ISSN 0974 - 9373

Vol. 22 No. 1 (2018) **Journal of International Academy of Physical Sciences** pp. 25- 50

Soret, Dufour and Hall Effects on Unsteady Mhd Flow Past a Semi-Infinite Vertical Plate by The Presence of Heat Source

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(Received November 10, 2017)

Abstract: In the present study a numerical attempt is made to analyze the effects Hall current, Soret and Dufour in the presence of heat source, on unsteady flow of a chemically reacting incompressible viscous fluid along a semi-infinite vertical plate with viscous dissipation and thermal radiation. The governing non-linear differential equations are solved using implicit finite difference formulae, for which numerical simulation is carried out by coding in C-Program. The results obtained for Skin-friction, Nusselt and Sherwood numbers in the absence of Soret and Dufour parameters, show a good agreement with previously published data.

Keywords: Hall current, Soret and Dufour, Magnetic field, Implicit finite difference scheme, Thomas algorithm.

1. Introduction

A special attention has been given to the unsteady free-convection flow of an incompressible, electrically conducting viscous fluid in the presence of magnetic field in connection with the theory of fluid motion in the liquid core of the earth, oceanographic and meteorological applications. Due to the gyration and drift of charged particles, the conductivity is reduced parallel to the electric field and the current is induced perpendicular to both electric and magnetic fields. This phenomenon is called as the ‘Hall current effect’. This effect on the fluid flow with variable concentration has a lot of applications in hydromagnetic power generators, general astrophysical and meteorological studies and it can be taken into account within the range of magneto hydro dynamical approximations. Sato[1] has studied the effect of Hall current on the steady hydro magnetic flow between two parallel plates. Katagiri[2] studied the steady incompressible boundary layer flow past a semi infinite flat plate in a transverse magnetic field at small magnetic Reynolds number considering with the effect of Hall current. On the other hand Hossain[3] studied the effect of Hall current on unsteady flow of incompressible fluid along an infinite vertical porous flat plate subjected to suction/injection velocity proportional to $(\text{time})^{-1/2}$. Hossain et al.[4] investigated the effect of Hall current on the unsteady MHD free convection flow of an incompressible viscous fluid along a vertical porous plate subjected to a time dependent transpiration velocity. Agarwal[5] discussed the effect of Hall current on the unsteady hydro magnetic free convection flow of viscous stratified fluid through a porous medium. The steady hydromagnetic free convective mass transfer flow past a vertical plate with Hall current, viscous dissipation and joule heating, taking in to account the thermal diffusion effect was studied by Singh[6]. The effects of heat Source/Sink on free-convective MHD heat transfer flow of viscous incompressible fluid along a vertical porous plate with hall current effect was analyzed by Srihari et al.[7] analyzed. The effect of Hall current on unsteady hydromagnetic mixed convection heat and mass transfer flow past a vertical porous plate immersed in a porous medium was discussed and analyzed by Sharma and Chaudhary[8].

In the above all stated studies, Soret and Dufour's effects were neglected. However, in nature and technology, many transport processes can be found in different ways in which the heat and mass transfer occur due to buoyancy forces which are caused by temperature and differences. When heat and mass transfer occurs simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are more intricate nature. Energy fluxes are generated by both temperature and concentration gradients. The energy flux is created by a composition gradient is called as Dufour effect whereas mass fluxes caused by temperature a gradients known as Soret effect. Such effects play an important role when density differences exist in the flow regime. For example, when species are introduced at a surface in a fluid domain, both Soret and Dufour effects are become influential, with a different (lower) density than the surrounding fluid. In heat and mass transfer problems, Soret and Dufour effects are very important for intermediate molecular weight gases in fluid binary systems, which are often encountered in high-speed aerodynamics and chemical process engineering. In view of these applications, ,Dursunkaya and Worek[9], Anghel et al.[10] discussed the Soret and Dufour effects on transient and steady natural convection flow from vertical surface. Postelnicu[11] studied the effects of a magnetic field, Soret and Dufour on heat and mass transfer by natural convection from vertical surfaces in porous media.

