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Quaternionic Formulation in Symmetry Breaking Mechanism

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Abstract: In this formalism the covariant derivative contains the four potentials associated with four charges and thus leads the different gauge strength for the particles containing electric, magnetic, gravitational and Heavisidian charges. Quaternions representation in spontaneously symmetry of breaking and Higg's mechanics and the equation of motion are derived for free particles (i.e. electric, magnetic, gravitational and Heavisidian charges). The quaternionic gauge theory of quantum electrodynamics has also been developed in presence of electric, magnetic, gravitational and Heavisidian charges.
Keywords: dyons, quaternions, symmetry breaking, gauge fields.

1. Introduction

The asymmetry between electricity and magnetism became very clear at the end of 19th century with the formulation of Maxwell's equations. Magnetic monopoles were advocated[1, 2] to symmetrize theseequations in a manifest way that the existence of an isolated magnetic charge implies the quantization of electric charge and accordingly the considerable literature[3-14] has come in force. The fresh interests in this subject have been enhanced by’t Hooft[15] and Polyakov[16] with the idea that the classicalsolutions having the properties of magnetic monopoles may be found in Yang-Mill's gauge theories. Now it has become clear that monopoles are better understood in grand unified theories and super-symmetric gauge theories. Julia and Zee[17] extended the’t Hooft-Polyakov theory[15, 16] of monopolesand constructed the theory of non-Abelian dyons (particles carrying simultaneously electric and magnetic charges). The quantum mechanical excitation of fundamental monopoles includes dyons which are automatically arisen[18] from the semi-

classical quantization of global charge rotation degree offreedom of monopoles. In view of the explanation of CP-violation in terms of non-zero vacuumangle of world[19], the monopoles are necessary dyons and Dirac quantization condition permits dyons tohave analogous electric charge. Renewed interests in the subject of monopole has gathered enormous potential importance in connection of quark confinement problem[20] in quantum chromodynamics,possible magnetic condensation[21,22] of vacuum leading to absolute color confinement in QCD, itsrole as catalyst in proton decay[23,24], CP-violation[19], current grand unified theories[25] and super-symmetric gauge theories[26-29]. There has been a revival in the formulation of natural laws withinthe frame work of general quaternion algebra and basic physical equations. Quaternions[30] were very first example of hyper complex numbers having the significant impacts on Mathematics and Physics.
Moreover, quaternions are already used in the context of special relativity[31], electrodynamics[32,3] Maxwell's equation[34], quantum mechanics[35,36], gauge theories[37,38], supersymmetry39,40 and other branches of Physics[41] and Mathematics[42]. Symmetry plays the central role in determining its dynamical structure.

The Lagrangian exhibits invariance under gauge transformations for the electroweak interactions. Since the imposition of local symmetry implies the existence of mass less vector particles[43], Higg's mechanism is used for the spontaneous breaking of gauge sym metry to generate masses for the weak gauge bosons charged as well as neutral particle[44]. If thesefeatures of the gauge theory are avoided, we obtain massive vector bosons and hence the gauge symmetry must be broken. In the Higg's mechanism a larger symmetry is spontaneously broken into a smaller symmetry through the vacuum expectation value of the Higg's field and accordingly gauge bosons become massive. The simplest way of introducing spontaneous symmetry breakdown is to include scalar Higg's fields by hand into the Lagrangian[45]. Recently, we have made an attemptto develop the quaternionic formulation of Yang-Mill's field equations and Octonion reformulation of quantum chromodynamics (QCD) by taking magnetic monopole into account[46,47]. The quaternion gauge theory of spontaneously symmetry breaking mechanism already developed by others[48-52] in terms of gauge groups and methodology adopted by them in different manners. Starting with the definition of quaternion gauge theory, we have undertaken the study of $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$ in terms of the simultaneous existence of electric and magnetic charges along with their Yang-Mill's counterparts[53-57].

As such, we have developed the gauge theory in terms of four coupling constants associated with four-gauge symmetry $SU(2)_e \times SU(2)_m \times U(1)_e \times U(1)_m$.

