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μ-Binary Topological Space (μBTS)

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Abstract: Topology is playing a key role in almost every field of applied sciences and several branches of mathematics. This is very useful concept especially in information system, particle physics, quantum physics, high energy physics etc. In 2002 Csaszar[4] introduced and studied the concept of generalized topology and generalized continuity. In 2011 Nithyanantha Jothi S and Thangavelu P[2] have introduced the concept of binary topology between two sets and studied different properties. In this paper we introduce and studied μ-binary topological spaces and investigate its basic properties

Keywords: Binary topological spaces, μ-binary topological spaces, μB-closure (μBI), μB-interior (μBI).

1. Introduction

In 2012 the authors[3] have introduced the concept of binary closure, binary interior and binary continuity and binary separation axioms. Further in 2014, they have introduced and studied generalized binary closed sets and generalized binary open sets that are analogous to the generalized closed sets and generalized open sets in point set topology. They[4] have further shown that binary closed sets in binary topological space are generalized binary closed but the converse is not necessarily true and obtained necessary and sufficient conditions for generalized binary closed sets to be binary closed. In this chapter we introduce $\mu-$binary topological spaces and $\mu-$binary continuous maps and studied different properties of these maps. In section 2, we study basic definitions and properties of binary topological spaces. In section 3 we introduce the concept of $\mu-$binary topological spaces and studied various characterizations. Throughout the chapter $\wp(X)$ and $\wp(X)$ denote the power sets of X and Y respectively.

2 . Preliminaries

Definition 2.1. *Let X and Y are any two non-empty sets. A binary topology from X to Y is a binary structure $M \subseteq \wp(X) \times \wp(X)$ that satisfies the following axioms:*

(i) (ϕ, ϕ) *and* $(X, Y) \in M$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$, *whenever* $(A_1, B_1) \in M$ *and* $(A_2, B_2) \in M$.

(iii) If $\{(A_\alpha, B_\alpha);\ \alpha \in \Delta\}$ *is a family of members of* M *, then*

$$\left(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha\right) \in M$$

Definition 2.2. *If M is a binary topology from X to Y, then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X, Y, M). The elements of $X \times Y$ are called the binary points of binary topological space (X, Y, M). If $Y = X$, then M is called binary topology on X in which case we write (X, M) for binary topological space.*

Definition 2.3. *Let (X, Y, M) be a binary topological space and let $(x, y) \in X \times Y$. The binary open set (A, B) is called a binary neighborhood of (x, y) if $x \in A$ and $y \in B$.*

Example 2.1. $I = \{(\phi, \phi), (X, Y)\}$ is called the indiscrete binary topology from X to Y and (X, Y, I) is called the indiscrete binary topological space.

Example 2.2. Let $D = \wp(X) \times \wp(Y)$. The binary topological space (X, Y, D) is called the discrete binary topological space.

Remark 2.1. As $\wp(X\times Y)\neq\wp(X)\times\wp(Y)$, the concept of binary topology from X to Y and the concept of topology on $X\times Y$ are independent. It is noteworthy to see that the product topology of topologies of X and Y is independent from the binary topology from X to Y as seen from the following Example 2.3.

Example 2.3. Suppose (X,τ) and (Y,σ) are topological spaces. Let $\rho=\{A\times B: A\in\tau \text{ and } B\in\sigma\}$. Then ρ is the product topology for $X\times Y$. However this cannot be the binary topology from X to Y as (ϕ,ϕ) does not belong to ρ. ρ is a binary topology if we identify (ϕ,ϕ) with the empty set ϕ.

Now let $X=\{a,b,c\}$ and $Y=\{1,2,3\}$. Clearly $M=\{(\phi,\phi),(\{a\},\phi),(X,Y)\}$ is a binary topology from X to Y. Since $(\{a\},\phi)$ cannot be identified with $\{a\}$ or ϕ, M is not a product topology on $X\times Y$. As the examples shows, the sets X and Y may have many binary topologies. By regarding each binary topology from X to Y as a subset of $\wp(X)\times\wp(Y)$, the binary topologies from X to Y are partially ordered by set inclusion.

Proposition 2.1. *Let $\{\phi_\alpha:\alpha\in\Omega\}$ be any family of binary topologies from X to Y. Then $\bigcap_{\alpha\in\Omega}\phi_\alpha$ is also a binary topology from X to Y.*

Remark 2.2. Let $\{\phi_\alpha:\alpha\in\Omega\}$ be any family of binary topologies from X to Y. Then $\bigcup_{\alpha\in\Omega}\phi_\alpha$ need not be a binary topology as shown in Example 2.3.