Sparrow et al.[12] reported the effects of transpiration induced buoyancy, Soret and Dufour effects in a helium–air free convection boundary layer flow. Dursunkaya and Worek[13] studied the Soret and Dufour effects on transient and steady natural convection flow from a vertical surface. Alam and Rahman[14] studied the Dufour and Soret effects on steady magneto-hydrodynamic free convective heat and mass transfer flow past a semi-infinite vertical porous plate. Alam and Rahman[15] investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. The effects of temperature dependent viscosity, Soret and Dufour number variations on free convective heat and mass transfer flow over a vertical isothermal flat plate is studied by Afify[16]. The influence of chemical reaction, Soret and Dufour effects on natural convective heat and mass transfer flow from vertical surfaces in porous media was analyzed Postelnicu [17]. Tsai and Huang [18] focused on effects of Soret and Dufour on Hiemenz heat and mass transfer flow through porous medium onto a stretching surface. The effects Soret and Dufour on natural convection flow past a vertical surface in a porous medium with variable surface temperature was investigated by Arabawy[19]. Soret and Dufour effects on steady hydromagnetic free convective boundary layer flow past a semi-infinite moving vertical plate embedded in a porous medium, taking viscous dissipation term into account was examined Reddy and Reddy[20]. Hydro-magnetic mixed convection flow past a vertical plate embedded in a porous medium with Soret and Dufour effects was discussed by Olanrewaju et al.[21], Makinde[22] and Sharma et al.[23].

Due to the various applications of radiative heat transfer in the field of nuclear power plants, gas turbines, recently, several authors have reported the effect of radiation on the fluid flow by considering different flow conditions. Elbashbeshby and Bazid[24] have reported the effect of radiation on forced convection flow of a micro polar fluid over a horizontal plate. Chamkha et al.[25] analyzed the effect of radiation on free convection flow past a semi infinite vertical plate with mass transfer. Ganesan and Loganathan [26] studied the radiation and mass transfer effects on flow of a viscous incompressible fluid past a moving cylinder. Kim et al.[27] analysed the effect of radiation on transient mixed convection flow of a micropolar fluid past a moving semi infinite vertical porous plate. Makinde[28] examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid. Prasad et al.[29] considered the effects radiation and mass transfer on two dimensional flow past an infinite vertical plate. The effect of thermal radiation, time-dependent suction and chemical reaction on the two-dimensional flow of an incompressible Boussinesq fluid, was studied by Prakash and Ogulu[30]. Later, for this same study a numerical investigation has been carried out by Rajireddy and Srihari[31]. A numerical analysis, using Keller-box method on unsteady laminar magneto hydrodynamics boundary-layer flow and heat transfer of incompressible, viscous and electrically conducting fluid is made by Ibrahim and Shanker[32]. The flow is considered over a stretching sheet in the presence of transverse magnetic field with heat source/sink. Hall current in the presence of radiation on hydromagnetic flow of a dissipative and chemically reacting fluid along a semi-infinite vertical plate with heat source/sink is analysed by Shivaiah et al.[33]. Several authors [34-38] have dealt the unsteady heat transfer flows on different geometry and considering various flow conditions. Recently, steady convection flow of a viscous incompressible electrically conducting fluid along a semi infinite vertical plate in the presence of internal heat generation and a convective surface boundary condition is investigated by Sharma and Yadav[39].