Accordingly, we have made an attempt to obtain the Abelian and non-Abelian gauge structures for the particles carrying simultaneously the electric and magnetic charges (namely dyons). In this paper the covariant derivative contains the four-potentials associated with these four charges and thus leads the different gauge strength for the particles containing electric, magnetic, gravitational and Heavisidian charges. Quaternion's representation in spontaneously symmetry of breaking and Higg's mechanics and the equation of motion are derived for free particles (i.e. electric, magnetic, gravitational and Heavisidian charges). We have extended the local gauge invariance in order to explain spontaneous symmetry breaking mechanism. The quaternionic gauge theory of quantum electrodynamics has also been developed in presence of electric, magnetic, gravitational and Heavisidian charges.

2. Spontaneous Symmetry Breaking in the Form of Quaternions

The Lagrangian of a complex scalar field, which carries a scalar electric charge (e), and magnetic charge (g), gravitational (m) and Heavisidian (h) must be gauged with respect to both the vector and pseudo vector potentials $(A_\mu, B_\mu, C_\mu, D_\mu)$ is[47,]

(2.1) $$L = \left(\overline{D_\mu \phi}\right)\left(D_\mu \phi\right) - V(\phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} M_{\mu\nu} M^{\mu\nu}$$
$$\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} N_{\mu\nu} N^{\mu\nu}.$$

Then the Lagrangian for unified charges (electric, magnetic, gravitational, Heavisidian) of the scalar field is[47],

(2.2) $$L_s = \left(\partial_\mu + ieA_\mu + igB_\mu + imC_\mu + ihD_\mu\right)$$
$$\times \bar{\phi} \left(\partial_\mu + ieA_\mu + igB_\mu + imC_\mu + ihD_\mu\right)\phi$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} M_{\mu\nu} M^{\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} N_{\mu\nu} N^{\mu\nu},$$

where $\bar{\phi}$ is the complex conjugate of ϕ. The electric, magnetic, gravitational and Heavisidian four currents $J_{\mu}^{(e)}$, $J_{\mu}^{(m)}$, $J_{\mu}^{(G)}$, $J_{\mu}^{(H)}$ of scalar field can be written in terms as,

$$(2.3)\qquad \begin{cases} J_{\mu}^{(e)} = ie\left[\bar{\phi}\left(D_{\mu}\phi\right)-\phi\left(\overline{D_{\mu}\phi}\right)\right], \\ J_{\mu}^{(m)} = ig\left[\bar{\phi}\left(D_{\mu}\phi\right)-\phi\left(\overline{D_{\mu}\phi}\right)\right], \\ J_{\mu}^{(G)} = im\left[\bar{\phi}\left(D_{\mu}\phi\right)-\phi\left(\overline{D_{\mu}\phi}\right)\right], \\ J_{\mu}^{(H)} = ih\left[\bar{\phi}\left(D_{\mu}\phi\right)-\phi\left(\overline{D_{\mu}\phi}\right)\right]. \end{cases}$$

Since e, g, m, h are scalar quantities, then the potential terms described as[47],

$$(2.4)\qquad V(\phi)^2 = m^2\left(\bar{\phi}\,\phi\right)+\lambda\left(\bar{\phi}\,\phi\right)^2,$$

where m^2 and are real constant parameters and should be positive to ensure the stable vacuum. If the potential energy in the vacuum state of minimum energy can be found by minimizing potential $V(\phi)$. Then for the vacuum state,

$$(2.5)\qquad \frac{dV}{d\phi}=0\Rightarrow\frac{dV}{d\bar{\phi}}=0\Rightarrow m^2\bar{\phi}+2\lambda\left(\bar{\phi}\,\phi\right)\bar{\phi}=0,$$

where

$$(2.6)\qquad \phi=\pm\frac{v}{\sqrt{2}}=\sqrt{\frac{-m^2}{2\lambda}}\,.$$

Substituting the value of $V(\phi)^2$ in the equation (2.1), then equation (2.1) can be written as,

$$(2.7)\qquad L_s=\left(\partial_{\mu}+ieA_{\mu}+igB_{\mu}+imC_{\mu}+ihD_{\mu}\right)$$
$$\times\bar{\phi}\left(\partial_{\mu}+ieA_{\mu}+igB_{\mu}+imC_{\mu}+ihD_{\mu}\right)\phi-\frac{1}{4}M_{\mu\nu}M^{\mu\nu}$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}-\frac{1}{4}N_{\mu\nu}N^{\mu\nu}-m^2\left(\bar{\phi}\phi\right)-\lambda\left(\bar{\phi}\phi\right)^2,$$

