ISSN 0974 - 9373

Vol. 23 No. 1 (2019) **Journal of International Academy of Physical Sciences** pp. 13- 24

Cosmological Models with Variable Deceleration Parameter in Scalar Tensor Theory of Gravitation

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(Received December 20, 2018)

Abstract: In this communication we have presented the study of Bianchi type-III space-time dark energy cosmological model with the time-dependent deceleration parameter in the scalar tensor theory of Gravitation proposed by Sàez-Ballester theory (*Phys. A* 113: 467, 1986). We have calculated the exact solution of Einstein's field equations by using a mathematical relation $a(t)=\left[\text{Sinh}\,\alpha t\right]^{\frac{1}{n}}$, here α and n are positive constants. As per need of the study we have also presented the physical and geometrical aspects along with properties in this paper.
Keywords: Bianchi type-III space time; EOS parameter; Variable Deceleration parameter; Accelerating universe; Saez -Ballester Theory.

1. Introduction

In recent research finding the study regarding the effect of cosmic string has an important area of cosmology. As the vacuum string[1] generate density perturbations which are useful for the formation of galaxies. The general relativistic treatment of a string was given by Letelier[2,3] and Stachel[4] formulated the energy momentum tensor for massive string. The discovery of accelerating expansion of the universe[5-10] has supported many speculations about the present contest of this paper. It has been also observed that at present many cosmologists has shown their interest in scalar tensor of gravitation proposed by several researcher as Brans and Dicke[11], Nordtvedt[12], Lyra[13], Sen and Dunn[14] and Sàez and Ballester[15]. It is further mentioned that Brans and Dicke theory including a long-range scalar field interacting equally with all forms of matter with the exception of electromagnetism while in Sàez-Ballester scalar-tensor theory the metric is coupled with a dimensionless scalar field in a simple manner. In present paper Einstein field equations for scalar tensor theory proposed Sàez-Ballester has been solved. The Sàez-Ballester theory field equations are;

(1.1) $$R_{\nu}^{\mu}-\frac{1}{2}Rg_{\nu}^{\mu}-\omega\phi^{m}\left(\phi^{,\mu}\phi_{,\nu}-\frac{1}{2}g_{\mu}^{\nu}\phi_{,l}\phi^{,l}\right)=-T_{\nu}^{\mu},$$

(1.2) $$2\phi^{m}\phi_{;\mu}^{,\mu}+m\phi^{l-1}\phi_{,k}\phi^{,}k=0.$$

The energy conservation equations as

(1.3) $$T_{;\mu}^{\mu}=0,$$

where R_{ν}^{μ} is Ricci tensor, R is the curvature scaler ω and m are constants, T_{ν}^{μ} is energy momentum tensor, Commas denote partial derivative and semicolon denote covariant derivative.

Under motivation of the above study in the present paper Einstein field equation have been solved along with mathematical relations with time dependent deceleration parameter (DP) in Bianchi type III space-time. The scenario may facilitate to describe the dynamistic nature of the universe specially from early decelerated phase to present accelerated phase. This paper has been divided into five sections, Section 1 is introduction , the metric and field equation in section 2, solution of Einstein field equation is presented in section 3 along with various physical and geometrical properties of the model, the result and discussion has been made in section 4, conclusion has been made in section 5.

2. The Metric and Field Equations

We consider totally anisotropic Bianchi type-III space-time described by the line element

(2.1) $$ds^{2}=-dt^{2}+A^{2}dx^{2}+B^{2}dy^{2}+C^{2}d^{2},$$

where A, B and C are the functions of cosmic time t only. The energy-momentum tensor of the fluid is taken as

(2.2) $$T_\nu^\mu = diag\left(T_0^0, T_1^1, T_2^2, T_3^3\right),$$

By considering EoS parameter for anisotropic in the fluid we may insight certain new possibilities for the evolution of dark energy source. To obtain this first we parameterized the energy momentum tensor given by equation (2.2) as

(2.3) $$T_\nu^\mu = \mathrm{diag}\left[\rho,\ p_x, p_y, p_z\right],$$

(2.4) $$T_\nu^\mu = diag\left[1,\ \omega_x, \omega_y, \omega_z\right]\rho = daig\left[1,\ -\omega_x, -(\omega+\delta)_y, -(\omega+\delta)_z\right]\rho,$$

where ρ is the energy-density, p_x, p_y, p_z are the pressure and ω_x, ω_y, ω_z are the directional EoS parameter along the x, y, z axis respectively. Now on simplification with equations (1.1)-(2.3) we have the following set of equations.

