ISSN 0974 - 9373

Vol. 23 No. 1 (2019) **Journal of International Academy of Physical Sciences** pp. 25-40

Fuzzified Analysis of Deteriorating Inventory Model with Decreasing Demand and Changing Holding Cost

S. S. Mishra, S. Rawat and Abhishek Singh

Department of Mathematics and Statistics (Centre of Excellence)
Dr. Rammanohar Lohia Avadh University, Ayodhya-224001, UP (India)
Email: sant_x2003@yahoo.co.in

S. K. Yadav

Department of Statistics
Babasaheb Bhimrao Ambedkar University, Lucknow-226025, UP (India)

(Received January 19, 2019)

Abstract: This paper intends to analyze an inventory model for perishable items with decreasing demand and changing holding cost in fuzzy environment by using fuzzy paradigm in order to obtain more realistic performance measure in the form of optimal total cost of the system. A non-linear equation has been developed and solved by MATHEMATICA to provide optimal total cost of the model to compare the same result obtained in crisp environment. A sensitivity analysis has also been carried out to demonstrate the use of the model under consideration.

Keywords: Cost Optimization, Inventory Control, Signed Distance Method, Fuzzy Paradigm.

1. Introduction

Inventory models with deteriorating items with their inherent characteristics have been reported by eminent researchers, for example, Raafat[1], Wee[2], Goyal and Giri[3] and Li et al[4]. For the first time in literature the inventory model was given by Harris[5], later it was generalized by Wilson[6]. Whitin[7] and Ghare and Schrader[8] presented their observations connected with deteriorating items with various distribution of time. Dave and Patel[9] were the first who developed the inventory model on perishable items with linearly magnify demand with no scarcity. Various authors have contributed significantly in this direction and some may be referred to as Chung and Ting[10], Wee[11], Mishra et al.[12], Mishra and Mishra[13] and Mishra[14].

Fuzzy inventory models with various characterized properties were developed and discussed by eminent researchers engaged in the fields whose contributions are cited as Abad[15-16], Liu and Liu[17], Lengari[18], Mishra and Mishra[19-20], Mishra and Singh[21-22], Pathak and Sarkar[23-24], Prasath and Seshaiah[25], Singh and Pattnayak[26], Mishra et al.[27], Kozarevic and Puska[28].

Zimmerman[29] classified the various types of uncertainties in two forms- stochastic uncertainties and fuzziness. Whenever, the process of a series of tasks including mathematical modeling, optimization, computing; expert and control systems is developed and carried out under fuzziness or fuzzy environment, it is defined as fuzzy paradigm (FP). Fuzzy paradigm in some part or as a whole may be used to analyze the model for its optimum performance measures in fuzzy environment. Of late, Dash et al.[33] has suggested an inventory model for perishable items with digressive demand as exponential function and time-varying holding cost in crisp environment.

In the present paper, the model suggested by Dash et al.[30] is further subjected to investigate by using FP in order to realize more realistic results with increased computational efficiency of the model. The performance measures particularly optimal total cost of the model have been computed in both of environments of crisp and fuzziness, and finally subjected to the comparison. A system of non-linear equation has been developed and its computational solution has been obtained using MATHEMETICA. A sensitivity analysis has been also carried out to test the model under consistent variation of various parameters involved in the model. The manuscript has been divided in various sections such as introduction, assumptions and notations; mathematical formulation of the model; model development and its solution in fuzzy environment; numerical computing and sensitivity analysis; and conclusion.

2. Assumptions Notations

The following assumptions and notations have been used in this manuscript for the development of the model.

Assumptions:

(i) The inventory system is based on the single homogeneous item.

(ii) Demand rate is exponential with respect to time and deterministic.

(iii) Item perishes at constant rate.

(iv) Time horizon is finite.

(v) Lead time is considered as zero.

(vi) There is no shortage in the system.

Notations:

(i) A_0 is per order ordering cost.

(ii) $I(t)$ is level of inventory at time t, $0 \le t \le T$.

(iii) $D(t)$ is demand rate following exponential distribution where $D(t) = Ke^{-\gamma t}$, $K > 0$, $K >> \gamma$ are constants, K is constant demand rate and γ the demand rate change.

(iv) $\theta(t)$ is perishable rate of an item as constant where $\theta(t) = \theta\,(0 < \theta < 1)$ and $0 < \gamma < \theta$.

(v) $h(t)$ is holding cost per unit time which is time dependent where $h(t) = h + \alpha t$, $h > 0$ and $\alpha > 0$.

(vi) C_d is cost of per perishable unit.

(viii) T is length of the ordering cycle.

(ix) I_0 is economic order quantity.

(x) TAC is total average cost per unit time.