The self interaction coupling constant λ is taken to be positive definite and for $m^2 \geq 0$ the potential acquires a vacuum expectation value of

(2.8) $$\phi(x)=\frac{1}{\sqrt{2}}\left(v+\eta(x)+i\xi(x)\right)=\frac{1}{\sqrt{2}}\left(v+\eta(x)\right)e^{i\xi(x)/\sqrt{2}}.$$

One can transform the $\xi(x)$ field away by making the gauge transformation under the following conditions,

(2.9) $$\phi'(x)=\phi(x)=\frac{1}{\sqrt{2}}\left(v+\eta(x)\right)$$

and the potentials has the following form

(2.10) $$\begin{cases} A'_\mu(x)=A_\mu(x)-\frac{1}{2ev}\partial_\mu\kappa(x), \\ B'_\mu(x)=B_\mu(x)-\frac{1}{2gv}\partial_\mu\kappa(x), \\ C'_\mu(x)=C_\mu(x)-\frac{1}{2mv}\partial_\mu\kappa(x), \\ D'_\mu(x)=D_\mu(x)-\frac{1}{2hv}\partial_\mu\kappa(x). \end{cases}$$

The Lagrangian L_s is invariant under the above transformations, substituting these unitary gauge transformations in the equation (2.7), the Lagrangian becomes [47]

(2.11) $$L_s=\frac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right)+\frac{1}{2}\left(2m^2\right)\eta^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}M_{\mu\nu}M^{\mu\nu}$$

$$-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}-\frac{1}{4}N_{\mu\nu}N^{\mu\nu}+\frac{1}{2}v^2\Big(e^2A_\mu A^\mu+g^2B_\mu B^\mu$$

$$+m^2C_\mu C^\mu+h^2D_\mu D^\mu+2egA_\mu B^\mu+2emA_\mu C^\mu+2ehA_\mu D^\mu$$

$$+2gmB_\mu C^\mu+2ghB_\mu D^\mu+2mhC_\mu D^\mu\Big)-\lambda v\eta^3-\frac{\lambda}{4}\eta^4$$

$$+v\eta\left(e^2A_\mu A^\mu+g^2B_\mu B^\mu+m^2C_\mu C^\mu+h^2D_\mu D^\mu\right)+\left(2egA_\mu B^\mu\right.$$

$$\left.+2emA_\mu C^\mu+2ehA_\mu D^\mu+2gmB_\mu C^\mu+2ghB_\mu D^\mu+2mhC_\mu D^\mu\right),$$

where

(2.12) $$L_s=L_0+L_I\,,$$

If the Lagrangian is free from kinetic and mass terms, then[47]

(2.13) $$L_0=\frac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right)+\frac{1}{2}\left(2m^2\right)\eta^2-\frac{1}{4}M_{\mu\nu}M^{\mu\nu}-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}$$

$$-\frac{1}{4}N_{\mu\nu}N^{\mu\nu}+\frac{1}{2}v^2\left(e^2A_\mu A^\mu+g^2B_\mu B^\mu+m^2C_\mu C^\mu\right.$$

$$+h^2D_\mu D^\mu+2egA_\mu B^\mu+2emA_\mu C^\mu+2ehA_\mu D^\mu+2gmB_\mu C^\mu$$

$$\left.+2ghB_\mu D^\mu+2mhC_\mu D^\mu\right)$$

and the interaction terms of Lagrangian L_I becomes[47]

(2.14) $$L_I=-\lambda v\eta^3-\frac{\lambda}{4}\eta^4+v\eta\left(e^2A_\mu A^\mu+g^2B_\mu B^\mu+m^2C_\mu C^\mu\right.$$

$$+h^2D_\mu D^\mu+2egA_\mu B^\mu+2emA_\mu C^\mu+2ehA_\mu D^\mu+2gmB_\mu C^\mu$$

$$\left.+2ghB_\mu D^\mu+2mhC_\mu D^\mu\right)+\frac{\eta^2}{2}\left(e^2A_\mu A^\mu+g^2B_\mu B^\mu\right.$$