In most of the previous investigations, the effects of Hall current, Soret and Dufour in the presence of heat source has not been considered. But it plays an important role in maintaining the heat transfer at desired level in the fields of gas turbines, Nuclear power plants, and the various propulsion devices for aircraft, missiles, satellites and space vehicles**.** Due to the coupled non-linearity of the problem in most of the earlier investigations, analytical or perturbation methods were applied to obtain the solution of the problem. However, in the present paper a numerical attempt has been made to study the effects of Hall current, Soret and Dufour in the presence of heat source on unsteady boundary layer flow of a chemically reacting incompressible viscous fluid past g a semi-infinite vertical plate with viscous dissipation and thermal radition. A magnetic field of uniform strength is applied normal to the fluid flow. In order to obtain the approximate solution and to describe the physics of the problem, the present non-linear boundary value problem is solved numerically using implicit finite difference formulae known as Crank-Nicholson method. The obtained results are discussed in detail and compared with the results of Skin-friction, Nusselt and Sher-wood numbers, presented by Shivaiah et al[33] in the absence of Soret and Dufour prameters. The present study is used as a bridge to fill the knowledge gap among the researchers.

2. Formulation of the problem

An unsteady laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate is considered in the presence of heat source. The x'-axis considered along the plate in the vertically upward direction and y'-axis normal to it. A magnetic field is applied normal to the plate. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. A time dependent suction velocity is assumed normal to the plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The equation of conservation of electric charge $\nabla . \vec{J} = 0$, *gives* $j_y =$ constant, where $\vec{J} = (j_x,\ j_y,\ j_z)$.We further assume that the plate is non-conducting. This implies $j_y = 0$ at the plate and hence zero everywhere. When the strength of magnetic field is very large the generalized Ohm's law, in the absence of electric field takes the following form

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}(\vec{J} \times \vec{B}) = \sigma\left(\vec{V} \times \vec{B} + \frac{1}{en_e}\nabla P_e\right) \tag{2.1}$$

Where $\vec{V}$ is the velocity vector, σ is the electric conductivity, ω_e is the electron frequency, τ_e is the electron collision time, e is the electron charge, n_e is the number density of the electron and P_e is the electron pressure. Under the assumption that the electron pressure (for weakly ionized gas), the thermo-electric pressure and ion-slip are negligible, equation (2.1) becomes:

$$J_x = \frac{\sigma B_0}{1+m^2}(mu - w) \text{ and } j_z = \frac{\sigma B_0}{1+m^2}(u + mw) \tag{2.2}$$

where u is the x-component of V, w is the z component of V and $m\ (= w_e \tau_e\ )$ is the Hall parameter.

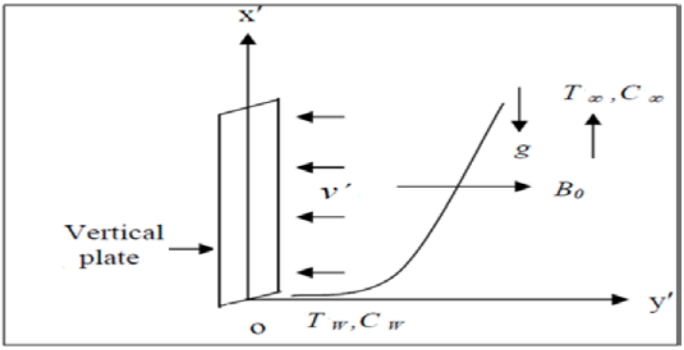


Figure 2.1: Schematic diagram of flow geometry

Within the above framework, the equations which govern the flow under the usual Boussinesq approximation are as follows:

- Continuity

(2.3) $$\frac{\partial v'}{\partial y'} = 0$$

- Momentum equations

(2.4) $$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho(1+m^2)}(u' + mw')$$

(2.5) $$\frac{\partial w'}{\partial t'} + v'\frac{\partial w'}{\partial y'} = \upsilon\frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(w' - mu')$$

- Energy

(2.6) $$\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{D_m k_T}{C_S C_P T_m}\frac{\partial^2 C}{\partial y'^2} + \frac{Q(T - T_\infty)}{\rho c_p}$$