3. Mathematical Formulation of the Model

According to Dash et.al[30], since the inventory is reduced during the time interval $[0,T]$ due to demand rate and the deterioration rate, therefore the inventory model for deteriorating items with declining demand rate is represented by the following differential equation as,

(3.1) $$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t),\ 0 \le t \le T,$$

where $\theta(t) = \theta$ and $D(t) = Ke^{-\gamma t}$.
The solution of the above differential equation (3.1) with the boundary condition $I(t) = I_0$ is given by

(3.2) $$I(t) = \frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T-\theta t} - e^{-\gamma t}\right],\ 0 \le t \le T.$$

The preparatory order quantity is obtained with the boundary condition $I(0) = I_0$ for $t = 0$ in equation (3.2) as

(3.3) $$I_0 = I(0) = \frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T} - 1\right].$$

During the period of cycle $[0,T]$, the total demand is given by

$$\int_0^T D(t)dt = \int_0^T Ke^{-\gamma t}dt = \frac{K}{\gamma}[1-e^{-\gamma T}].$$

The number of deteriorated units of the inventory item is

$$I_0 - \int_0^T D(t)dt = \frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T}\right] + \frac{K}{\gamma}[e^{-\gamma T} - 1]$$

Thus the deteriorating cost (DC) for the period of cycle $[0,T]$ is given by

(3.4) $$DC = C_d \times \text{ number of deteriorating units}$$

$$= C_d\left[\frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T} - 1\right] + \frac{K}{\gamma}[e^{-\gamma T} - 1]\right]$$

$$= C_d Ke^{-\gamma T}\left[\frac{1}{(\theta-\gamma)}\left[e^{\theta T} - e^{\gamma T}\right] + \frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

The total inventory holding cost (IHC) for the period of cycle $[0,T]$ is given by

(3.5) $$IHC = \int_0^T (h+\alpha t)I(t)dt$$

$$= \int_0^T (h+\alpha t)\frac{K}{(\theta-\gamma)}\left[e^{(\theta-\gamma)T-\theta t} - e^{-\gamma t}\right]dt$$

$$= \frac{Ke^{-\gamma T}}{\theta\gamma(\theta-\gamma)}\left[(h+\alpha T)(\theta-\gamma) - h(\theta e^{\gamma T} - \gamma e^{\theta T})\right]$$

$$+ \frac{\alpha}{\theta\gamma}\left[(\theta^2-\gamma^2) - (\theta^2 e^{\gamma T} - \gamma^2 e^{\theta T})\right].$$

Thus the total variable cost (TVC) is given by

$$TVC = OC + DC + IHC$$

Thus, (TVC/T) is given by,

(3.6) $$TVC/T = \frac{A_0}{T} + \frac{C_d K e^{-\gamma T}}{T}\left[\frac{1}{(\theta-\gamma)}\left[e^{\theta T} - e^{\gamma T}\right] + \frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta\gamma(\theta-\gamma)T}\left[\left\{(h+\alpha T)(\theta-\gamma) - h(\theta e^{\gamma T} - \gamma e^{\theta T})\right\}\right.$$

$$\left.+\frac{\alpha}{\theta\gamma}\left\{(\theta^2-\gamma^2)-(\theta^2 e^{\gamma T} - \gamma^2 e^{\theta T})\right\}\right] \ .$$

Here, we are basically interested in finding the minimum variable cost per unit of time. Therefore, the necessary and sufficient conditions for the minimization of (TVC/T) for the given value of T are respectively given by

$$\frac{\partial(TVC/T)}{\partial T} = 0 \text{ and } \frac{\partial^2(TVC/T)}{\partial T^2} > 0$$

Now the solution of the equation $\frac{\partial(TVC/T)}{\partial T} = 0$ is given by

(3.7) $$\frac{Ke^{-\gamma T}}{\theta^2\gamma T(\theta-\gamma)}\left[\alpha\theta(\theta-\gamma) - h\theta^2\gamma(e^{\gamma T} - e^{\theta T})\alpha\theta(\gamma e^{\gamma T} - \theta e^{\theta T})\right.$$

$$\left.-\gamma\theta(\theta-\gamma)(h+\alpha T) + h\theta\gamma(\theta e^{\gamma T} - \gamma e^{\theta T}) - \alpha(\theta^2-\gamma^2-\theta^2 e^{\gamma T} + \gamma^2 e^{\theta T})\right]$$

$$+\frac{C_d K e^{-\gamma T}}{(\theta-\gamma)T}\left[\theta e^{\theta T} - \gamma e^{\gamma T} - \gamma(e^{\theta T} - e^{-\gamma T}) - \theta + \gamma\right] - \frac{A_0}{T} = 0\,.$$

