

Casson Fluid Flow with Magnetic Field Effects over a Non-Linear Stretching Sheet

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Abstract: Boundary layer flow of an MHD non-Newtonian Casson fluid and the transfer of heat and mass due to a non-linearly stretching sheet in the presence of heat source/sink and chemical reaction influences are considered. Appropriate similarity transformations are employed to convert the governing partial differential equations to a set of highly non-linear ordinary differential equations and solved numerically by using fourth order Runge Kutta method along with shooting technique. In the current work, the impacts of the governing non-dimensional parameters on the velocity, temperature and concentration profiles have been talked about and presented graphically. The profiles of temperature and concentration are found rising to a higher value when the Casson parameter raises but the opposite is true for velocity distribution. Also, the velocity profile decreases at a fixed point with an increase of the non-linear stretching parameter, but the temperature profile is enhanced when the non-linear stretching parameter rises. In addition, to check the validity of the method the coefficient of skin friction and temperature gradient has been compared with the previously published work and it is found in a good agreement.

Keywords: MHD, non-linear stretching, Casson fluid, heat source, \sink.

1. Introduction

The behavior of non-Newtonian fluid flow over a stretching sheet on a boundary layer have been investigated by several researchers over the past few decades because of its great importance in the industrial and technological applications. Some of its application in the industrial process is like drilling muds, manufacturing coated sheets, polymer sheet extruded

unendingly from a die, spinning of fibers, etc. The rate of cooling or heating at the non-linear stretching sheet has an expectant significant to produce the quality of the final product. The Casson fluid is a class of non-Newtonian fluid model. Although, this fluid can be assumed as a fluid shear thinning. Since at zero shear rate, there is an infinite viscosity no flow occurs, but it reveals the zero viscosity at an infinite shear rate. Such type of fluid model can be conceived for the flow of tomato sauce, honey, different kind concentrated soups, jelly, etc. For certain cases to analysis the human blood flow, it is addressed with a Casson fluid model. A lot of studies has been examined on the boundary layer flow of non-Newtonian fluid both theoretically and experimentally.

The first person who examined a problem on a stretching sheet was Crane¹ by considering the flow of a fluid over a linearly stretched surface. Bhattacharyya² examined a flow on a boundary layer and the transfer of heat over an exponentially shrinking sheet. Soon after, a Casson fluid flow problem with MHD over an exponentially shrinking sheet has been solved analytically using the Adomian Decomposition Method (ADM) by Nadeem et al.³. Pramanik⁴ investigated a non-Newtonian fluid flow on a boundary layer conveyed by heat transfer due to an exponentially stretching surface in the presence of Suction or blowing at the surface. He found that an increase of the suction parameter enhances the coefficient of skin friction. Furthermore, Bala and Reddy⁵ studied an MHD convective, two-dimensional steady, a Casson fluid flow on a boundary layer over an exponentially inclined permeable stretching surface. MHD flow of a Casson fluid model with inconsistent viscosity on a stretching surface with changeable thickness, Cattaneo-Christov heat flux model is employed instead of Fourier's law to discover the characteristics of heat transmit has been studied numerically using Keller box method by Malik et al.⁶. Alternatively, Bhattacharyya et al.⁷ studied an MHD Casson fluid flow on a boundary layer on a stretching/shrinking permeable sheet in the occurrence of wall mass transfer analytically. Kameswaran et al.⁸ investigated a non-Newtonian incompressible Casson fluid flow on a stagnation point over a stretching plate in the existence of Soret and Dufour effects.

A non-Newtonian Casson fluid flow in a boundary layer followed by the transmit of heat and mass towards a porous exponentially stretching sheet amid by velocity and thermal slip state in the attendance of thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction effects was investigated by Saidulu and Lakshimi⁹. They obtained that the velocity distribution decreases with an increase of Casson

parameter, but the reverse process happens for concentration profile and temperature profile. Shehzad et al.¹⁰ examined mass transfer effects in the MHD flow of a Casson fluid over a porous stretching sheet in the incidence of a chemical reaction. They found that both Hartman number and Casson parameter have the same impacts on the velocity in a qualitative sense. Recently, Jakati et al.¹¹ solved an MHD non-Newtonian Casson nanofluid flow over a stretching sheet using HAM.

