Effect of Magnetic Field on an Unsteady Flow of a Dusty Viscous Fluid Between Two Infinite Plates Having Porous Medium

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Abstract: The flow of a dusty viscous fluid between two infinite plates filled with porous medium is of great importance in the field of liquid metal. cooling of nuclear reactors. plasma confinement. electromagnetic casting, fluidization process. gas purification. sedimentation, pipe flow and transport process. In this paper the effect of inclined magnetic on the flow of a dusty fluid between two parallel plates having porous medium has been considered. After forming the governing equations of the flow under the assumed conditions and solving them using non-dimensional parameters, the expressions for the flow of fluid phase and dust phase have been found. Later on they have been used to draw the graphs between magnetic field and velocity of both phases .It has been found that velocity of dust phase decreases very rapidly while the velocity of fluid phase increases very slowly for everv set of values.

Keywords: Dusty, plates, porous, magnetic, phase.

1. Formulation of the Problem

Consider an unsteady laminar flow of a dusty, incompressible, electrically conducting, and viscous fluid through a channel filled porous medium of uniform cross section h, when one wall of the channel is fixed and the other is oscillating with time about a constant non-zero mean. Let x-axis be along the fluid flow at the fixed wall and y-axis perpendicular to it. An inclined magnetic field is applied to the flow occurring in y direction.

Assumptions: The governing equations are written based on the following assumptions:

(i) The dust particles are solid, spherical, non-conducting, and equal in size and uniformly distributed in the flow region.

- (ii) The density of dust particle is constant and a particle is uniform throughout the motion.
- (iii) The iteration between the particles, chemical reaction between the particles and liquid has not been considered to avoid multiple equations.
- (iv) The volume occupied by the particles per unit volume of the mixture, (i.e., volume fraction of dust particles) and mass concentration have been taken into consideration.
- (v) The dust concentration is so small and the continuity equation is satisfied.

Governing equations of the flow: The study of the dusty viscous fluid through an open channel has been carried out by many mathematicians; Attia¹ studied the flow between two infinite plates in the presence of magnetic field through porous medium. Then the study was further carried out by Sandeep N. (2013) under the effect of temperature variant plates one of which was oscillatinting. Further by keeping the temperature fixed, the effect of magnetic field on the fluid flow was studied when the channel was inclined at angle θ with *X*-axis. Modifying their equations suitable for the present flow, they are written as:

(1.1)
$$\frac{\partial u}{\partial t} = \left[-\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{\partial^2 u}{\partial y^2} \right] + \frac{1}{1 - \phi} \left[\frac{KN_0}{\rho} (v - u) - \frac{\mu}{K_1} u - \frac{KN_0 \sigma \mu_c^2 H_0^2}{\rho} u \sin^2 \theta \right],$$

(1.2)
$$\frac{\partial v}{\partial t} = \frac{\varphi}{N_0 m} \left[-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{K}{m} (v - u) \right]$$

The boundary conditions of the problem are:

$$t = 0, \ u(y, t) = v(y, t) = 0,$$

$$t > 0, \ u(y, t) = v(y, t) = 0,$$

$$u(y, t) = v(y, t) = 1 + \varepsilon e^{int},$$

where,

u(y, t) = velocity of the fluid particle, v(y, t) = velocity of the dust particle, m = mass of each dust particle, N_0 = Number density of dust particle,

 ϕ = Volume fraction of dust particle,

f = Mass concentration of dust particle,

 K_1 = Porous parameter,

K = Stake's Resistance Coefficient,

 σ = Electrical conductivity of the fluid,

 μ_c = Magnetic Permeability,

 H_0 = Magnetic field induction.

The problem is simplified by writing the equation in the following non dimensional. Here the characteristic length is taken to be h and the characteristic velocity is v.