For (TVC/T) to be minimum, we should have $\frac{\partial^2(TVC/T)}{\partial T^2} > 0$ and we have

$$\frac{\partial^2(TVC/T)}{\partial T^2} =$$

$$\frac{Ke^{-\gamma T}}{T^2}\Big[\theta(\theta-\gamma)(h\gamma^2T+\alpha T^2\gamma^2-2\alpha\gamma T)\theta^2\gamma(e^{\gamma T}-e^{\theta T})(2\gamma hT-\alpha T+2h)$$

$$+\theta(\gamma e^{\theta T}-\theta e^{\gamma T})(\gamma T-2\alpha+2h\gamma)+\alpha(\gamma T+2)(\theta^2-\gamma^2-\theta^2e^{\gamma T}+\gamma^2e^{\theta T})$$

$$-hT\theta^2\gamma(\gamma e^{\gamma T}-\theta e^{\theta T})+2\theta\gamma(\theta-\gamma)(h+\alpha T)\Big]+\frac{2A_0}{T^2}$$

$$+\frac{C_dKe^{-\gamma T}}{(\theta-\gamma)T^2}\Big[(\theta-\gamma)[(\theta-\gamma)Te^{\theta T}+\gamma T+2]$$

$$-2\gamma(e^{\gamma T}-e^{\theta T})-2\gamma^2Te^{-\gamma T}-2\theta e^{\theta T}+2\gamma e^{\gamma T}\Big].$$

As we see from the numerical study that $\frac{\partial^2(TVC/T)}{\partial T^2}>0$ for the optimal value of T obtained from equation (3.7), therefore the equation (3.6) has unique optimal solution.

4. Model Development and its Solution in Fuzzy Environment

Now, we fuzzify the parameters involved as under:
Let $\tilde{h}=(h_1,h_2,h_3,h_4)$, $\tilde{C}_d=(C_{d1},C_{d2},C_{d3},C_{d4})$, $\tilde{\theta}=(\theta_1,\theta_2,\theta_3,\theta_4)$ be the trapezoidal fuzzy numbers. Then, we have fuzzified TVC as,

$$T\tilde{V}C/T=\frac{A_0}{T}+\frac{C_dKe^{-\gamma T}}{T}\left[\frac{1}{(\theta-\gamma)}\Big[e^{\theta T}-e^{\gamma T}\Big]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\tilde{\theta}\gamma(\tilde{\theta}-\gamma)}\Big[(\tilde{h}+\alpha T)(\tilde{\theta}-\gamma)-\tilde{h}(\tilde{\theta}e^{\gamma T}-\gamma e^{\tilde{\theta}T})\Big]$$

$$+\frac{\alpha}{\tilde{\theta}\gamma}\Big[(\tilde{\theta}^2-\gamma^2)-(\tilde{\theta}^2e^{\gamma T}-\gamma^2e^{\tilde{\theta}T})\Big]$$

$$T\tilde{V}C/T=\frac{A_0}{T}+\frac{(C_{d1},C_{d2},C_{d3},C_{d4})Ke^{-\gamma T}}{T}$$

$$\times\left[\frac{1}{\{(\theta_1,\theta_2,\theta_3,\theta_4)-\gamma\}}\Big[e^{(\theta_1,\theta_2,\theta_3,\theta_4)T}-e^{\gamma T}\Big]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{(\theta_1,\theta_2,\theta_3,\theta_4)\gamma\{(\theta_1,\theta_2,\theta_3,\theta_4)-\gamma\}}$$

$$\times\begin{bmatrix}\{(h_1,h_2,h_3,h_4)+\alpha T\}\{(\theta_1,\theta_2,\theta_3,\theta_4)-\gamma\} \\ -(h_1,h_2,h_3,h_4)\{(\theta_1,\theta_2,\theta_3,\theta_4)e^{\gamma T}-\gamma e^{(\theta_1,\theta_2,\theta_3,\theta_4)T}\}\end{bmatrix}$$

$$+\frac{\alpha}{(\theta_1,\theta_2,\theta_3,\theta_4)\gamma}\Big[\{(\theta_1^2,\theta_2^2,\theta_3^2,\theta_4^2)-\gamma^2\}$$

$$-\{(\theta_1^2,\theta_2^2,\theta_3^2,\theta_4^2)e^{\gamma T}-\gamma^2 e^{(\theta_1,\theta_2,\theta_3,\theta_4)T}\}\Big]$$

$$T\tilde{V}C/T=(W,X,Y,Z)$$

where,

$$W=\frac{A_0}{T}+\frac{C_{d1}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_1-\gamma)}\left[e^{\theta_1 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_1\gamma(\theta_1-\gamma)}\left[(h_1+\alpha T)(\theta_1-\gamma)-h_1(\theta_1 e^{\gamma T}-\gamma e^{\theta_1 T})\right]$$