Example 2.4. Let $X=\{a,b,c\}$ and $Y=\{1,2,3,4\}$. Then $M_1=\{(\phi,\phi),(\{a\},\{1\}),(\{a,b\},\{1,2\}),(X,Y)\}$ and $M_2=\{(\phi,\phi),(\{a,b\},\{1,3\}),(X,Y)\}$ are two binary topologies from X to Y. Clearly $M_1\cup M_2$ is not binary topology from X to Y.

Definition 2.4. *Let (X,Y,M) be a binary topological space and $A\subseteq X$, $B\subseteq Y$. Then (A,B) is binary closed in (X,Y,M) if $(X\setminus A, Y\setminus B)\in M$.*

3. $\mu-$Binary Topological Space (μBTS)

Definition 3.1. *Let X and Y are any two non-empty sets. A generalize-d binary topology (μBT) from X to Y is a binary structure*

$\mu \subseteq \wp(X) \times \wp(Y)$ *that satisfies the following axioms:*

(i) (ϕ, ϕ) *and* $(X, X) \in \mu$

(ii) If $\{(L_\alpha, M_\alpha) : \alpha \in \Delta\}$ *is a family of members of* μ, *then* $\left(\bigcup_{\alpha\in\Delta} L_\alpha, \bigcup_{\alpha\in\Delta} M_\alpha,\right) \in \mu$.

If μ is μBT from X to Y, then the triplet (X, Y, μ) is called generalized binary topological space (μBTS) and the members of μ are called the $\mu -$binary open subsets (μBOS) of $\mu BTS(X, Y, \mu)$. The elements of $X \times Y$ are called the $\mu -$binary points (μBP) of $\mu BTS(X, Y, \mu)$.

Example 3.1. Let $X = \{1, 2\}$ and $Y = \{a, b, c\}$. Then $\mu = \{(\phi, \phi), (\{1\}, \{a, c\}), (\{2\}, \{a, b\}), (X, Y)\}$ is μBT satisfying the conditions mentioned above.

Definition 3.2. Let X and Y be any two non-empty sets and (L, M), (N, P) belongs to $\wp(X) \times \wp(Y)$, then $(L, M) \subseteq (N, P)$ if $L \subseteq N$ and $M \subseteq P$.

Proposition 3.1. Let $\{\psi_\Delta : \Delta \in \Lambda\}$ be any family of $\mu BT'S$ from X to Y. Then, $\bigcap_{\Delta\in\Lambda} \psi_\Delta$ is also μBT from X to Y.

Proof: Let ψ_Δ is μBT from X to Y. Then (ϕ, ϕ) and $(X, X) \in \psi_\Delta$ for all $\Delta \in \Lambda$. This implies $(\phi, \phi) \in \bigcap_{\Delta\in\Lambda} \psi_\Delta$ and $(X, X) \in \bigcap_{\Delta\in\Lambda} \psi_\Delta$. Let $\bigcap_{\Delta\in\Lambda} \psi_\Delta = \psi$ and $(L_\beta, M_\beta) \in \psi, \forall \beta \in \Omega$, where Ω is arbitrary index. Then $(L_\beta, M_\beta) \in \psi_\Delta, \forall \Delta \in \Lambda$. Since ψ_Δ is μBT from X to Y. It follows $\left(\bigcup_{\beta\in\Omega} L_\beta, \bigcup_{\beta\in\Omega} M_\beta\right) \in \psi_\Delta, \forall \Delta \in \Lambda$. This implies that $\left(\bigcup_{\beta\in\Omega} L_\beta, \bigcup_{\beta\in\Omega} M_\beta\right) \in \bigcap_{\Delta\in\Lambda} \psi_\Delta$. Hence $\bigcap_{\Delta\in\Lambda} \psi_\Delta$ is μBT from X to Y.

Remark 3.1. Let $\{\psi_\Delta : \Delta \in \Lambda\}$ be any family of μBT 's from X to Y. Then $\bigcup_{\Delta\in\Lambda} \psi_\Delta$ need not be μBT as shown in Example 3.2.