(2.5) $$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = \omega\rho,$$

(2.6) $$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -(\omega+\delta)\rho,$$

(2.7) $$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}\frac{\omega}{2}\phi^m\dot{\phi}^2 = -(\omega+\delta)\rho,$$

(2.8) $$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} + \frac{\omega}{2}\phi^m\dot{\phi}^2 = \rho,$$

(2.9) $$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = 0,$$

(2.10) $$\ddot{\phi} + \dot{\phi}\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right] + \frac{m\dot{\phi}^2}{2\phi} = 0,$$

(2.11) $$\ddot{\rho} + \dot{\rho}(\omega+\delta) + \left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right] = 0.$$

Here dot indicates derivative with respect to t. On integrating equation (2.9), we obtain,

(2.12) $$A = B,$$

By considering integrating constant as unity, now putting this value $A = B$ and $\delta = 0$ into the above set of equations we may re-formulating the set of equations (2.5)−(2.11) as

(2.13) $$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\omega}{2}\phi^m \dot{\phi}^2 = -\omega\rho ,$$

(2.14) $$\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{\omega}{2}\phi^m \dot{\phi}^2 = -\omega\rho ,$$

(2.15) $$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2}\frac{\omega}{2}\phi^m \dot{\phi}^2 = -\omega\rho ,$$

(2.16) $$2\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} - \frac{\omega}{2}\phi^m \dot{\phi}^2 = \rho ,$$

(2.17) $$\ddot{\phi} + \dot{\phi}\left[2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C}\right] + \frac{m\dot{\phi}^2}{2\phi} = 0 ,$$

(2.18) $$\ddot{\rho} + \dot{\rho}\,\omega\left[2\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C}\right] = 0 .$$

3. Solution of Field Equations

Equations (2.12)−(2.16) are the set of four equations along with five unknowns viz. A, C, ϕ, ω and ρ. For explicit solution of field equations solution (2.12)-(2.16), we need one more condition. For this we introduce deceleration parameter q as a function to time t, i.e.

(3.1) $$q = \frac{a\ddot{a}}{\dot{a}^2} = -\frac{\left(\dot{H} + H^2\right)}{H^2} = b\left(t\right) \quad \text{(say).}$$

It is noticed in literature that deceleration parameter is constant quantity and several authors have been cosmological models with constant deceleration parameter[16-19]. Also, it suggests a power law for metric function or corresponding quantity, The motivation to choose such time dependent deceleration parameter (DP) is behind the fact that the universe exhibits phase transition from the past deceleration to present acceleration one as revealed by the recent observations of SNe Ia[20-24] and CMB anisotropy[25].
In this section our focus on the evolution of universe with variable mean deceleration parameter as suggested by our peer group Mishra et al.[26-30];

(3.2) $$\frac{\ddot{a}}{a}+b\frac{\dot{a}}{a^2}=0,$$

here $b=b(a)=b(a(t))$.