All the above studies are concentrated on a linear stretching surface. Thus, the studies on a non-linearly stretching surface are explained as follows. Alinejad and Samarbakhsh¹² investigated numerically the flow and the properties of heat transport of incompressible viscous flow over a nonlinearly stretching sheet with the attendance of viscous dissipation. Sumalatha and Bandari¹³ studied a flow over a non-linearly stretching surface of Casson fluid with an impact of heat source/sink and radiation. More recently, a flow on a boundary layer and heat transport of a Casson fluid over a non-linearly permeable stretching surface with the effect of chemical reaction in the occurrence of a variable magnetic field was investigated by Reza et al.¹⁴. In their study, thermal radiation effects were considered so as to control the rate of heat transfer at the surface. Furthermore, the following researchers did their study on a non-linearly stretching surface¹⁵⁻¹⁹.

Finally, the purpose of this work is to analyze the impacts of a magnetic field and heat source/sink of Casson fluid flow over a nonlinearly stretching sheet. The governing PDEs are changed into nonlinear DEs by using suitable similarity transformations and those equations are then solved by Runge Kutta fourth order method.

2. Mathematical Formulations

In this work, an MHD incompressible viscous, steady, laminar, two-dimensional flow of a Casson fluid bounded by a non-linearly stretching sheet at $y=0$ is considered. The flow is detained to $y > 0$ and it is directed in the x -axis and normal to the y -axis. The equation of rheological state for an anisotropic and incompressible flow of the Casson fluid is given by

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_y / \sqrt{2\pi})e_{ij}, & \pi > \pi_c, \\ 2(\mu_B + P_y / \sqrt{2\pi_c})e_{ij}, & \pi < \pi_c, \end{cases}$$

where μ_B and P_y are the plastic dynamic viscosity, yield stress, respectively. Correspondingly, π is the product of the constituent of deformation rate with itself, $\pi = e_{ij} e_{ij}$, e_{ij} is the $(i, j)^{\text{th}}$ component of the deformation rate and π_c is a critically value of this product based on the non-Newtonian model.

The equations of continuity, momentum, energy, and concentration governing such types of flow are given as

$$(2.1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$(2.2) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = V \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u,$$

$$(2.3) \quad u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q^*(x)}{\rho c_p} (T - T_\infty),$$

$$(2.4) \quad u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty),$$

where u and v are the velocity components in the x and y directions, respectively, V is the kinematic viscosity, ρ is the fluid density, $\beta = \mu_B \sqrt{2\pi_c} / P_y$ is the parameter of the Casson fluid, and k is the thermal conductivity of the fluid, D is the mass diffusivity coefficient and k_0 is the coefficient rate of chemical reaction.

The appropriate boundary conditions for the problem are given by

$$(2.5) \quad u = U = c x^n, v = 0, T = T_\omega, C = C_\omega \text{ at } y = 0,$$

$$(2.6) \quad u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty.$$

Here, $c(c > 0)$ is a parameter related to the surface stretching speed, T_ω is the uniform temperature at the sheet, C_ω is the uniform concentration at the sheet, T_∞ and C_∞ are the free stream temperature and concentration, respectively, and n is the power index related to the surface stretching speed.

Introducing the following similarity transformations

$$(2.7) \quad \begin{cases} \eta = y \sqrt{\frac{c(n+1)}{2V}} x^{\frac{n-1}{2}}, \\ v = -\sqrt{cV(\frac{n+1}{2})} x^{\frac{n-1}{2}} \left[f + \frac{n-1}{n+1} \eta f' \right], \\ u = cx^n f', \quad \frac{T-T_\infty}{T_\omega-T_\infty} = \theta, \quad \frac{C-C_\infty}{C_\omega-C_\infty} = \phi \end{cases}$$

Substituting Eq. (2.6) into Eqs. (2.2)-(2.4), we obtain the reduced governing equations

$$(2.8) \quad \left(1 + \frac{1}{\beta}\right) f''' + ff'' - \frac{2n}{n+1} f'^2 - \frac{2}{n+1} Mf' = 0,$$

$$(2.9) \quad \theta'' + Pr \left(f\theta' + \frac{2}{n+1} \delta\theta \right) = 0,$$

$$(2.10) \quad \phi'' + Sc \left(f\phi' - \frac{2}{n+1} k_1\phi \right) = 0.$$

The boundary conditions take the following form:

$$(2.11) \quad f' = 1, f = 0, \theta = 1, \phi = 1, \text{ at } \eta = 0,$$

$$(2.12) \quad f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty,$$

where the prime indicates the differentiation with respect to η and $Pr = \frac{V}{k}$ is

the Prandtl number, $\delta = \frac{Q_0 x}{U \rho c_p}$ is the heat source/sink parameter,

$M = \frac{\sigma B_0^2}{\rho c x^{n-1}}$ is the magnetic field parameter, $Sc = \frac{V}{D}$ is the Schmidt number,

and $k_1 = \frac{k_0 x}{U}$ is the chemical reaction parameter.

