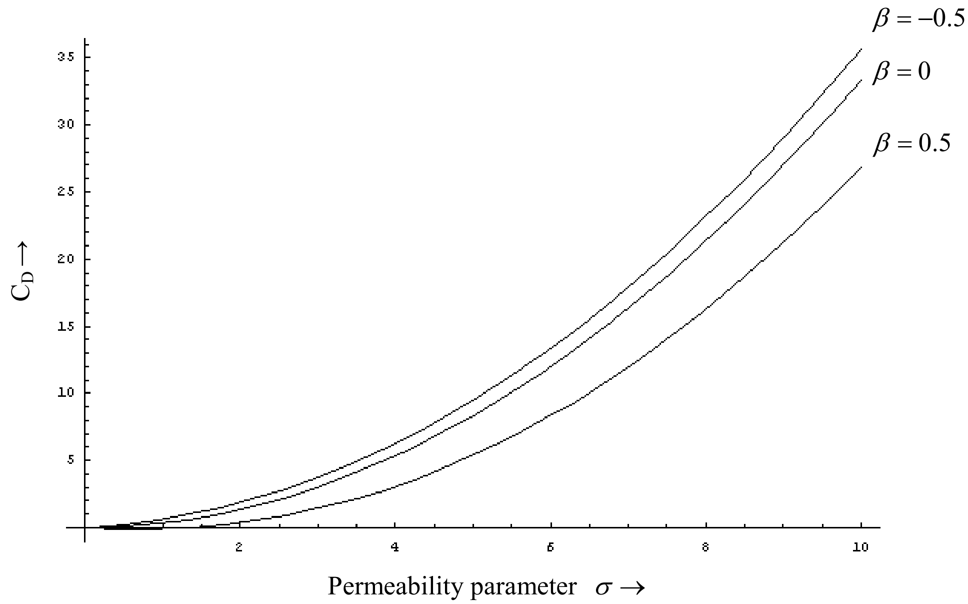


Fig. -2: Plot of drag coefficient C_D of a porous sphere in a cell versus permeability parameter σ for various values of β when cell size $\ell = 1$ and $\gamma = 1$.

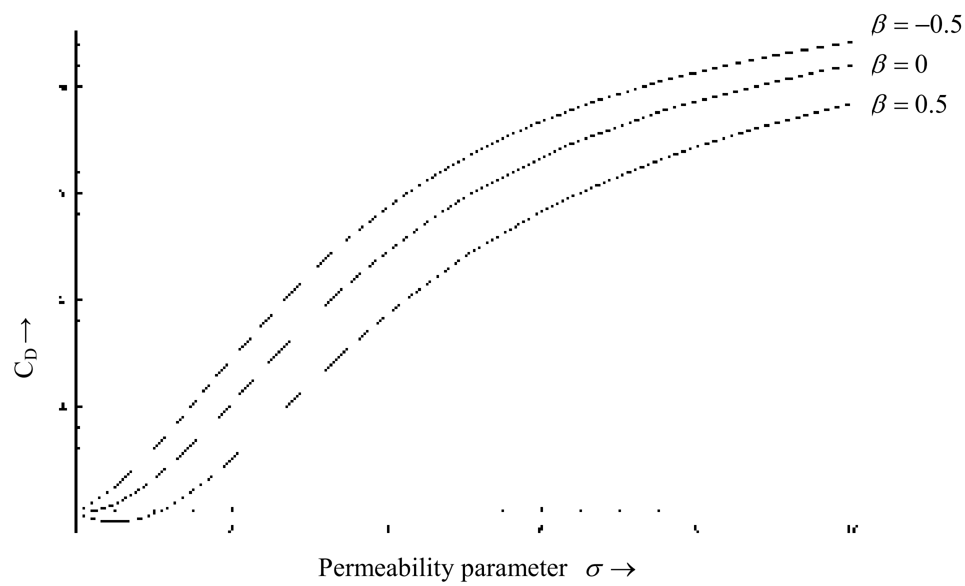
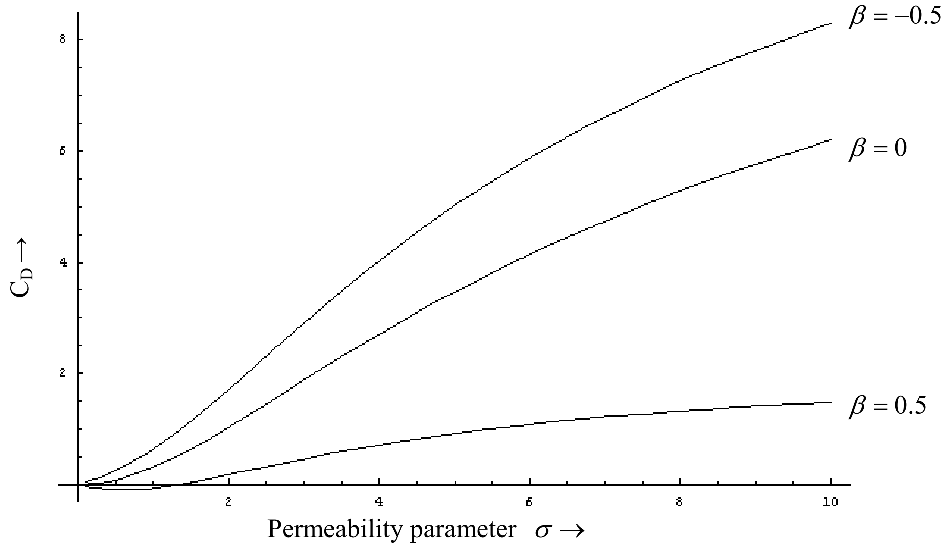


Figure -3: Plot of drag coefficient C_D of a porous sphere in a cell versus permeability parameter σ for various values of β when cell size $\ell = 2$ and $\gamma = 2$.