$$+\frac{\alpha}{\theta_1\gamma}\left[(\theta_1^2-\gamma^2)-(\theta_1^2 e^{\gamma T}-\gamma^2 e^{\theta_1 T})\right]$$

$$X=\frac{A_0}{T}+\frac{C_{d2}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_2-\gamma)}\left[e^{\theta_2 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_2\gamma(\theta_2-\gamma)}\left[(h_2+\alpha T)(\theta_2-\gamma)-h_2(\theta_2 e^{\gamma T}-\gamma e^{\theta 2T})\right]$$

$$+\frac{\alpha}{\theta_2\gamma}\left[(\theta_2^2-\gamma^2)-(\theta_2^2 e^{\gamma T}-\gamma^2 e^{\theta_2 T})\right]$$

$$Y=\frac{A_0}{T}+\frac{C_{d3}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_3-\gamma)}\left[e^{\theta_3 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_3\gamma(\theta_3-\gamma)}\left[(h_3+\alpha T)(\theta_3-\gamma)-h_3(\theta_3 e^{\gamma T}-\gamma e^{\theta_3 T})\right]$$

$$+\frac{\alpha}{\theta_3\gamma}\left[(\theta_3^2-\gamma^2)-(\theta_3^2e^{\gamma T}-\gamma^2e^{\theta_3 T})\right]$$

$$Z=\frac{A_0}{T}+\frac{C_{d4}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_4-\gamma)}\left[e^{\theta_4 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_4\gamma(\theta_4-\gamma)}\left[(h_4+\alpha T)(\theta_4-\gamma)-h_4(\theta_4e^{\gamma T}-\gamma e^{\theta_4 T})\right]$$

$$+\frac{\alpha}{\theta_4\gamma}\left[(\theta_4^2-\gamma^2)-(\theta_4^2e^{\gamma T}-\gamma^2e^{\theta_4 T})\right].$$

The α -Cuts, $C_L(\alpha)$ and $C_R(\alpha)$ of trapezoidal fuzzy number ($T\tilde{V}C/T$) are given by,

$$C_L(\alpha)=W+(X-W)\alpha$$

$$=\frac{A_0}{T}+\frac{C_{d1}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_1-\gamma)}\left[e^{\theta_1 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_1\gamma(\theta_1-\gamma)}\left[(h_1+\alpha T)(\theta_1-\gamma)-h_1(\theta_1e^{\gamma T}-\gamma e^{\theta_1 T})\right]$$

$$+\frac{\alpha}{\theta_1\gamma}\left[(\theta_1^2-\gamma^2)-(\theta_1^2e^{\gamma T}-\gamma^2e^{\theta_1 T})\right]$$

$$+[\frac{C_{d2}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_2-\gamma)}\left[e^{\theta_2 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$-\frac{C_{d1}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_1-\gamma)}\left[e^{\theta_1 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_2\gamma(\theta_2-\gamma)}\left[(h_2+\alpha T)(\theta_2-\gamma)-h_2(\theta_2e^{\gamma T}-\gamma e^{\theta 2T})\right]$$

$$-\frac{\alpha}{\theta_2\gamma}\left[(\theta_2^2-\gamma^2)-(\theta_2^2 e^{\gamma T}-\gamma^2 e^{\theta_2 T})\right]$$

$$\left.-\frac{\alpha}{\theta_1\gamma}\left[(\theta_1^2-\gamma^2)-(\theta_1^2 e^{\gamma T}-\gamma^2 e^{\theta_1 T})\right]\right]\alpha$$

and $$C_R(\alpha)=Z-(Z-Y)\alpha$$

$$=\frac{A_0}{T}+\frac{C_{d4}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_4-\gamma)}\left[e^{\theta_4 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_4\gamma(\theta_4-\gamma)}\left[(h_4+\alpha T)(\theta_4-\gamma)-h_4(\theta_4 e^{\gamma T}-\gamma e^{\theta_4 T})\right]$$

$$+\frac{\alpha}{\theta_4\gamma}\left[(\theta_4^2-\gamma^2)-(\theta_4^2 e^{\gamma T}-\gamma^2 e^{\theta_4 T})\right]$$

$$-\left[\frac{C_{d4}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_4-\gamma)}\left[e^{\theta_4 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]\right.$$

$$-\frac{C_{d3}Ke^{-\gamma T}}{T}\left[\frac{1}{(\theta_3-\gamma)}\left[e^{\theta_3 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{\theta_4\gamma(\theta_4-\gamma)}\left[(h_4+\alpha T)(\theta_4-\gamma)-h_4(\theta_4 e^{\gamma T}-\gamma e^{\theta_4 T})\right]$$