$$+m^2C_\mu C^\mu+h^2D_\mu D^\mu+2egA_\mu B^\mu+2emA_\mu C^\mu+2ehA_\mu D^\mu$$

$$\left.+2gmB_\mu C^\mu+2ghB_\mu D^\mu+2mhC_\mu D^\mu\right).$$

One can write the gauge boson mass and their scalar interaction terms in the form of 4×4 matrices as, (2.15) $$\beta=\kappa\left(A_{\mu}B_{\mu}C_{\mu}D_{\mu}\right)=\begin{pmatrix} e^{2} & eg & em & eh \\ ge & g^{2} & gm & gh \\ me & mg & m^{2} & mh \\ he & hg & hm & h^{2} \end{pmatrix}\begin{pmatrix} A^{\mu} \\ B^{\mu} \\ C^{\mu} \\ D^{\mu} \end{pmatrix},$$

$\kappa=\frac{v^{2}}{2}$, v, $\frac{1}{2}$ for the gauge boson mass, tri-linear interaction and quaternionic action terms respectively. Now applying the duality transformation $E\Rightarrow E\cos\eta+H\sin\eta$, $H\Rightarrow E\cos\eta-\sin\eta$, and $G\Rightarrow G\cos\eta$ $-H\sin\eta$ and $G\Rightarrow G\cos\eta+M\sin\eta$, $M\Rightarrow G\cos\eta-H\sin\eta$. If the mass and integration matrices are diagonalized. Then

(2.16) $$\beta=\kappa\left(A_{\mu}B_{\mu}C_{\mu}D_{\mu}\right)=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \begin{matrix} e^{2}+g^{2} \\ +m^{2}+h^{2} \end{matrix} \end{pmatrix}\begin{pmatrix} A^{\mu} \\ B^{\mu} \\ C^{\mu} \\ D^{\mu} \end{pmatrix}.$$

The equation (2.7) can also be written as,

(2.17)) $$L_{s}=\overline{D_{\mu}\phi}D^{\mu}\phi-m^{2}\left(\bar{\phi}\,\phi\right)-\lambda\left(\bar{\phi}\,\phi\right)^{2}$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}M_{\mu\nu}M^{\mu\nu}-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}-\frac{1}{4}N_{\mu\nu}N^{\mu\nu}.$$

Applying the following gauge transformations,

(2.18) $$\phi\rightarrow\omega\phi\bar{\omega}.$$

The unified gauge fields for electric, magnetic, gravitational and Heavisidian A_{μ}, B_{μ}, C_{μ}, D_{μ} transformed[53] as,

(2.18) $$\begin{cases} A_{\mu}=\omega A_{\mu}\bar{\omega}+\omega\partial_{\mu}\bar{\omega}, \\ B_{\mu}=\omega B_{\mu}\bar{\omega}+\omega\partial_{\mu}\bar{\omega}, \\ C_{\mu}=\omega C_{\mu}\bar{\omega}+\omega\partial_{\mu}\bar{\omega}, \\ D_{\mu}=\omega D_{\mu}\bar{\omega}+\omega\partial_{\mu}\bar{\omega}. \end{cases}$$

Applying the condition equation (2.18), the covariant derivatives for unified charges reduces to,

(2.19) $$\Delta_{\mu}\phi = \partial_{\mu}\phi - ieA_{\mu}\phi - igB_{\mu}\phi - imC_{\mu}\phi - ihD_{\mu}\phi\,,$$

Taking the variation in the covariant derivative, the equation (1.19) becomes,

(2.20) $$\begin{aligned}\delta\left(\Delta_{\mu}\phi\right) &= \partial_{\mu}\delta\phi - ieA_{\mu}\delta\phi - igB_{\mu}\delta\phi - imC_{\mu}\delta\phi - ihD_{\mu}\delta\phi \\ &\quad - ie\delta A_{\mu}\phi - ig\delta B_{\mu}\phi - im\delta C_{\mu}\phi - ih\delta D_{\mu}\phi \\ &= \left(\partial_{\mu} - ieA_{\mu} - igB_{\mu} - imC_{\mu} - ihD_{\mu}\right)\delta\phi - ie\delta A_{\mu}\phi \\ &\quad - ig\delta B_{\mu}\phi - im\delta C_{\mu}\phi - ih\delta D_{\mu}\phi \\ &= \Delta_{\mu}\delta\phi - ie\delta A_{\mu}\phi - ig\delta B_{\mu}\phi - im\delta C_{\mu}\phi - ih\delta D_{\mu}\phi\,.\end{aligned}$$