3. Numerical Solution

The transformed non-linear ordinary differential equations (2.8)-(2.10) together with boundary conditions (2.11) and (2.12) are solved by Runge-

Kutta fourth order method along with shooting technique. Applying this method on the given problem the following hypothesis are made

$$\begin{cases} f \rightarrow f(1), f' \rightarrow f(2), f'' \rightarrow f(3), \theta \rightarrow f(4), \\ \theta' \rightarrow f(5), \phi \rightarrow f(6), \phi' \rightarrow f(7). \end{cases}$$

And rewriting equations (2.8)-(2.10) with their boundary conditions in the form of

$$(3.1) \quad f''' = \left(-ff'' + \frac{2n}{n+1} f'^2 + \frac{2}{n+1} Mf' \right) / \left(1 + \frac{1}{\beta} \right),$$

$$(3.2) \quad \theta'' = -Pr \left(f\theta' + \frac{2}{n+1} \delta\theta \right),$$

$$(3.3) \quad \phi'' = -Sc \left(f\phi' - \frac{2}{n+1} k_1\phi \right).$$

The appropriate boundary conditions are:

$$(3.4) \quad \begin{cases} f_a(1) = 0, f_a(2) = 1, f_a(4) = 1, f_a(6) = 1, \\ f_b(2) = 0, f_b(4) = 0, f_b(6) = 0. \end{cases}$$

The initial and boundary condition points are denoted by a and b i.e. $a=0, b=\infty$. We assumed the step size $\Delta\eta=0.01$ and the accuracy convergence criteria to be a five decimal value before solving the problem by the explained method using Matlab program.

4. Results and Discussion

The effect of all the parameters involved in the governing equations for the non-linear stretching parameter on a Casson fluid flow has been discussed in this work. The Runge Kutta fourth order method along with shooting technique is applied to solve the ordinary differential equations. The validity of this method is checked by comparing the coefficient of skin friction and temperature gradient for a viscous Newtonian fluid with the previously published papers as shown in Table-1 and Table-2, and the result is found in an excellent agreement.

Table 1. The evaluation of skin friction coefficient $f''(0)$ for $M = 0$

n	$f''(0)$			
n	Present result	Vajravelu [20]	Cortel [21]	Swati [15]
1	-1.000063	-1.0000	-1.0000	-1.0000
5	-1.194541	-1.1945	-	-1.1944
10	-1.234926	-1.2348	-1.234875	-1.2348

Table 2. Temperature gradient comparison $\theta'(0)$ when $M = \delta = 0$

n	$\theta'(0)$		
	Pr = 7		
	Present Results	Vajravelu [20]	Swati [15]
1	-1.895402	-1.8953	-1.8953
5	-1.861581	-1.8610	-1.8611
10	-1.854637	-1.8541	-1.8541

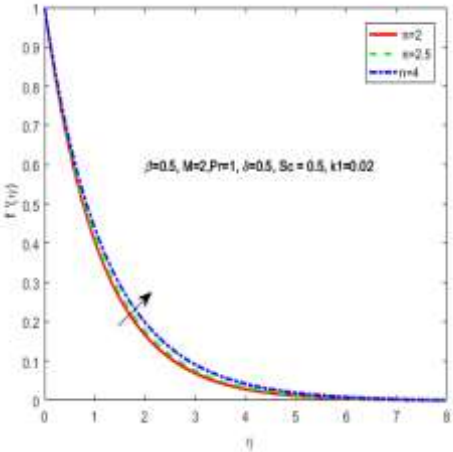


Figure 1.(a) Velocity profile with variation of n

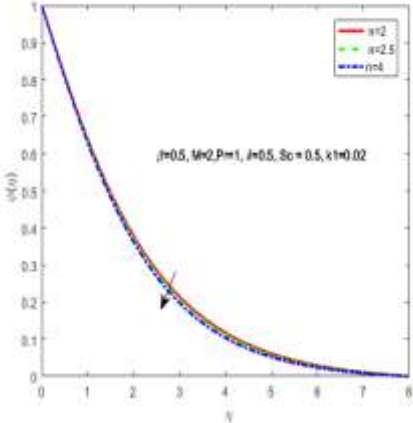


Figure 1.(c) Concentration profile with variation of n .