$$-\frac{Ke^{-\gamma T}}{\theta_3\gamma(\theta_3-\gamma)}\left[(h_3+\alpha T)(\theta_3-\gamma)-h_3(\theta_3 e^{\gamma T}-\gamma e^{\theta_3 T})\right]$$

$$+\frac{\alpha}{\theta_4\gamma}\left[(\theta_4^2-\gamma^2)-(\theta_4^2 e^{\gamma T}-\gamma^2 e^{\theta_4 T})\right]$$

$$\left.-\frac{\alpha}{\theta_3\gamma}\left[(\theta_3^2-\gamma^2)-(\theta_3^2 e^{\gamma T}-\gamma^2 e^{\theta_3 T})\right]\right]\alpha$$

The defuzzified value of the fuzzy number ($T\tilde{V}C/T$), using signed distance method is given by,

$$TVC/T=\frac{1}{2}\int_0^1[C_L(\alpha)+C_R(\alpha)]d\alpha$$

$$=\frac{A_0}{T}+\frac{C_{d1}Ke^{-\gamma T}}{2T}\left[\frac{1}{(\theta_1-\gamma)}\left[e^{\theta_1 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{Ke^{-\gamma T}}{2\theta_1\gamma(\theta_1-\gamma)}\left[(h_1+\frac{T}{2})(\theta_1-\gamma)-h_1(\theta_1 e^{\gamma T}-\gamma e^{\theta_1 T})\right]$$

$$+\frac{1}{4\theta_1\gamma}\left[(\theta_1^2-\gamma^2)-(\theta_1^2 e^{\gamma T}-\gamma^2 e^{\theta_1 T})\right]$$

$$-\frac{C_{d2}Ke^{-\gamma T}}{2T}\left[\frac{1}{(\theta_2-\gamma)}\left[e^{\theta_2 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{C_{d1}Ke^{-\gamma T}}{2T}\left[\frac{1}{(\theta_1-\gamma)}\left[e^{\theta_1 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$-\frac{Ke^{-\gamma T}}{\theta_2\gamma}\left(\frac{h_2}{2}+\frac{T}{4}\right)+\frac{Ke^{-\gamma T}}{\theta_1\gamma}\left(\frac{h_1}{2}+\frac{T}{4}\right)$$

$$-\frac{Ke^{-\gamma T}h_1}{2\theta_1\gamma(\theta_1-\gamma)}\left[(\theta_1 e^{\gamma T}-\gamma e^{\theta 1T})\right]$$

$$+\frac{1}{6\theta_2\gamma}\left[(\theta_2^2-\gamma^2)-(\theta_2^2 e^{\gamma T}-\gamma^2 e^{\theta_2 T})\right]$$

$$-\frac{1}{6\theta_1\gamma}\left[(\theta_1^2-\gamma^2)-(\theta_1^2 e^{\gamma T}-\gamma^2 e^{\theta 1T})\right]+\frac{C_{d4}Ke^{-\gamma T}}{2T}$$

$$\left[\frac{1}{(\theta_4-\gamma)}\left[e^{\theta 4T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]+\frac{Ke^{-\gamma T}}{2\theta_4\gamma(\theta_4-\gamma)}$$
$$\left[(h_4+\frac{T}{2})(\theta_4-\gamma)-h_4(\theta_4 e^{\gamma T}-\gamma e^{\theta 4T})\right]$$
$$-\frac{C_{d4}Ke^{-\gamma T}}{2T}\left[\frac{1}{(\theta_4-\gamma)}\left[e^{\theta_4 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$+\frac{C_{d3}Ke^{-\gamma T}}{2T}\left[\frac{1}{(\theta_3-\gamma)}\left[e^{\theta_3 T}-e^{\gamma T}\right]+\frac{1}{\gamma}[1-e^{-\gamma T}]\right]$$

$$-\frac{Ke^{-\gamma T}}{2\theta_4\gamma}\left(\frac{h_4}{2}+\frac{T}{4}\right)+\frac{Ke^{-\gamma T}h_4}{\theta_4\gamma(\theta_4-\gamma)}\left[(\theta_4 e^{\gamma T}-\gamma e^{\theta 4T})\right]$$

$$+\frac{Ke^{-\gamma T}}{2\theta_3\gamma}\left(\frac{h_3}{2}+\frac{T}{4}\right)-\frac{Ke^{-\gamma T}h_3}{\theta_3\gamma(\theta_3-\gamma)}\left[(\theta_3 e^{\gamma T}-\gamma e^{\theta 3T})\right]$$

$$+\frac{1}{6\theta_4\gamma}\left[(\theta_4^2-\gamma^2)-(\theta_4^2 e^{\gamma T}-\gamma^2 e^{\theta_4 T})\right]$$

$$-\frac{1}{6\theta_3\gamma}\left[(\theta_3^2-\gamma^2)-(\theta_3^2 e^{\gamma T}-\gamma^2 e^{\theta_3 T})\right].$$

The optimal value of T is obtained using the method of maxima-minima by solving the equation $\frac{\partial TVC_{ds}(T)}{\partial T}=0$ for T with the sufficient condition $\frac{\partial^2 TVC_{ds}(T)}{\partial T^2}>0$.