(1.3)
$$\begin{cases} x^* = \frac{x}{h}, \ y^* = \frac{y}{h}, \ p^* = \frac{h^2 p}{\rho v^2}, \\ t^* = \frac{vt}{h^2}, \ u^* = \frac{uh}{v}, \ v^* = \frac{vh}{v}. \end{cases}$$

Substituting the above non dimensional parameters equation (1.3) in the governing equation (1.1 and (1.2)). and after removing asterisks it is found that,

(1.4)
$$\begin{cases} \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \varepsilon_1 v - \left[\varepsilon_1 + \varepsilon_2 M + \varepsilon_3\right] u\\ \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \varepsilon_1 v - \lambda u, \end{cases}$$

where $\lambda = \varepsilon_1 + \varepsilon_2 M + \varepsilon_3$

Equation (1.2) becomes

(1.5)
$$f\frac{\partial v}{\partial t} = \varphi \left[-\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} \right] + \beta \left(v - u \right),$$

where,

$$\varepsilon_1 = \frac{f}{\sigma_1(1-\varphi)}, \ \sigma_1 = \frac{mv}{Kh^2}$$

$$arepsilon_{2} = rac{1}{\left(1 - arphi
ight)}, \quad M' = \mu_{c}^{2} h^{2} H_{0}^{2} \; rac{\sigma}{\mu},$$

 $M = M' \sin^2 \theta$ (Magnetic parameter) $f = \frac{mN_0}{\rho}$ (Mass Concentration of dust particles),

 $\varepsilon_2 = \frac{\mu h^2}{K_1(1-\varphi)}$ (Porous Parameter), $\beta = \frac{f}{\sigma_1}$ (Concentration resistance ratio)

The corresponding non-dimensional boundary conditions are:

(1.6)
$$\begin{cases} t \le 0, \ u(y,t) = v(y,t) = 0, \ for \ 0 \le y \le 1, \\ t > 0, \ u(y,t) = v(y,t) = 0, \ at \ y = 0, \\ u(y,t) = v(y,t) = 1 + \varepsilon \ e^{int}, \ at \ y = 1. \end{cases}$$

2. Solution of the Problem

To solve the equations (1.4) to (1.5) the below equations introduced by Soundal Gekar and Bhat equations has been used. When $\varepsilon < < 1$ On solving equation (2.1), (2.2) and using these solution in to equation (1.4), (1.5)

(2.1)
$$u(y, t) = u_0(y) + \varepsilon u_1(y) e^{int},$$

(2.2)
$$v(y,t)=v_0'(y)+\varepsilon v_1(y)e^{int},$$

then becomes equation:

(2.3)
$$P = u_0^{"}(y) - \varepsilon u_1(y) \text{ in } e^{\text{int}} + \varepsilon u_1^{"}(y) e^{\text{int}} - \varepsilon_1 v_0(y) \varepsilon \varepsilon_1 v_1(y) e^{\text{int}} + (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) u_0(y) + (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) \varepsilon u_1(y) e^{\text{int}}.$$

(2.4)
$$u_0''(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3) u_0(y) + \varepsilon_1 v_0(y) = p,$$

(2.5)
$$u_1'(y) - (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3 + in) u_1(y) + \varepsilon_1 v_1(y) = 0.$$

From equation (1.5)

(2.6)
$$\beta v_0(y) = \varphi \Big[u_0'(y) - p \Big] + \beta u_0(y),$$

(2.7)
$$(\beta + nif)v_1(y) = \beta u_1(y) + \varphi u_1'(y).$$

The corresponding boundary conditions becomes

(2.8) $u_0(y) = u_1(y) = v_0(y) = v_1(y) = 0$, at y = 0,

(2.9)
$$u_0(y) = u_1(y) = v_0(y) = v_1(y) = 1$$
, at $y = 1$.

Substituting equation (2.6) in equation (2.4), we get from equation (2.6)

(2.10)
$$v_0(y) = u_0(y) + \frac{\varphi}{\beta} \left[u_0^{"}(y) - p \right]$$
$$\Rightarrow u_0^{"}(y) - A^2 u_0(y) = p.$$

Solving second order partial differential equations and using above boundary condition then obtained the solution of the differential equation:

(2.11)
$$u_0(y) = \frac{P}{A^2} \left(e^{Ay} \cos h \, Ay - 1 \right) + e^{A(y-1)} \frac{\sinh Ay}{\sin h \, A} \cdot \left(1 + \frac{P}{A^2} \right)$$
$$- \frac{P}{A^2} e^{Ay} \cos h \, A \frac{\sin h \, Ay}{\sin h \, A} \cdot$$

The first order partial deviations of $u_0(y)$ are

(2.12)
$$u_0'(y) = \frac{P}{A}e^{Ay}\left(\sin hAy + \cos hAy\right) - \frac{1}{e^{A(1-y)}}\left(1 + \frac{P}{A^2}\right)$$
$$\times \left(A\cos hAy - A(1-y)\right)\frac{\sin hAy}{\sin hA} + \frac{P}{A}e^{Ay}\left(\frac{\cos hAy}{\sin hA} - \cos A\frac{\sin hAy}{\sin hA}\right).$$