$\beta = 0$

$\beta = 0.5$

Figure-4: Plot of drag coefficient C_D of a porous sphere in a cell versus permeability parameter σ for various values of β when cell size $\ell = 1.5$ and $\gamma = 5$.

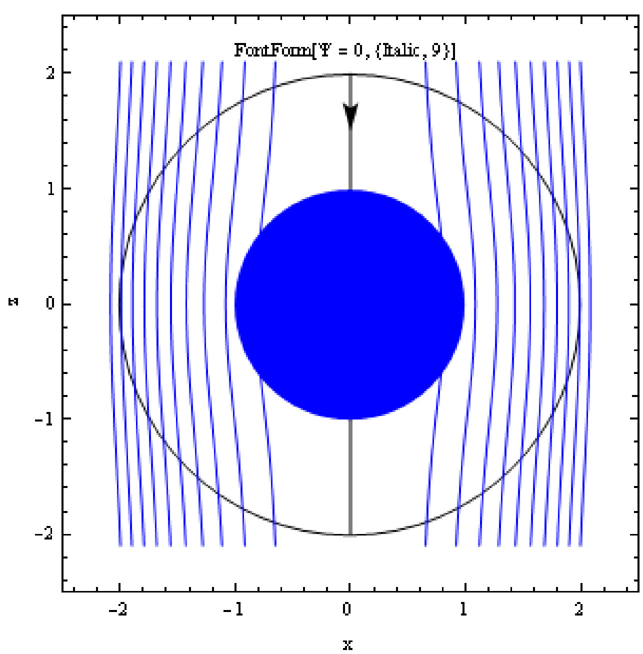


Figure-5: Stream lines of flow pattern for a porous sphere in a cell when $\sigma = 2$, $\gamma = .125$ and $a = 1$.

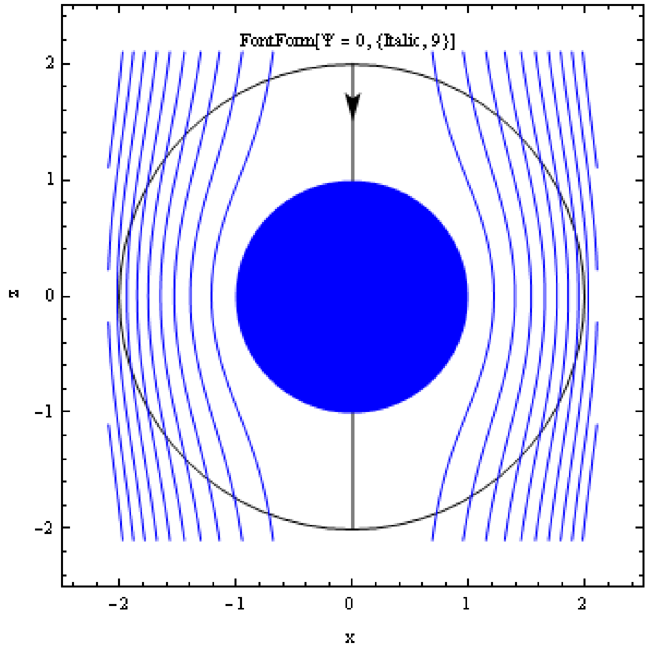


Figure-6: Stream lines of flow pattern for a porous sphere in a cell when $\sigma = 8, \gamma = .125$ and $a = 1$.

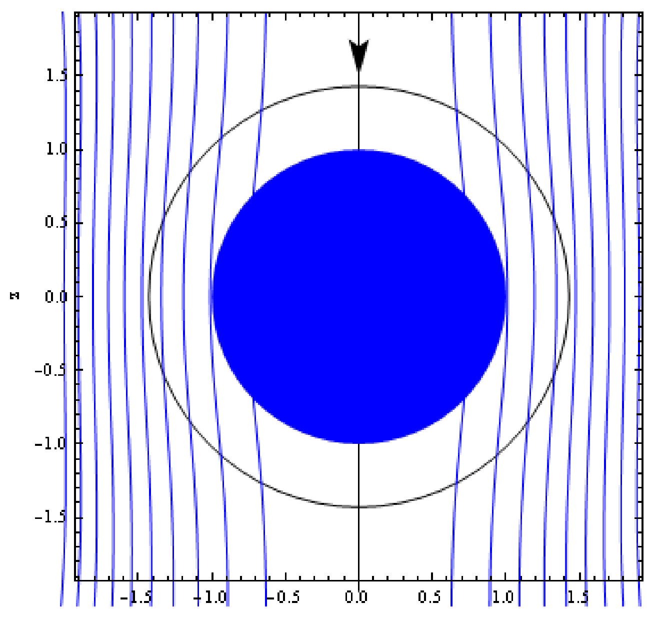


Figure-7: Stream lines of flow pattern for a porous sphere in a cell when $\sigma = 2, \gamma = .343$ and $a = 1$

The case when β=0, corresponds to the continuity of the stresses. The effect of stress jump coefficient β is observed in all the cases. The stream lines of flow patterns for a porous sphere in a cell without jump are also plotted for various parameters as shown in Figures 4, 5 and 6. Here, the numerical results and figures for the given input parameter values, have been evaluated by using Mathematica 5.2 software.

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