Thus, $\frac{\partial TVC_{ds}(T)}{\partial T}=$

$$\frac{C_{d1}K}{2T^2}\left[T\,e^{T(\theta_1-\gamma)}-\frac{2\,e^{-\gamma T}}{(\theta_1-\gamma)}\left(e^{\theta_1 T}-e^{\gamma T}\right)\right]$$

$$+\frac{e^{-\gamma T}}{2\gamma T^2}\left[2\gamma T\left(2e^{-\gamma T}-1\right)-\left(1-e^{-\gamma T}\right)\right]$$

$$+\frac{K\,e^{-\gamma T}}{4\gamma\,\theta_1}\left(1-\gamma\left(h+\frac{T}{2}\right)\right)-\frac{K\,h_1\,e^{-\gamma T}}{2\gamma\,\theta_1(\theta_1-\gamma)}$$

$$\times\left[\theta_1\,\gamma\left(e^{\gamma T}-e^{\theta_1 T}\right)+\left(\theta_1\,e^{\gamma T}-\gamma\,e^{\theta_1 T}\right)\right]$$

$$+\theta_2\,\gamma\left(\theta_2\,e^{\gamma T}-\gamma\,e^{\theta_2 T}\right)\frac{C_{d2}K}{2T^2}\left[T\,e^{T(\theta_2-\gamma)}-\frac{2\,e^{-\gamma T}}{(\theta_2-\gamma)}\left(e^{\theta_2 T}-e^{\gamma T}\right)\right]$$

$$+\frac{e^{-\gamma T}}{2\gamma\,T^2}\left[2\gamma\,T\left(2\,e^{-\gamma T}-1\right)-\left(1-e^{-\gamma T}\right)\right]+\frac{K\,e^{-\gamma T}}{4\gamma\,\theta_2}$$

$$\left(1-\gamma\left(h+\frac{T}{2}\right)\right)-\frac{2\,e^{-\gamma T}}{2\gamma\,\theta_2(\theta_2-\gamma)}\left[\theta_2\,\gamma\left(e^{\gamma T}-e^{\theta_2 T}\right)\right]$$

$$+\left(\theta_2\,e^{\gamma T}-\gamma\,e^{\theta 2T}\right)+\left(\theta_2\,e^{\gamma T}-\gamma\,e^{\theta_2 T}\right)$$

$$+\frac{C_{d1}K}{2T^2}\left[T\,e^{T(\theta_1-\gamma)}-\frac{2\,e^{-\gamma T}}{(\theta_1-\gamma)}\left(e^{\theta_1 T}-e^{\gamma T}\right)\right]+\frac{e^{-\gamma T}}{2\gamma\,T^2}$$

$$\times\left[2\gamma\,T\left(2\,e^{-\gamma T}-1\right)-\left(1-e^{-\gamma T}\right)\right]+\frac{K\,e^{-\gamma T}}{4\gamma\,\theta_1}$$

$$\times\left(1-\gamma\left(h+\frac{T}{2}\right)\right)-\frac{K\,h_1\,e^{-\gamma T}}{2\gamma\,\theta_1(\theta_1-\gamma)}\left[\theta_1\,\gamma\left(e^{\gamma T}-e^{\theta_1 T}\right)+\left(\theta_1\,e^{\gamma T}-\gamma\,e^{\theta_1 T}\right)\right]$$

$$+\theta_2\,\gamma\left(\theta_2\,e^{\gamma T}-\gamma\,e^{\theta_2 T}\right)\frac{C_{d1}K}{2T^2}\left[T\,e^{T(\theta_1-\gamma)}-\frac{2\,e^{-\gamma T}}{(\theta_1-\gamma)}\left(e^{\theta_1 T}-e^{\gamma T}\right)\right]$$

$$+\frac{e^{-\gamma T}}{2\gamma\,T^2}\left[2\gamma\,T\left(2\,e^{-\gamma T}-1\right)-\left(1-e^{-\gamma T}\right)\right]+\frac{K\,e^{-\gamma T}}{4\gamma\,\theta_1}$$

$$\times\left(1-\gamma\left(h+\frac{T}{2}\right)\right)-\frac{K\,h_1\,e^{-\gamma T}}{2\gamma\,\theta_1(\theta_1-\gamma)}\left[\theta_1\,\gamma\left(e^{\gamma T}-e^{\theta_1 T}\right)+\left(\theta_1\,e^{\gamma T}-\gamma\,e^{\theta_1 T}\right)\right]$$

$$+\theta_2\,\gamma\left(\theta_2\,e^{\gamma T}-\gamma\,e^{\theta_2 T}\right)-\frac{A_0}{T^2}=0.$$