- Mass transfer

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y'^2} - k_r^2 C \tag{2.7}$$

The radiative flux q_r by using the Rosseland approximation [39], is given by

$$q_r = -\frac{4\sigma^*}{3a_R}\frac{\partial T^4}{\partial y'} \tag{2.8}$$

The boundary conditions suggested by the physics of the problem are

$$\begin{aligned} &u' = U_0,\ w' = 0,\ T = T_W + \varepsilon(T_W - T_\infty)e^{n't'}, \\ &C = C_W + \varepsilon(C_W - C_\infty)e^{n't'} \quad at\ \ y' = 0 \\ &u' \to 0,\ \ w' = 0,\ \ T \to T_\infty, \quad C \to C_\infty \qquad as\ \ y' \to \infty \end{aligned} \tag{2.9}$$

It has been assumed that the temperature differences within the flow are sufficiently small and T^4 may be expressed as a linear function of the temperature T. This is accomplished by expanding T^4 in a Taylor series about T_∞, as follows. Let

$$f(T) = f(T_\infty) + (T - T_\infty)f'(T_\infty) + \frac{(T - T_\infty)^2}{2!}f''(T_\infty) + \ldots, \tag{2.10}$$

Where, $f(T) = T^4$, then $f'(T) = 4T^3$, $f''(T) = 12T^2$, simplifying (2.10), we get

$$T^4 = T_\infty^4 + 4(T - T_\infty)T_\infty^3 + 12\frac{(T - T_\infty)^2}{2!}T_\infty^2 + \ldots$$

In the above expansion, neglecting the higher order terms, we have

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{2.11}$$

Using (2.11) in (2.8) and then (2.8) in (2.6), equation of energy (2.6) gives

(2.12) $$\frac{\partial T}{\partial t'}+v'\frac{\partial T}{\partial y'}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y'^2}+\frac{16\sigma^* T_\infty^3}{3\rho c_p\, a_R}\frac{\partial^2 T}{\partial y'^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 +\frac{Q(T-T_\infty)}{\rho c_p}$$

Integration of continuity equation (2.3) for variable suction velocity normal to the plate gives

(2.13) $$v'=-U_0\left(1+\varepsilon\, A e^{n't'}\right)$$

where A is the suction parameter and εA is less than unity. Here U_0 is mean suction velocity, which is a non-zero positive constant and the minus sign indicates that the suction is towards the plate.
In order to get the non-dimensional partial differential equations, introducing the following non-dimensional quantities

(2.14) $$u=\frac{u'}{U_0},\quad w=\frac{w'}{U_0},\quad y=\frac{y'U_0}{\upsilon}\quad t=\frac{U_0^2 t'}{\upsilon},\quad \theta=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\phi=\frac{C-C_\infty}{C_w-C_\infty},\ \Pr=\frac{\mu C_p}{k},\ Sc=\frac{\upsilon}{D},\ M=\frac{\sigma B_0^2\,\upsilon}{\rho U_0^2}$$
$$Gr=\frac{g\beta\,\upsilon\left(T_w-T_\infty\right)}{U_0^3},\quad Gm=\frac{g\beta^*\upsilon\left(C_w-C_\infty\right)}{U_0^3},\ S=\frac{Q\upsilon}{\rho C_p U_0^2}$$

$$Ch=\frac{k_r'^2\upsilon}{U_0^2},\quad NR=\frac{16\sigma^* T_\infty^3}{3k a_R},\ Ec=\frac{U_0^2}{C_p(T_w-T_\infty)}\quad n=\frac{\upsilon n'}{U_0^2}.$$

$So=\dfrac{D_m k_T\left(T_w-T_\infty\right)}{\upsilon T_m\left(C_w-C_\infty\right)}$, $Du=\dfrac{D_m k_T\left(C_w-C_\infty\right)}{\upsilon C_S\, C_P\left(T_w-T_\infty\right)}$, into equations (2.4), (2.5), (2.7), (2.9) and (2.12), we obtain the following:

(2.15) $$\frac{\partial u}{\partial t}-\left(1+\varepsilon\, A e^{nt}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-\frac{M}{1+m^2}\left(u+mw\right)+Gr\theta+Gm\phi$$

(2.16) $$\frac{\partial w}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}-\frac{M}{1+m^2}\left(w-mu\right)$$