The second order partial differential equations (2.12) differentiate with respect to y

(2.13)
$$u_0''(y) = Pe^{Ay} (\sin h Ay + \cos h Ay) + Pe^{Ay} (\sin h Ay + \cos h Ay)$$

$$-\left(1+\frac{P}{A^{2}}\right)\frac{1}{e^{A(1-y)}}A^{2}\sin hAy.\frac{\sin hAy}{\sin hA} - \left(1+\frac{P}{A^{2}}\right)\frac{1}{e^{A(1-y)}}$$

$$\times \frac{\sin hAy}{\sin hA} + A^{2}\left(1-y\right)\left(1+\frac{P}{A^{2}}\right)\cos hAy\frac{\sin hAy}{\sin hA}e^{-A(1-y)}$$

$$-A^{2}\left(1-y\right)^{2}e^{-A(1-y)}\left(1+\frac{P}{A^{2}}\right)\frac{\sin hAy}{\sin hA} - \left(1+\frac{P}{A^{2}}\right)Ae^{-A(1-y)}$$

$$\times \frac{\sin hA\cos hAy}{\sin h^{2}A} - \left(1+\frac{P}{A^{2}}\right)e^{-A(1-y)} + Pe^{Ay}\frac{\cosh Ay}{\sin hA} + Pe^{Ay}\frac{\sin hAy}{\sin hA}$$

$$(2.14) + Pe^{Ay}\cos hA\frac{\cos hAy}{\sin hA} - Pe^{Ay}\cosh A\frac{\sin hAy}{\sin hA}$$

$$u_{0}^{'}(y) = Pe^{Ay}\left(\cos hAy + \sin hAy\right) - \left(1+\frac{P}{A^{2}}\right)\frac{1}{e^{A(1-y)}}A^{2}$$

$$\times \left(\frac{\sinh Ay}{\sin hA} + 1\right) - \left(1+\frac{P}{A^{2}}\right)A\cos hAy\left(1-\frac{1}{e^{A(1-y)}}\frac{1}{\sinh A}\right)$$

$$-A\left(1+\frac{P}{A^{2}}\right)\left(1-y\right)\left(1-\frac{1}{e^{A(1-y)}}\frac{\sinh Ay}{\sin hA}\right) + P\left(e^{Ay}\frac{\sinh Ay}{\sinh A}\right)$$

$$+ \frac{\cosh A\cos hAy}{\sinh A}\right).$$

Substituting equation (2.11) and (2.14) in equation (2.6) and obtain the equation,

(2.15)
$$v_{0}(y) = \frac{P}{A^{2}}e^{Ay}\cosh Ay \left(1 - \frac{\sinh Ay}{\sin hA}\right) + \frac{P}{A^{2}}e^{A(y-1)}$$
$$\times \frac{\sin hAy}{\sin hA} + \frac{\varphi}{\beta} \left[\left[Pe^{Ay} \left(\cosh Ay + \sin hAy \right) \right] - e^{-A(1-y)} \left(A^{2} + P\right) \right]$$
$$\times \frac{\sin hAy}{\sin hA} - e^{-A(1-y)} \left(P + A^{2}\right) + e^{-A(1-y)} \left(A + \frac{P}{A}\right) \frac{\cosh Ay}{\sin hA}$$
$$- \left(A + \frac{P}{A}\cosh Ay\right) - \left(1 - y\right) \left(A + \frac{P}{A}\right) + \left(1 - y\right)e^{-A(1-y)} \left(A + \frac{P}{A}\right)$$

$$\times \frac{\sin h \, Ay}{\sin hA} + P \Big(e^{Ay} + \cos hA \Big) \frac{\sin hAy}{\sin hA} \, .$$

On solving equation (2.5) and (2.6), we get

$$u_{\cdot}^{"}(y) - (\varepsilon_{1} + \varepsilon_{2}M + \varepsilon_{3} + in)u_{1}(y) + \varepsilon_{1}v_{1}(y) = 0,$$

$$(\beta + nif)v_{1}(y) = \beta u_{1}(y) + \varphi \left[u_{\cdot}^{"}(y)\right].$$

Solving above equation and obtain the equation

(2.16)
$$v_1(y) = \frac{\beta u_1(y)}{\beta + ni} + \frac{\varphi \lfloor u_1(y) \rfloor}{\beta + nif}$$

or

$$u_1''(y) - B^2 u_1(y) = 0,$$

where $B^2 = \frac{\varepsilon_1(1-\beta) + \varepsilon_2 M + \varepsilon_3 + in}{(\beta + nif) + \varepsilon_1(\varphi)}$.