(3.3) $$\int e^{\int\frac{b}{a}da}da=t+k,$$

(3.4) $$\int\frac{b}{a}da=\ln\left[f(a)\right],$$

(3.5) $$\int f(a)da=t+k.$$

Now, choice of $f(a)$ is in quite arbitrary, but sake of physically viable models of the universe with observations, we choose

(3.5) $$f(a)=\frac{na^{n-1}}{\sqrt{\left(1+a^{2n}\right)}},$$

(3.6) $$a(t)=\left[\text{Sinh}(\alpha t)\right]^{\frac{1}{n}}.$$

Since, the set of non-linear differential equations are always difficult to solve so remove this complication, we assume the shear scalar (σ) is proportional to expansion scalar (θ) i.e. $\theta \propto \sigma$ which lead to,

(3.7) $$2\frac{\dot{A}}{A}+\frac{C}{C}=K\left[\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right],$$

where, K is constant of proportionality

(3.8) $$A = C^K .$$

Now, we explain this study further in view of the reference of Thore[31] where the observation of the velocity red-shift relation for extragalactic sources suggested that Hubble expansion $\simeq$ 30 percent by Kantwoski & Sachs[32] and Kristian & Sachs[33]. The parameter to be analysis for spatial volume V are as

(3.9) $$V = ABC\,e^x .$$

Using equation (2.12) and (3.8) the above equation become

(3.10) $$V = A^{\frac{2K+1}{K}} e^x ,$$

Also,

(3.11) $$V = \left[\sinh(\alpha t)\right]^{\frac{3}{n}} ,$$

(3.12) $$A = \left[\sinh(\alpha t)\right]^{\frac{3K}{n(2K+1)}} .\mathrm{e}^{-\frac{Kx}{2K+1}} ,$$

(3.13) $$B = \left[\sinh(\alpha t)\right]^{\frac{3K}{n(2K+1)}} .\mathrm{e}^{-\frac{Kx}{2K+1}} ,$$

(3.14) $$C = \left[\sinh(\alpha t)\right]^{\frac{3}{n(2K+1)}.\mathrm{e}^{-\frac{x}{2K+1}}} .$$

The metric equation (1.4) can be written as

(3.15) $$ds^2 = -dt^2 + \left[\sinh(\alpha t)\right]^{\frac{6K}{n\,(2K+1)}} \mathrm{e}^{-\frac{2Kx}{2K+1}}.(\mathrm{dx}^2 + dy \wedge 2$$

$$+ \left[\sinh(\alpha t)\right]^{\frac{6}{n\,(2K+1)}} .\mathrm{e}^{-\frac{2x}{2K+1}} dz^2 ,$$

(3.16) $$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{3\alpha K}{n(2K+1)} \coth(\alpha t),$$

(3.17) $$\frac{\dot{C}}{C}=\frac{3K}{n(2K+1)}\coth(\alpha t).$$

Hubble parameter is defined as,

(3.18) $$H=\frac{1}{3}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right]=\frac{\alpha}{n}\coth(\alpha t).$$

Now, we have calculated some cosmological parameters for Bianchi type-III model for the analysis. The average scale factor $\alpha(t)$ and spatial volume V are defined as

(3.19) $$V=a^3(t)=A^2Ce^x.$$

The expansion scalar θ, anisotropic parameter A_m and shear scalar σ^2 are found to have the following expressions;

(3.20) $$\theta=\frac{2\dot{B}}{B}+\frac{\dot{C}}{C},$$

(3.21) $$\sigma^2=\frac{1}{3}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right).$$

Here, H_1, H_2 and H_3 are directional Hubble's parameter in the directions of x, y and z respectively. The physical quantity of observational interest in cosmology in the deceleration parameter q defined by

(3.22) $$q=-\frac{a\ddot{a}}{\dot{a}^2}=n(1-\tanh^2\alpha t)-1.$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponding to a standard decelerating model whereas the negative indicates acceleration.

(3.23) $$\theta=\frac{3\alpha}{n}\coth(\alpha t),$$

(3.24) $$A_m = 2\left[\frac{\alpha(K-1)}{n(2K-1)}.\coth(\alpha t)\right]^2,$$

(3.25) $$\sigma^2 = \frac{K-1}{n(2K+1)}.\coth^2(\alpha t),$$

(3.26) $$\frac{\sigma^2}{\theta^2} = \frac{n(K-1)}{9\alpha^2(2K+1)}.$$

From equation (2.17) the value of potential function ϕ is,

(3.27) $$\frac{\ddot{\phi}}{\phi} + \frac{m\dot{\phi}}{2\phi} = \frac{3\alpha}{n}\coth(\alpha t),$$