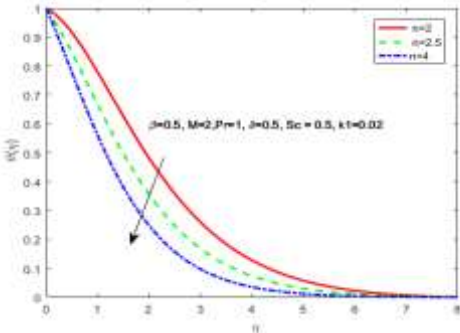


Figure 1.(b) Temperature profile with variation of n

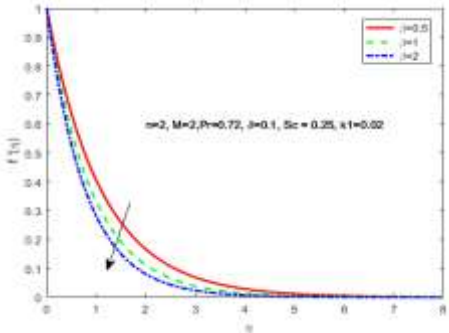


Figure 2.(a)Velocity profile with variation of β

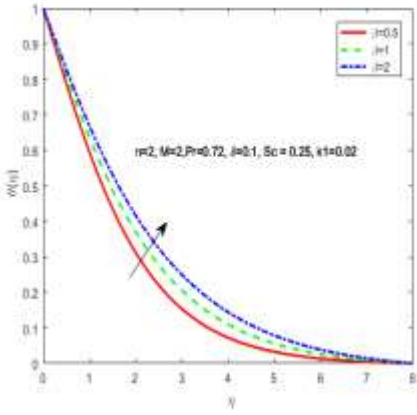


Figure 2.(b) Temperature profile with variation of β

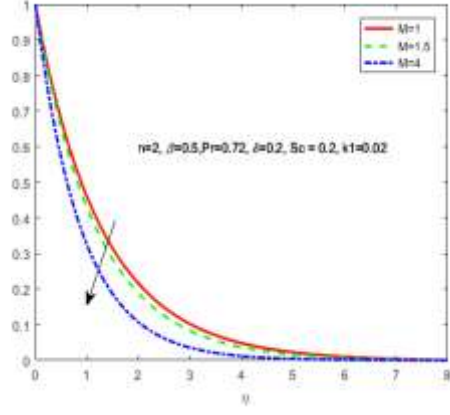


Figure 3.(a) Velocity profile with variation of M

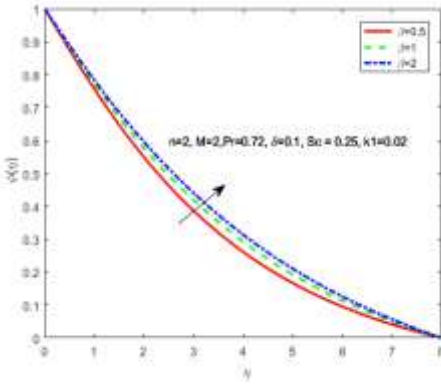


Figure 2.(c) Concentration profile with variation of β

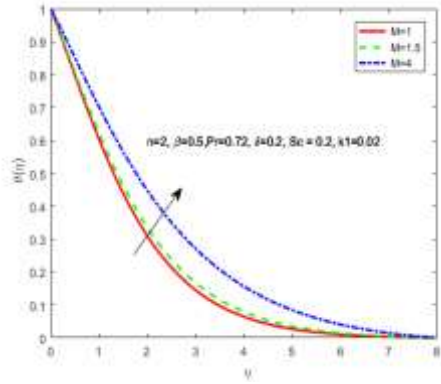


Figure 3.(b) Temperature profile with variation of M

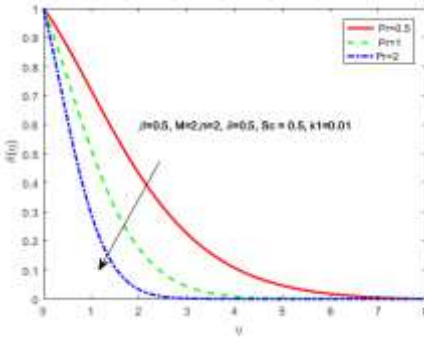


Figure 4. Temperature profile with variation of Pr

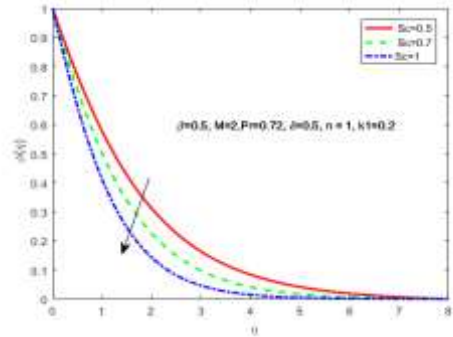


Figure 6. Concentration profile with variation of Sc

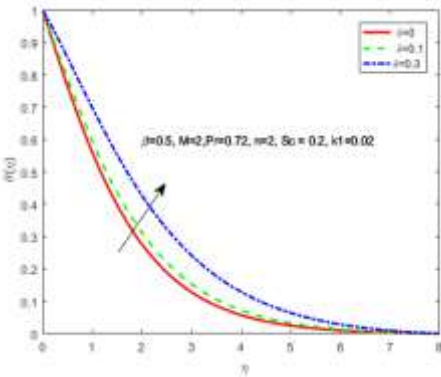


Figure 5. Temperature profile with variation of δ

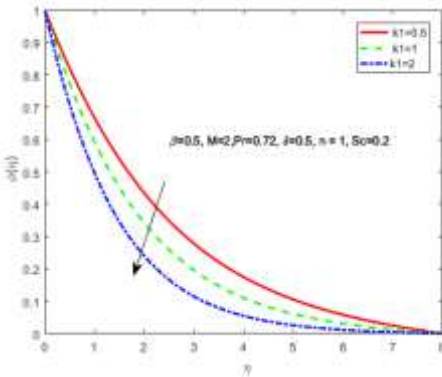


Figure 7. Concentration profile with variation of k_1

The results obtained are explained through the following figures. Figs. 1(a), 1(b) and 1(c) exhibit the effects of the non-linear stretching parameter n for the velocity profile, temperature and concentration. As shown in the figure, it is found that an increase n enhances the velocity profile, but the profiles of temperature, and concentration are reduced in the presence of a magnetic field parameter. Figs. 2(a), 2(b) and 2(c) display the influence of the Casson fluid parameter β for the velocity, temperature, and concentration. The results obtained have shown that for a non-linear stretching parameter the temperature and concentration profiles increase, while the velocity field decreases with an increase of the Casson fluid parameter β . This is due to the fact that an increase of β results a rise in the viscosity of a fluid, i.e., the yield stress will be reduced. As a result, the boundary layer thickness of momentum will be reduced. Figs. 3(a) and 3(b) demonstrate a magnetic field parameter effect on the velocity and temperature. The result described that the velocity profile decreases with an increase of M . Since the presence of M creates a Lorentz force that retards the motion of the fluid. This condition raises the thermal conductivity of the fluid and the momentum boundary layer thickness reduces.