(2.17) $$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial \theta}{\partial y}=\left(\frac{1+NR}{Pr}\right)\frac{\partial^2 \theta}{\partial y^2}+Du\frac{\partial^2 \phi}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2+S\theta$$

(2.18) $$\frac{\partial \phi}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}+S_0\frac{\partial^2 \theta}{\partial y^2}-Ch\phi$$

with the boundary conditions

(2.19) $$u=1,\ w=0,\ \theta=1+\varepsilon e^{nt},\ \phi=1+\varepsilon e^{nt} \quad \text{at } y=0$$
$$u\to 0,\ w=0,\ \theta\to 0,\ \phi\to 0 \quad \text{as } y\to\infty$$

In order to establish a finite condition, $\eta\to 1$ in equation (2.19) instead of an infinite boundary condition, $y\to\infty$, employing the transformation $\eta=1-e^{-y}$ on equations (2.15) - (2.19), the following are obtained.

(2.20) $$\frac{\partial u}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial u}{\partial \eta}=(1-\eta)^2\frac{\partial^2 u}{\partial \eta^2}-(1-\eta)\frac{\partial u}{\partial \eta}$$
$$-\frac{M}{1+m^2}\left(u+mw\right)+Gr\theta+Gm\phi$$

(2.21) $$\frac{\partial w}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial w}{\partial \eta}=(1-\eta)^2\frac{\partial^2 w}{\partial \eta^2}$$
$$-(1-\eta)\frac{\partial w}{\partial \eta}-\frac{M}{1+m^2}\left(w-mu\right)$$

(2.22) $$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon Ae^{nt}\right)(1-\eta)\frac{\partial \theta}{\partial \eta}=\frac{1+NR}{Pr}\left((1-\eta)^2\frac{\partial^2 \theta}{\partial \eta^2}-(1-\eta)\frac{\partial \theta}{\partial \eta}\right)$$
$$+Ec\left((1-\eta)\frac{\partial u}{\partial \eta}\right)^2+Du\left[(1-\eta)^2\frac{\partial^2 \phi}{\partial \eta^2}-(1-\eta)\frac{\partial \phi}{\partial \eta}\right]+S\theta$$

$$\frac{\partial\phi}{\partial t} - \left(1+\varepsilon\, Ae^{nt}\right)(1-\eta)\frac{\partial\phi}{\partial\eta} = \frac{1}{Sc}\left[(1-\eta)^2\frac{\partial^2\phi}{\partial\eta^2} - (1-\eta)\frac{\partial\phi}{\partial\eta}\right] + So\left[(1-\eta)^2\frac{\partial^2\theta}{\partial\eta^2} - (1-\eta)\frac{\partial\theta}{\partial\eta}\right] - Ch\phi \tag{2.23}$$

with boundary conditions

$$\begin{aligned} &u=1:\ \ w=0,\ \ \theta = 1+\varepsilon\, e^{nt},\quad \phi = 1+\varepsilon\, e^{nt} \quad at \quad \eta=0 \\ &u\to 0:\ w=0,\ \ \theta\to 0,\qquad \theta\to 1+\varepsilon\, e^{nt} \quad as \quad \eta\to 1 \end{aligned} \tag{2.24}$$

3. Method of solution

Equations (2.20) - (2.23) are non-linear coupled, differential equations whose exact solution is difficult to obtain, so they are solved numerically, using Crank-Nicholson method to obtain the following system of equations.