(3.28) $$\phi(t) = \left[\frac{m+2}{2}\int \phi_0\left[sinh(\alpha t)\right]^{-\frac{3}{n}} dt\right]^{\frac{2}{m+2}},$$

(3.29) $$\phi(t) = \left[\frac{m+2}{2}.\frac{(-1)^{\frac{n+3}{2n}}\phi_0}{\alpha}.\coth(\alpha t).F(t) + \phi_1\right]^{\frac{2}{m+2}},$$

where

$$F(t) = 1 + \frac{1}{6}\left(1+\frac{3}{n}\right)\cosh^2(\alpha t) + \frac{3}{40}\left(1+\frac{3}{n}\right)\left(1+\frac{1}{n}\right)\cosh^4(\alpha t) + O\left(\cosh(\alpha t)\right)^6$$

and ϕ_0, ϕ_1 are constant of integration. Now, expressions for energy density parameter ρ and EoS parameter ω are given by

(3.30) $$\rho(t) = \frac{9\alpha K}{n^2(2K+1)^2}\left[2Kcoth(\alpha t)+1\right] - \left[\sinh(\alpha t)\right]^{-\frac{6K}{n(2K+1)}}$$

$$e^{\frac{2K}{2K+1}} - \frac{\omega\phi_0}{2}\left[\sinh(\alpha t)\right]^{-\frac{6}{n}},$$

$$\omega(t)=\frac{1}{\rho}\left[\frac{(2-\alpha)}{\alpha}\left(\frac{3K\alpha}{n(2K+1)}\right)^2\coth^2(\alpha t)+\frac{6K\alpha^2}{n(2K+1)}cosech^2(\alpha t)\right]. \tag{3.31}$$

As per equation obtain (3.18) for Hubble parameter H, (3.22) for deceleration parameter (q), (3.24) for anisotropic parameter A_m and (3.30) for energy density parameter (q) with respect to time parameter we have drawn their graph in Fig. 1 Hubble parameter versus time, Fig. 2 deceleration parameter versus time, Fig. 3 anisotropic parameter versus time and Fig. 4 energy density parameter versus time. Based on mathematical equations and graphs, we summarized the results in section 4.

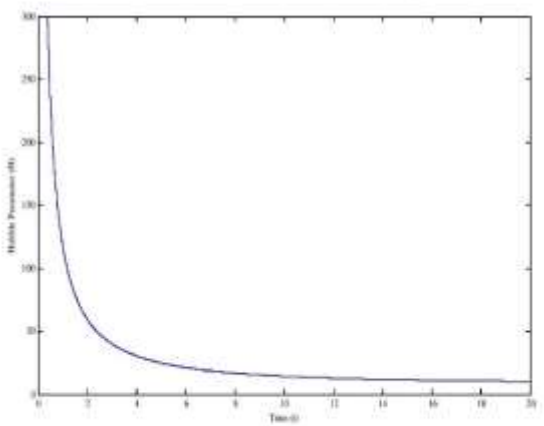

Figure 1. Plot of Hubble's parameter vs. cosmic time.

4. Results and Discussion

Spatial volume scale factor (V): It is clear from equation (3.11) spatial volume scale factor V is vanishing at $t=0$, increasing exponentially with time and approaches to infinity as $t\to\infty$.

Directional Hubble parameter (H): Fig.1 show that directional Hubble parameter is infinity at $t=0$ and decreasing with time, approaches to zero at late time. i.e. $H\to\infty$ as $t\to 0$ and $H\to 0$ a as t.

Deceleration parameter $q(t)$: It is clear from Fig. 2 that deceleration parameter q decrease rapidly with time and change the sign positive to negative, positive sign of $q(q>0)$ indicate the decelerating model in early history of the universe, whereas negative sign of $q(q<0)$ (particularly $-1 \le q < 0$) show that universe expansion is at accelerating rate. Also, when $t \cong 13.8GYr$ there is transition phase. During the mathematical analysis of equation (3.26) following observation be made:

$\frac{\sigma^2}{\theta^2} \to \infty$ as $t \to 0$ and $\frac{\sigma^2}{\theta^2} \to \frac{K-1}{3\alpha(2K+1)}$ as $t \to \infty$, provided $K \ne \frac{1}{2}$.