5. Numerical computing and Sensitivity Analysis

The numerical computing related to different results of the model is primarily become a basis for performing the sensitivity analysis of the model. The sensitivity mainly seeks to explore variations of various parameters involved in the model with respect to total optimal cost of the model as a main performance measure

Table1 shows the comparison of the total optimal cost in both environments- crisp as well as fuzzy. Tables 2-4 provide the variational study of total optimal cost with respect to rate of deterioration, total optimal cost with respect to rate of deterioration cost, total optimal cost with respect to rate of holding cost. Moreover, trend of correlation of total optimal cost with these parameters are positive as evident from tables 2-4. The parametric values for the inventory system are as follows:

Crisp Model: A_0 = 200 per order /unit, h =5 per year/unit, Cd = 20 per unit per year. The solution of crisp model is TAC = Rs 515.7690, T = 1.8643.
Fuzzy Model: The following are the two methods for obtaining the solution of the fuzzy model.

Table 1. Comparison of Optimal Results

Model	Optimal T	Opt I	Optimal TAC
Crisp Model	1.8643	67.4336	515.7690
Fuzzy Model	1.6592	72.3235	591.5366

Table 2. Sensitivity Analysis with respect to θ

DefuzzifIied θ	Fuzzified parameter	T (year)	$\widetilde{TAC}(t, T)$
0.005	(0.001, 0.004, 0.008, 0.012)	1.4832	512.411
0.007	(0.002, 0.006, 0.010, 0.014)	1.5530	536.572
0.009	(0.004, 0.008, 0.012, 0.016)	1.5967	577.553
0.011	(0.006, 0.010, 0.014, 0.018)	1.6592	591.566
0.013	(0.008, 0.012, 0.016, 0.020)	1.9660	611.809

Table 3. Sensitivity Analysis with respect to h

Defuzzified h	Fuzzified parameter	T (year)	$\tilde{TAC}$ (t , T)
3	(1, 2, 4, 6)	1.5552	544.099
4	(1, 3, 5, 7)	1.5596	555.762
5	(2, 4, 6, 8)	1.5898	567.658
6	(3, 5, 7, 9)	1.6592	591.536
7	(4, 6, 8, 10)	1.6921	655.063

6. Conclusion

This paper presents the comparison of results between crisp and fuzzy environments to comprehensively gain the realistic spectrum of the model for effective judgment of applications. Here, fuzzy paradigm is used to handle an uncertainty in the environment of fuzziness whereas other several techniques including dynamic, heuristic and stochastic process have been used to investigate the uncertainty other than fuzziness. Through numerical computing as given in tables 1-3, various performance measures including optimal total cost of the model as main result of the model have been computed in fuzzy environment. In tables 2-3, correlations between optimal total cost and other parameters of the model have been discussed. The model will be subjected to further investigation under intutionistic fuzzy paradigm as a future research.

Acknowledgement: The work is supported by the grant of a major research project of National Board for Higher Mathematics, Bombay, India (Ref. No. 2/48(6)/2012/NBHM/R&D II/13614).

References

1. F. Raafat, Survey of Literature on Continuously Deteriorating Inventory Model, *Journal of the Operational Research Society*, **42** (1991), 27-37.

2. H. M. Wee, Economic Production Lot Size Model for Deteriorating Items with Partial Back Ordering, *Computers and Indus- trial Engineering*, **24** (1993), 449-458.

3. S. K. Goyal, and B. C. Giri, Recent Trends in Modeling of Deteriorating Inventory, *European Journal of Operational Research*, **134** (2001), 1-16.

4. R. Li, H. Lan, and J. R. Mawhinney, A Review on Deteriorating Inventory Study, *Journal of Service Science and Management*, **3** (2010), 117-129.

5. F. W. Harris, *Operations and Cost*, A. W. Shaw Company, Chicago, 1915.

6. R. H. Wilson, A Scientific Routine for Stock Control, *Harvard Business Review*, **13** (1934), 116-128.

7. T. M. Whitin, *Theory of Inventory Management*, Princeton University Press, Princeton, 1957.

8. P. M. Ghare, and G. F. Schrader, A Model for an Exponentially Decaying Inventory, *Journal of Industrial Engineering*, **14** (1963), 238-243.