Similarly,

(2.21) $$\delta\left(\Delta_{v}\phi\right) = \Delta_{v}\delta\phi - ie\delta A_{v}\phi - ig\delta B_{v}\phi - im\delta C_{v}\phi - ih\delta D_{v}\phi\,,$$

where the energy momentum field strength for different charges such as electric, magnetic, gravitational and Heavisidian[53],

(2.22) $$\begin{cases} F_{\mu v} = \partial_{\mu}A_{v} - \partial_{v}A_{\mu} - ie\left[A_{\mu}, A_{v}\right], \\ M_{\mu v} = \partial_{\mu}B_{v} - \partial_{v}B_{\mu} - ie\left[B_{\mu}, B_{v}\right], \\ f_{\mu v} = \partial_{\mu}C_{v} - \partial_{v}C_{\mu} - ie\left[C_{\mu}, C_{v}\right], \\ N_{\mu v} = \partial_{\mu}D_{v} - \partial_{v}D_{\mu} - ie\left[D_{\mu}, D_{v}\right]. \end{cases}$$

and correspondingly the variation in equation (2.22) becomes,

(2.23) $$\begin{cases} \delta F_{\mu v} = \Delta_{\mu}\delta A_{v} - \Delta_{v}\delta A_{\mu}, \\ \delta M_{\mu v} = \Delta_{\mu}\delta B_{v} - \Delta_{v}\delta B_{\mu}, \\ \delta f_{\mu v} = \Delta_{\mu}\delta C_{v} - \Delta_{v}\delta C_{\mu}, \\ \delta N_{\mu v} = \Delta_{\mu}\delta D_{v} - \Delta_{v}\delta D_{\mu}. \end{cases}$$

Applying the variational principle, then the Lagrangian L in equation (2.17) becomes[53],

$$\delta L=\left(\overline{\Delta_{\mu}\delta\phi-ie\delta A_{\mu}\phi-ig\delta B_{\mu}\phi-im\delta C_{\mu}\phi-ih\delta D_{\mu}\phi}\right)\Delta^{\mu}\phi$$
$$+\overline{\Delta_{\mu}\phi}\left(\Delta^{\mu}\delta\phi-ie\delta A^{\mu}\phi-ig\delta B^{\mu}\phi-im\delta C^{\mu}\phi-ih\delta D^{\mu}\phi\right)$$
$$-\left(m^{2}+2\lambda\bar{\phi}\phi\right)\left(\phi\delta\bar{\phi}+\bar{\phi}\delta\phi\right)+\frac{1}{4}F_{\nu\mu}\left(\Delta^{\mu}\delta A^{\nu}-\Delta^{\nu}\delta A^{\mu}\right)$$
$$+\frac{1}{4}M_{\nu\mu}\left(\Delta^{\mu}\delta B^{\nu}-\Delta^{\nu}\delta B^{\mu}\right)+\frac{1}{4}f_{\nu\mu}\left(\Delta^{\mu}\delta C^{\nu}-\Delta^{\nu}\delta C^{\mu}\right)$$
$$+\frac{1}{4}N_{\nu\mu}\left(\Delta^{\mu}\delta D^{\nu}-\Delta^{\nu}\delta D^{\mu}\right), \tag{2.24}$$

where the transformations are $F_{\mu\nu}=-F_{\nu\mu}$, $M_{\mu\nu}=-M_{\nu\mu}$, $f_{\mu\nu}=-f_{\nu\mu}$, $N_{\mu\nu}=-N_{\nu\mu}$. Rearranging the terms of equation (2.24), we get