Solve ordinary differential equation (2.16)

(2.17)
$$u_1(y) = e^{By} (C_1 \cos hBy + C_2 \sin hBy)$$

Applying above boundary condition, and obtain the value

$$C_1 = 0$$
, $C_2 = \frac{1}{e^B \sin h B}$

Putting the value of C_1 and C_2 in equation (2.17)

(2.18)
$$u_1(y) = e^{By(y-1)} \frac{\sin h By}{\sin h B}$$
.

Differentiate Equation (2.18) with respect to y, we get

(2.19)
$$u_1'(y) = -e^{B(y-1)}B\frac{\cos h By}{\sin hB} + B(y-1)e^{B(y-1)}\frac{\sin h By}{\sin hB},$$

Again differentiating eq. (2.19) with respect to y

(2.20)
$$u_{1}^{"}(y) = -B^{2}e^{B(y-1)}\frac{\sin hBy}{\sin hB} - B^{2}(y-1)e^{B(y-1)}\frac{\cos hBy}{\sin hB}$$
$$-B^{2}(y-1)e^{B(y-1)}\frac{\cos hBy}{\sin hB} + \frac{\sin hBy}{\sin hB}B^{2}(y-1)^{2}e^{B(y-1)}$$
$$+Be^{B(y-1)}\frac{\sin hBy}{\sin hB}.$$

Substituting equation (2.18) and (2.20) in equation (2.7), we get

(2.21)
$$v_1(y) = B_0 e^{B(y-1)} \beta \left[\frac{\sin h By}{\sin h B} + \frac{B^2 \varphi}{\beta} \left\{ \left(y^2 - 2y - 1 \right) \frac{\sin h By}{\sin h B} -2 \left(y - 1 \right) \frac{\cosh h By}{\sin h B} \right\},$$

where
$$B_0 = \frac{(\beta - nif)}{\beta^2 + n^2 f^2}$$
.
(2.22) $v_1(y) = B_0 e^{B(y-1)} \beta \left[\frac{\sin h By}{\sin h B} + \frac{B^2 \varphi}{\beta} \left\{ (y^2 - 2y - 1) \frac{\sin h By}{\sin h B} -2(y-1) \frac{\cosh h By}{\sin h B} \right\}$.

Substituting the equation (2.11) and (2.18) in equations (2.2), it is obtain,

(2.23)
$$u(y,t) = \frac{P}{A^2} \left(e^{Ay} \cosh Ay - 1 \right) + e^{A(y-1)} \frac{\sinh Ay}{\sinh A} + e^{A(y-1)} \frac{\sinh Ay}{\sinh A}$$
$$-e^{Ay} \frac{P}{A^2} \cosh A \frac{\sinh AY}{\sinh A} + \varepsilon e^{B(y-1)} e^{int} \frac{\sinh By}{\sinh B}.$$

Substituting equation (2.15) and (2.22) in equation (2.3) and it is obtain

(2.24)
$$v(y,t) = \frac{P}{A^2} e^{Ay} \cos h Ay \left(1 - \frac{\sin h Ay}{\sin hA}\right) + \frac{P}{A^2} e^{A(y-1)} \frac{\sin h Ay}{\sin hA}$$
$$+ \frac{\varphi}{\beta} \left[P e^{Ay} \left(\cos h Ay + \sin Ay\right) \right] - e^{A(y-1)} \left(A^2 + P\right) \frac{\sin h Ay}{\sin hA}$$

$$-e^{A(y-1)}\left(P+A^{2}\right)+e^{A(y-1)}\left(A+\frac{P}{A}\right)\frac{\cos hAy}{\sin hA}-\left(A+\frac{P}{A}\right)\cosh Ay$$
$$-\left(1-y\right)\left(A+\frac{P}{A}\right)+\left(1-y\right)e^{A(y-1)}\left(A+\frac{P}{A}\right)\frac{\sinh Ay}{\sinh A}$$
$$+P\left(e^{Ay}+\cosh A\right)\frac{\sinh Ay}{\sinh A}+\varepsilon\left[B_{0}e^{B(y-1)}\beta\left\{\frac{\sin hBy}{\sin hB}\right.$$
$$+\frac{\varphi}{\beta}B^{2}\left(y^{2}-2y-1\right)\frac{\sin hBy}{\sin hB}-2\left(y-1\right)\frac{\cos hBy}{\sin hB}\right\}\right]e^{\operatorname{int}}.$$

Hence the equations (2.23) and (2.24) represent velocity of the fluid, and velocity of the dust particle. Now choosing suitable values of the parameters and draw the graphs.