The influence of Prandtl number Pr on the temperature profile is displayed using Fig. 4. Thus, the distribution of temperature is reduced due to an increase of Pr . Since Pr has an inverse relationship with the thermal conductivity of the fluid and the thickness of thermal boundary layer also decreases. The impact of heat source/sink parameter δ on the thermal boundary layer thickness is demonstrated through Fig. 5. The result described that the thickness of the thermal boundary layer increases with an increase of δ . Physically, an increase in the heat source parameter can add

more heat to the stretching sheet which increases its temperature. The influence of Schmidt number Sc and chemical reaction k_1 for $n=1$ on the concentration profile is displayed by using Figs. 6 and 7. The result has shown that the concentration profile decreases in both cases. In the case of Schmidt number, an increase of Sc signifies that there is a decrease of mass diffusivity of a fluid that results in the fluid concentration to be reduced.

5. Conclusions

The numerical solution of a steady, laminar two-dimensional boundary layer flow of a Casson fluid and the characteristics of heat and mass transfer over a nonlinearly stretching sheet is obtained. Hence, based on the numerical results obtained, the following conclusions are drawn.

- (i) An increase of the non-linear stretching parameter enhances the velocity at a fixed point, while the profiles of temperature and concentration reduces.
- (ii) Velocity profiles reduces with an increase of magnetic field parameter.
- (iii) The thermal boundary layer thickness decreases with an increase of heat source parameter and decreases with an increase of Prandtl number.
- (iv) Concentration profile reduces with an increment of both Schmidt number and chemical reaction parameter.

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