Example 3.2. Let $X = \{a, b, c\}$ and $Y = \{1, 2, 3\}$. Then $\mu_1 = \{(\phi, \phi), (X, Y)$

$(\{a\},\{1,3\}),(\{X\},\{1,2\})\}$ and $\mu_2=\{(\phi,\phi),(\{X\},\{1,2\}),(\{b,c\},\{3\}),(X,Y)\}$ are two μBT 's from X to Y. Clearly $\mu_1 \cup \mu_2$ is not μBT from X to Y.

Definition 3.3. *Let* (X, Y, μ) *be* μBTS *and* $L \subseteq X$, $M \subseteq Y$. *Then* (L, M) *is* μ*–binary closed set* (μBCS) *in* (X, Y, μ) *if* $(X \setminus L, Y \setminus M) \in \mu$.

Proposition 3.2. *Let* (X, Y, μ) *be* μBTS. *Then*
(i) (X, Y) *and* (ϕ, ϕ) *are* $\mu BCS's$.
(ii) If $\{(L_\alpha, M_\alpha): \alpha \in \Delta\}$ *is a family of* $\mu BCS's$, *then* $(\bigcap_{\alpha\in\Delta} L_\alpha, \bigcap_{\alpha\in\Delta} M_\alpha)$ *is* μBCS.

Definition 3.4. *Let* (X, Y, μ) *be* μBTS *and* $(L, M) \subseteq (X, Y)$. *Let* $(L, M)_\mu^{1^*} = \bigcap\{L_\alpha : (L_\alpha, M_\alpha)\}$ *is* μBCS *and* $(L, M) \subseteq (L_\alpha, M_\alpha)$ *and Let* $(L, M)_\mu^{2^*} = \bigcap\{M_\alpha : (L_\alpha, M_\alpha)\}$ *is* μBCS *and* $(L, M) \subseteq (L_\alpha, M_\alpha)$. *Then* $\left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$ *is* μBCS *and* $(L, M) \subseteq \left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$. *The ordered pair* $\left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$ *is called* μB*–closure* (μBC) *of* (L, M) *and is denoted* $Cl_\mu(L, M)$ *in the* $\mu BT(X, Y, \mu)$ *where* $(L, M) \subseteq (X, Y)$.

Proposition 3.3. *Let* $(A, B) \subseteq (X, Y)$ *Then* (A, B) *is* μBCS *in* (X, Y, μ) *iff* $(A, B) = Cl_\mu(A, B)$.

Proof: Suppose (L, M) is μBCS in (X, Y, μ). Then by the definition, the order pair $\left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$ is μB– closure of (L, M) and $(L, M) \subseteq \left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$. Since (L, M) is μBCS containing (L, M) i.e. $\left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right) \subseteq (L, M)$. Therefore we get $(L, M) = \left((L, M)_\mu^{1^*}, \left((L, M)_\mu^{2^*}\right)\right)$.

Conversely, suppose $(L, M)=\left((L, M)_{\mu}^{1^*},\left((L, M)_{\mu}^{2^*}\right)\right)$. Let (X, Y, μ) be μBTS and $(L, M)\subseteq(X, Y)$. Then $(L, M)_{\mu}^{1^*}=\bigcap\{L_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ is μBCS and $(L, M)\subseteq(L_{\alpha}, M_{\alpha})$ and $(L, M)_{\mu}^{2^*}=\bigcap\{M_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ is μBCS and $(L, M)\subseteq(L_{\alpha}, M_{\alpha})$. Therefore $\left((L, M)_{\mu}^{1^*},\left((L, M)_{\mu}^{2^*}\right)\right)$ is μBCS and $(L, M)\subseteq\left((L, M)_{\mu}^{1^*},\left((L, M)_{\mu}^{2^*}\right)\right)$. Therefore $Cl_{\mu}(L, M)$ is μBCS and hence (L, M) is μBCS.

Proposition 3.4. *Suppose* $(L, M)\subseteq(N, P)\subseteq(X, Y)$ *and* (X, Y, μ) *is* μBTS. *Then*

(i) $Cl_{\mu}(\phi, \phi)=(\phi, \phi)$, $Cl_{\mu}(X, Y)=(X, Y)$

(ii) $(L, M)\subseteq Cl_{\mu}(L, M)$

(iii) $(L, M)_{\mu}^{1^*}\subseteq(N, P)_{\mu}^{1^*}$

(iv) $(L, M)_{\mu}^{2^*}\subseteq(N, P)_{\mu}^{2^*}$

(v) $Cl_{\mu}(L, M)\subseteq Cl_{\mu}(N, P)$

(v) $Cl_{\mu}(Cl_{\mu}(L, M))=Cl_{\mu}(L, M)$

Proof: The properties (i) and (ii) follows easily.