$$-P_3\, r\, u_{i-1}^{j+1} + \left(1+2P_3\, r\right)u_i^{j+1} - P_3\, r\, u_{i+1}^{j+1} = C_{1i}^{j} \tag{3.1}$$

$$-P_3\, r\, w_{i-1}^{j+1} + \left(1+2P_3\, r\right)w_i^{j+1} - P_3\, r\, w_{i+1}^{j+1} = C_{2i}^{j} \tag{3.2}$$

$$-P_3P_4\, r\,\theta_{i-1}^{j+1} + \left(1+2P_3P_4\, r\right)\theta_i^{j+1} - P_3P_4\, r\,\theta_{i+1}^{j+1} = C_{3i}^{j} \tag{3.3}$$

$$-\frac{P_3 r}{Sc}\phi_{i-1}^{j+1} + \left(1+\frac{2P_3 r}{Sc}\right)\phi_i^{j+1} - \frac{P_3 r}{Sc}\phi_{i+1}^{j+1} = C_{4i}^{j} \tag{3.4}$$

with boundary conditions in finite difference form

$$\begin{aligned} &u\,(0,j)=1,\quad \theta(0,j)=1+\varepsilon\exp(n.\,j.\Delta t),\quad \phi = 1+\varepsilon\exp(n.\,j.\,\Delta t),\ \ \forall\ j \\ &u(10,j)\to 0,\quad \theta(10,j)\to 0,\qquad \phi(10,j)\to 1 \quad \forall\ j \end{aligned} \tag{3.5}$$

where

$$E_i^j = P_3 r\, u_{i-1}^j - \left(1 - P_1 P_2\, r\Delta\eta - 2P_3 r + P_2 r\Delta\eta - \frac{M\,m}{1+m^2}\Delta t\right) u_i^j$$

$$+\left(P_1\, P_2 r\Delta\eta + P_3 r - P_2 r\Delta\eta\right) u_{i+1}^j + Gr\,\Delta t\theta_i^j + Gm\,\Delta t\phi_i^j - \frac{M\,m}{1+m^2}\Delta t\, w_i^j$$

$$D_i^j = P_3 r\, w_{i-1}^j - \left(1 - P_1 P_2\, r\Delta\eta - 2P_3 r + P_2 r\Delta\eta - \frac{M\,m}{1+m^2}\Delta t\right) w_i^j$$

$$+\left(P_1\, P_2 r\Delta\eta + P_3 r - P_2 r\Delta\eta\right) w_{i+1}^j + \frac{M\,m}{1+m^2}\Delta t\, u_i^j$$

$$F_i^j = P_3 P_4 r\,\theta_{i-1}^j + \left(1 - P_1 P_2 r\Delta\eta - 2P_3 P_4 r + P_2 P_4 r\Delta\eta + S\Delta t\right)\theta_i^j$$

$$+\left(P_1 P_2 r\Delta\eta + P_3 P_4 r - P_2 P_4 r\Delta\eta\right)\theta_{i+1}^j + \left(2P_3 Du - DuP_1 r\Delta\eta\right)\phi_{i+1}^j$$

$$+\left(DuP_1 r\Delta\eta - 4P_3 rDu\right)\phi_i^j + 2P_3 rDu\phi_{i-1}^j + 2P_3 Ec\left(\frac{u_{i+1}^j - u_i^j}{\Delta\eta}\right)^2$$

$$H_i^j = \frac{P_3 r}{Sc}\,\phi_{i-1}^j + \left(1 + P_1 P_2\, r\Delta\eta - \frac{2P_3 r}{Sc} + \frac{P_2 r\Delta\eta}{Sc} - Ch.\Delta t\right)\phi_i^j$$

$$+\left(\frac{P_3 r}{Sc} - P_1\, P_2 r\Delta\eta - \frac{P_2 r\Delta\eta}{Sc}\right)\phi_{i+1}^j + \left(2P_3 rS_0 - S_0 P_1 r\Delta\eta\right)\theta_{i+1}^j$$

$$+\left(S_0 P_1 r\Delta\eta - 4P_3 rS_0\right)\theta_i^j + 2P_3 rS_0\theta_{i-1}^j$$

$$P_1 = 1 + \in Ae^{n\, j\Delta t},\ P_2 = 1 - i\,\Delta\eta,\ P_3 = \frac{(1 - i\,\Delta\eta)^2}{2},\ P_4 = \frac{1 + NR}{\Pr}.$$

Here $\Delta\eta$ and Δt are mesh sizes along η and t (time) direction, respectively. Index i refers to space and j for time.