Anisotropy parameter (A_m)**:** It is observed from Fig. 3 that A_m is very large at the early stage of the universe. A_m is decreasing with time and approaches to very small positive value which means the anisotropy parameter model may indicate the isotropy at late time. i.e. $A_m \to 0$ as $t \to 0$ and $A_m \to 2\left[\frac{\alpha(K-1)}{n(2K+1)}\right]$ as $t \to \infty$ provided that $K \ne \frac{1}{2}$. Also, $\frac{\sigma^2}{\theta^2} =$ constant, which show that model does not approaches to isotropy at any time.

Energy density parameter (ρ)**:** From Fig. 4 it is observed that energy density decreases with time, when $t \to 0$, $\rho \to \infty$ and when $t \to \infty$ then $\rho \to 0$.

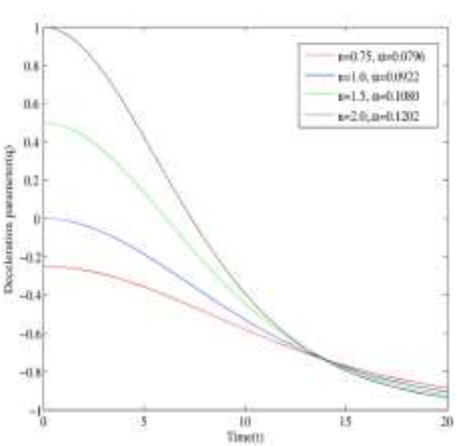

Figure 2. Plot of deceleration parameter vs. cosmic time

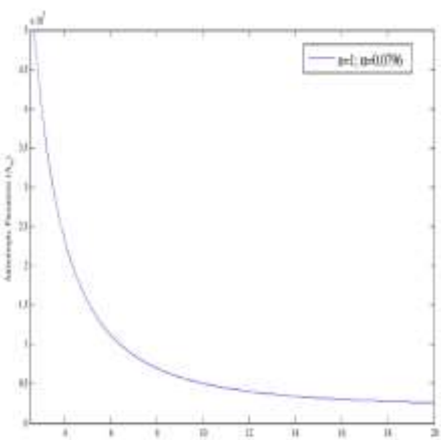

Figure 3. Plot of anisotropy parameter vs. cosmic time

5. Concluding Remarks

In this communication Bianchi III space time dark energy cosmological model along with time dependent deceleration parameter have been present with the help of scaler tensor theory of gravitation as proposed by Sàez and Ballester. It is observed that for $\phi \to 0$, Sàez-Ballester theory does not hold for later time as the discussed model represented shearing non-rotating and expanding universe which exhibits point type singularity with Big-Bang origin at $t = 0$ and approaches to isotropy at late time.

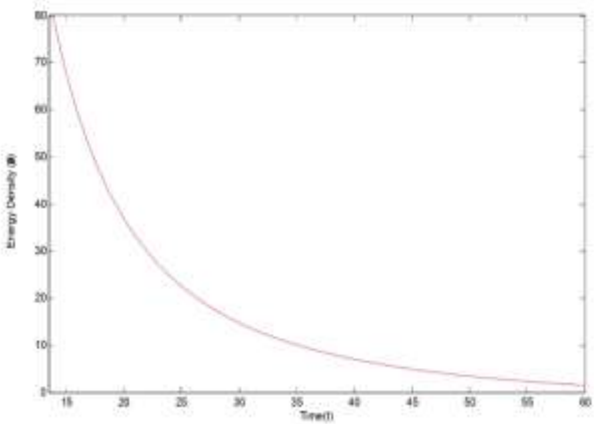

Figure 4. Plot of energy density parameter vs. cosmic time

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