Now $(L, M)_{\mu}^{1^*}=\bigcap\{L_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ is μBCS and $(L, M)\subseteq(L_{\alpha}, M_{\alpha})$, which implies $(L, M)_{\mu}^{1^*}\subseteq\bigcap\{L_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ is μBCS and $(N, P)\subseteq(L_{\alpha}, M_{\alpha})$ $=(N, P)_{\mu}^{1^*}$. This proves (iii). Proof of (iv) is analogous and (v) and (vi) follows from Proposition 3.2.

Definition 3.5. *Let* (X, Y, μ) *be* μBTS *and* $(L, M)\subseteq(X, Y)$ *Let* $(L, M)_{\mu}^{1^0}=\bigcup\{L_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ *is* μBOS *and* $(L_{\alpha}, M_{\alpha})\subseteq(L, M)$ *and Let* $(L, M)_{\mu}^{2^0}=\bigcup\{M_{\alpha}:(L_{\alpha}, M_{\alpha})\}$ *is* μBOS *and* $(L_{\alpha}, M_{\alpha})\subseteq(L, M)$. *Then* $\left((L, M)_{\mu}^{1^0},(L, M)_{\mu}^{2^0}\right)$ *is* μBOS *and* $\left((L, M)_{\mu}^{1^0},(L, M)_{\mu}^{2^0}\right)\subseteq(L, M)$. *The* *ordered pair* $\left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$ *is called* $\mu B-interior$ (μBI) *of* (L, M) *and is denoted by* $I_{\mu}(L, M)$.

Proposition 3.6. *Let* $(L, M) \subseteq (X, Y)$. *Then* (L, M) *is* μBOS *in* (X, Y, μ) *iff* $(L, M) = I_{\mu}(L, M)$.

Proof: Suppose (L, M) is μBOS in (X, Y, μ). Then by the definition of μBI, the order pair $\left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$ is μBI of (L, M) and $(L, M) \subseteq \left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$. Since (L, M) is μBOS containing $(L, M)_{\mu}^{1^0}$ and $(L, M)_{\mu}^{2^0}$ i.e. $\left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right) \subseteq (L, M)$. Therefore we get $(L, M) = \left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$.

Conversely Suppose $(L, M) = \left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$. Let (X, Y, μ) be μBTS and $(L, M) \subseteq (X, Y)$. Then $(L, M)_{\mu}^{1^0} = \bigcup\left\{L_{\alpha} : \left(L_{\alpha}, M_{\alpha}\right)\right\}$ is μBOS and $\left(L_{\alpha}, M_{\alpha}\right) \subseteq (L, M)$ and $(L, M)_{\mu}^{2^0} = \bigcup\left\{M_{\alpha} : \left(L_{\alpha}, M_{\alpha}\right)\right\}$ is μBOS and $\left(L_{\alpha}, M_{\alpha}\right) \subseteq (L, M)$. Therefore $\left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right)$ is μBOS and $\left((L, M)_{\mu}^{1^0}, (L, M)_{\mu}^{2^0}\right) \subseteq (L, M)$. Therefore $I_{\mu}(L, M)$ is μBOS and hence (L, M) is μBOS.

Proposition 3.7. *Suppose* $(L, M) \subseteq (N, P) \subseteq (X, Y)$ *and* (X, Y, μ) *is* μBTS. *Then*

(i) $I_{\mu}(\phi, \phi) = (\phi, \phi)$, $I_{\mu}(X, Y) = (X, Y)$

(ii) $(L, M)_{\mu}^{1^0} \subseteq (N, P)_{\mu}^{1^0}$

(iii) $(L, M)_{\mu}^{2^0} \subseteq (N, P)_{\mu}^{2^0}$

(iv) $I_{\mu}(L, M) \subseteq I_{\mu}(N, P)$

(v) $I_{\mu}\left(I_{\mu}(L, M)\right) = I_{\mu}(L, M)$

Proof: Quite easy

4. Conclusion

In this paper we introduce and studied $\mu-$binary topological spaces, $\mu B-$closure (μBI) and $\mu B-$interior (μBI) and investigate several properties

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