9. U. Dave, and L. K. Patel, (T, Si) Policy Inventory Model for Deteriorating Items with Time Proportional Demand, *Journal of the Operational Research Society*, **32** (1981), 137-142.

10. K. J. Chung, and P. S. Ting, A Heuristic for Replenishment for Deteriorating Items with a Linear Trend in Demand, *Journal of the Operational Research Society*, **44** (1993), 1235-1241.

11. H. M. Wee, A deterministic Lot-Size Inventory Model for Deteriorating Items with Shortages and a Declining Market, *Computers and Operations Research*, **22** (1995), 345-356.

12. S. S. Mishra, H. Pandey and R. S. Singh, A Fuzzified Deteriorating Inventory Model with Breakdown of Machine and Shortage Cost, *International Journal of Mathematical Science*, **3** (2004), 241-255.

13. S. S. Mishra and P. P. Mishra, A (Q, R) Model For Fuzzified Deterioration under Cobweb Phenomenon and Permissible Delay in Payment, *Computers and Mathematics with Applications*, **61** (2011), 921-932.

14. S. S. Mishra, Developing Intelligent Index for Fuzzified Inventory of Supply Chain by Using Neural Computing, *International Journal of Operations Research*, *Inderscience*, **29** (2017), 460-477.

15. P. L. Abad, Optimal Pricing and Lot-Sizing under Conditions of Perishability and Partial Backordering, *Management Science*, **42** (1996), 1093-1104.

16. P. L. Abad, Optimal Price and Order-Size for a Reseller under Partial are Backlogging, *Computers and Operations Research*, **28** (2001), 53-65.

17. B. Liu and Y. K. Liu, Expected Value of Fuzzy Variable and Fuzzy Expected Value Model, *IEEE Trans. Fuzzy Syst.*, **10** (2002), 445-450.

18. J. Y. R. Lengari, *Fuzzy Logic Intelligence, Control an Information*, Pearson Prentice Hall, Pearson education, Inc., 2005.

19. S. S. Mishra and P. P. Mishra, Price Determination for an EOQ Model for Deteriorating Items Under Perfect Competition, *Computer and mathematics with application*, **56** (2008), 1082-1101.

20. S. S. Mishra and P. P. Mishra, Phase Wise Supply Chain Model of EOQ with Normal life Time for Queued Customers: A Computational Approach, *American Journal of Operations Research*, **2** (2012), 296-307.

21. S. S. Mishra and P. K. Singh, Computing of Total Optimal Cost of an EOQ Model with Quadratic Deterioration and Occurrence of Shortages, *International journal*

of Management Science and Engineering Management, World Academic Press, **7** (2012), 243-252.

22. S. S. Mishra and P. K. Singh, Computational Approach to an Inventory Model with Ramp-Type Demand and Linear Deterioration, *International Journal of Operations Research, inderscience*, **15** (2012), 337-357.

23. S. Pathak, and S. Sarkar, A Fuzzy EOQ Inventory Model for Random Weibull Deterioration with Ramp-Type Demand, Parial Backlogging and Inflation Under Trade Credit Financing, *Int. J. Res. Commer. IT Manag.*, **2** (2012), 8-18.

24. S. Pathak and S. Sarkar, Fuzzy Production Inventory Model for Deteriorating Items with Shortage under the Effect of the Time Dependent Learning and Forgetting: a Possibility/ Necessity Approach, *OPSEARCH*, **50** (2013), 149-181.

25. G. M. A. Prasath and C. V. Seshaiah, A fuzzy production-Distribution Inventory Model with Shortage, *International Journal of Innovation and Applied Studies*, **2** (2013), 131-137.

26. T. Singh, and H. Pattnayak, An EOQ Model for Deteriorating Items with Linear Demand, Variable Deterioration and Partial Backlogging, *Journal of Service Science and Management*, **6** (2013), 186-190.

27. S. S. Mishra, D. K. Sharma, S. K. Yadav, and S. Rawat, Computational Approach to Fuzzified Profit Optimization of Inventory Flow in Supply Chain with Deteriorating Items, *International Journal Operational Research, Inderscience*, **30** (2017), 83-98.

28. S. Kozarević, and A. Puška, , Use of Fuzzy Logic for Measuring Practices and Performances of Supply Chain, *Operations Research Perspectives*, **5** (2018), 150–160.

29. H. J. Zimmermann, *Fuzzy Set Theory and its Applications,* Springer, 2001.

30. B. P. Dash, T. Singh and H. Pattnayak, An Inventory Model for Deteriorating Items with Exponential Declining Demand and Time-Varying Holding Cost, *American Journal of Operations Research*, **4** (2014), 1-7.