$$\delta L=\left[\left\{\delta\bar{\phi}\left(\Delta_{\mu}\Delta^{\mu}\phi-\left(m^{2}+2\lambda\bar{\phi}\phi\right)\right)\phi\right\}\right]$$
$$+\left[\left\{\delta\phi\left(\Delta_{\mu}\Delta^{\mu}\phi-\left(m^{2}+2\lambda\phi\bar{\phi}\right)\right)\bar{\phi}\right\}\right]$$
$$+\left[\delta A^{\mu}\left(\Delta_{\mu}\phi\right)\bar{\phi}-\phi\left(\overline{\Delta_{\mu}\phi}\right)+\frac{1}{ie}\Delta^{\mu}F_{\nu\mu}\right]$$
$$+\left[\delta B^{\mu}\left(\Delta_{\mu}\phi\right)\bar{\phi}-\phi\left(\overline{\Delta_{\mu}\phi}\right)+\frac{1}{ig}\Delta^{\mu}M_{\nu\mu}\right]$$
$$+\left[\delta C^{\mu}\left(\Delta_{\mu}\phi\right)\bar{\phi}-\phi\left(\overline{\Delta_{\mu}\phi}\right)+\frac{1}{im}\Delta^{\mu}f_{\nu\mu}\right]$$
$$+\left[\delta\Delta^{\mu}\left(\Delta_{\mu}\phi\right)\bar{\phi}-\phi\left(\overline{\Delta_{\mu}\phi}\right)+\frac{1}{ie}\Delta^{\mu}N_{\nu\mu}\right], \tag{2.25}$$

We get the equation of motion for unified charge as,

$$\Delta_{\mu}\Delta^{\mu}\phi-\left(m^{2}+2\lambda\bar{\phi}\phi\right)\phi=\Delta_{\mu}\Delta^{\mu}\bar{\phi}-\left(m^{2}+2\lambda\phi\bar{\phi}\right)\bar{\phi}, \tag{2.26}$$

Then the current equation for the unified charges are described as,

$$(2.27)\qquad \begin{cases} \Delta^{v} F_{v\mu} = J_{\mu}^{(e)} = ie\left[\phi\left(\overline{\Delta_{\mu}\phi}\right) - \left(\Delta_{\mu}\phi\right)\bar{\phi}\right], \\ \Delta^{v} M_{v\mu} = J_{\mu}^{(m)} = ig\left[\phi\left(\overline{\Delta_{\mu}\phi}\right) - \left(\Delta_{\mu}\phi\right)\bar{\phi}\right], \\ \Delta^{v} f_{v\mu} = J_{\mu}^{(G)} = im\left[\phi\left(\overline{\Delta_{\mu}\phi}\right) - \left(\Delta_{\mu}\phi\right)\bar{\phi}\right], \\ \Delta^{v} N_{v\mu} = J_{\mu}^{(H)} = ih\left[\phi\left(\overline{\Delta_{\mu}\phi}\right) - \left(\Delta_{\mu}\phi\right)\bar{\phi}\right]. \end{cases}$$

References

1. P. A. M. Dirac, *Proc. Roy. Soc. London*, **A133** (1931), 60.
2. P. A. M. Dirac, *Phys. Rev.*, **74** (1948), 817.
3. J. Schwinger, *Science*, **151** (1966), 1048.
4. J. Schwinger, *Science*, **151** (1966), 1055.
5. J. Schwinger, *Phys. Rev.*, **144** (1966), 1087.
6. J. Schwinger, *Science*, **173** (1968), 1536.
7. J. Schwinger, *Science*, **165** (1969), 757.
8. D. Zwanzinger, *Phys. Rev.*, **176** (1968), 1489.
9. D. Zwanzinger, *Phys. Rev.*, **D(3)** (1971), 880.
10. C. N. Yang and T. T. Wu, *Nucl. Phys.*, **B107** (1976), 365.
11. C. N. Yang and T. T. Wu, *Phys. Rev.*, **D14** (1976), 437.
12. F. Rhrlich, *Phys. Rev.*, **150** (1966), 1104.
13. P. Goddard and D. Olive, *Rep. on Progress in Phys.*, **41** (1978), 1357.
14. N. Craigie, World Scientic, Singapur, 1986.
15. G.' t Hooft, *Nucl. Phys.*, **B79** (1974), 276.
16. A. M. Polyakov, *JETP lett.*, **20** (1974), 194.
17. B. Julia and A. Zee, *Phys. Rev.*, **D11** (1975), 2227.
18. J. Preskill, *Ann. Rev. Nucl. Sci.*, **34** (1984), 461.
19. E. Witten, *Phys. Lett.*, **B86** (1979) 283.
20. Y. M. Cho, *Phys. Rev.*, **D21** (1980), 1080.
21. G. ' t Hooft, *Nucl. Phys.*, **B153** (1979), 141.
22. G. ' t Hooft, *Nucl. Phys.*, **B190** (1981), 455.
23. V. A. Rubakov, *JETP Lett.*, **33** (1981), 644.
24. V. A. Rubakov, *Nucl. Phys.*, **B203** (1982), 311.
25. C. Dokos and T. Tomaras, *Phys. Rev.*, **D21** (1980), 2940.
26. N. Seiberg and E. Witten, *Nucl. Phys.*, **B426** (1994) 19.
27. K. Intriligator and N. Seiberg, *Nucl. Phys.(Proc. Suppl.)*, **BC(45)** (1996), 1.
28. M. E. Peskin, *Dualities InSupersymmetric Yang-Mills Theories.*
29. A. Kapustin and E. Witten, *Electric-Magnetic Duality And The Geometric Lang lands Program.*
30. W. R. Hamilton, *Elements of Quaternions*, Chelsea Publications Co., New York, 1969.
31. L. Silberstein, *Phil. Mag.*, **23** (1912), 790.
32. P. G. Tait, *An elementary Treatise on Quaternions*, Oxford Univ. Press, 1875.