3. Result and Discussion

Table1. Effect of Magnetic field on Velocity of fluid phase and Velocity of dust phase at distance from initial point (y) = 0.01.

 $(\varepsilon_1 = 0.201, \varepsilon_2 = 1.005, \varepsilon_3 = 1, \beta = 2, \varphi = 0.005, A = 0.9997, 1.4156, 1.7344, ..., y = 0.01, P = 1)$

Sr.No.	Magnetic field(M)	Velocity of fluid Particle(U)	Velocity of dust particle(V)
1	0	0.00499768	0.992939
2	1	0.00542017	0.48785
3	2	0.00634819	0.31609
4	3	0.00717712	0.227999
5	4	0.00795143	0.17331
6	5	0.00868178	0.135311
7	6	0.0093738	0.106851
8	7	0.01003172	0.084367
9	8	0.01065923	0.06589

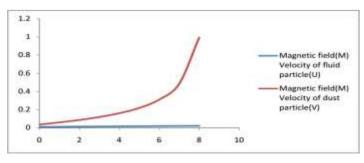


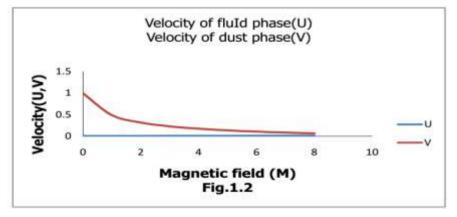
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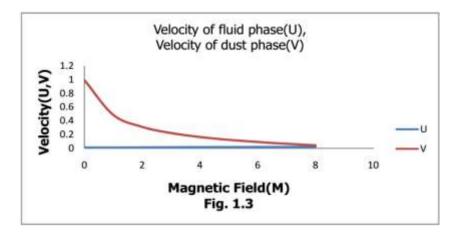
Table 2. Magnetic field (M), Velocity of fluid phase (U) and Velocity of dust phase (V) $(\varepsilon_1 = 0.201, \varepsilon_2 = 1.005, \varepsilon_3 = 1, \beta = 2, \varphi = 0.005, A = 0.9997, 1.4156, 1.7344, ..., y = 0.005, P = 1)$

Table 3. Magnetic field (M), Velocity of fluid phase (U) and Velocity of dust phase (V) $(\varepsilon_1 = 0.201, \varepsilon_2 = 1.005, \varepsilon_3 = 1, \beta = 2, \varphi = 0.005, A = 0.9997, 1.4156, 1.7344, ..., y = 0.009, P = 1)$

Sr.No.	Magnetic field(M)	Velocity of fluid particle(U)	Velocity of dust particle(V)
1	0	0.011016	0.991191
2	1	0.011941	0.483767
3	2	0.013979	0.308461
4	3	0.0158	0.21658
5	4	0.017501	0.157739
6	5	0.019107	0.115176
7	6	0.020628	0.081712
8	7	0.022075	0.053755
9	8	0.023455	0.029313



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All the graphs are showing the same trend regarding velocity of fluid phase (U) and velocity of dust phase (V) with respect to magnetic field (M). This means that the magnetic field is a very dominating factor in the motion. It is evident that these expressions are containing hyperbolic terms which are more effective in V than in U. The magnetic field is more effective on particle phase as is expected by its physical nature. The inclination of channel is more effective on particle phase rather than fluid phase as the magnetic field weakens at higher values. The increasing value of y (distance from the initial point) decreases the nature of curvature of the graphs which is due to the combined effect of inclination and magnetic field. This can be explained of the basis of strength of applied magnetic field together with the effect of other factor. As field is more effective on dust phase, therefore they can be easily controlled by it. But the field is less effective on fluid phase, therefore the velocity gains a very little increment. Thus the model so evolved is justified on the physical nature.

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