33. B. S. Rajput, S. R. Kumar and O. P. S. Negi, *Lett. Nuovo Cimento,* **34** (1982), 180.
34. V. Majernik and M. Nagy, *Nuovo Comento Lett.,* **16** (1976), 265.
35. S. L. Adler, *Quaternion Quantum Mechanics and Quantum Fields*, Oxford Univ. Press, NY, 1995.
36. D. Finklestein, J. M. Jauch, S. Schiminovich and D. Speiser, *J. Math. Phys.,* **4** (1963), 788.
37. K. Morita, *Prog. Theor. Phys.*, **68** (1982), 2159.
38. P. S. Bisht, O. P. S. Negi and B. S. Rajput, *Int. J. Theor. Phys.*, **32** (1993), 2099.
39. J. Lukiersky and A. Nowicki and Fortsch, *Physik*, **30** (1982), 75.
40. A. J. Davies, *Phys. Rev.*, **A(49)** (1994), 714.
41. V. V. Kravchenkov, *Applied Quaternion Analysis*, HeldermanVerlag, Germany, 2003.
42. A. Waser, *Quaternions in Electrodynamics*, AW-Verlag; www.aw-verlag.ch, 2000.
43. T. P. Cheng and L. F. Li, *Gauge Theory of Elementary Particle Physics,* Clarendon, Oxford, 1984.
44. P. H. Frampton, *Gauge Field Theories*, Wiley-VCH Verlag, GmbH & Co. KGaA, Weinheim, 2008.
45. C. Quigg, *Gauge Theories of the Strong, Weak and Electro-magnetic Interaction,* Westview Press, Boulder, 1997.
46. P. S. Bisht, Pushpa, T. Li and O. P. S. Negi, *Int. J. Theor. Phys.*, **50** (2011), 594.
47. D. Singleton, *Magnetic Charge as a Hidden Gauge Symmetry*.
48. C. Nash and G. C. Joshi, *J. Math. Phys.,* **28** (1987), 463.
49. K. Morita, *Prog. Theor. Phys.*, **65** (1981), 2071.
50. F. N. Ndili, *Int. J. Theor. Phys.,* **15** (1976), 265.
51. S. De Leo, *Int. J. Theor. Phys.,* **35** (1996), 1821.
52. A. S. Rawat, O. P. S. Negi, P. S. Bisht and B. S. Rajput, *Indian J. Pure Appl. Phys.,* **38** (2000), 390.
53. Pushpa, P. S. Bisht and O. P. S. Negi, *Int. J. Theor. Phys.*, **50** (2011), 1927.
54. ShaliniDangwal, P. S. Bisht and O. P. S. Negi, Russian Physics Journal, 49 (2006), 1274.
55. P. S. Bisht, O. P. S. Negi and B. S. Rajput, *Ind. J. Pure and Applied Math.,* **31** (1993), 543.
56. Gaurav Karnatak, P. S. Bisht and O. P. S. Negi, *Inter. J. of Theor. Phys.,* **49** (2010), 1344.
57. O. P. S. Negi and H. Dehnen, *Intern. Journ. Theor. Phys.*, **50** (2011), 2446.