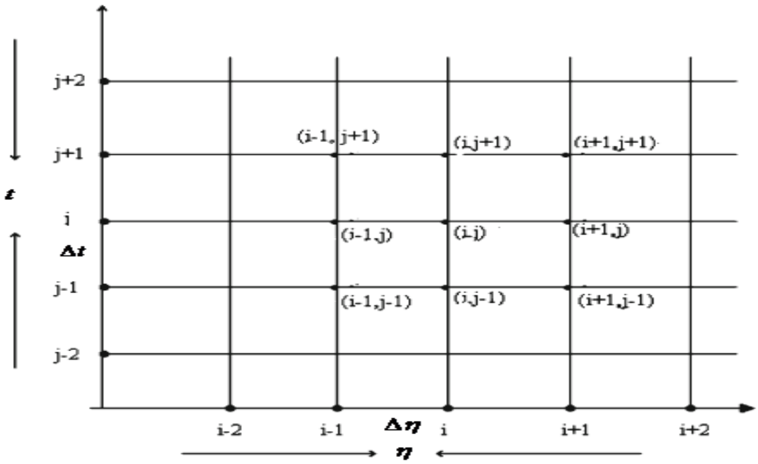


Figure 3.1: Grid meshing for finite difference method

To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to η and t axes as shown in the figure. The finite-difference equations at every internal nodal point on a particular *n*-level constitute a tri-diagonal system of equations. So, in the equations (3.1) to (3.4), taking $i = 1(1)n$ and using the boundary conditions (3.5), the following tri-diagonal system of equations are obtained.

(3.6) $DU = A$

(3.7) $ET = H$

(3.8) $FC = G$

(3.9) $MN = R$

Where D, E, F and M are the tri-diagonal matrices of order n whose elements are defined by

$$D_{i,i} = B_1; \quad E_{i,i} = B_2; \quad F_{i,i} = B_3; \quad M_{i,i} = B_4; \text{ at } i = 1(1)n$$
$$D_{i-1,i} = A_1; \quad E_{i-1,i} = A_2; \quad F_{i-1,i} = A_3; \quad M_{i-1,i} = A_4; \text{ at } i = 2(1)n$$
$$D_{i,i-1} = A_1; \quad E_{i,i-1} = A_2; \quad F_{i,i-1} = A_3; \quad M_{i,i-1} = A_4; \text{ at } i = 2(1)n$$

and U, A, T, H, C, G are column matrices having n components, namely u_i^{j+1}, C_{1i}^j, w_i^{j+1}, C_{2i}^j, θ_i^{j+1}, C_{3i}^j, ϕ_i^{j+1}, C_{4i}^j i=1(1)n respectively.

The above tri-diagonal system is solved by using the Thomas algorithm, for which a numerical simulation is carried out by coding in C-Program. In order to prove the convergence of present numerical scheme, the computation is carried out by slightly changed values of $\Delta\eta$ and Δt and the iterations on until a tolerance 10^{-8} is attained. No significant change is observed in the values of u, w, θ *and* ϕ. Thus, it is concluded that the finite difference scheme is convergent and stable. From the technological point of view, after knowing the velocity, temperature and concentration profiles, it is important to know the skin-friction, rate of heat and mass transfer between the plate and the fluid.

- **Skin-friction**

The Skin friction coefficient τ is given by

(3.10) $$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0},$$

- **Nusselt number**

The rate of heat transfer in terms of Nusselt number is given by

(3.11) $$Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0}$$

- **Sherwood number**

The coefficient of Mass transfer which is generally known as Sherwood number, Sh, is given by

(3.12) $$Sh = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = (1-\eta)\left.\frac{\partial \phi}{\partial \eta}\right|_{\eta=0}$$

Nomenclature		
ρ	-	Density
C_p	-	Specifi hceat at constant pressure
v	-	Kinematic viscosity
k	-	Thermal conductivity
Sc	-	Schmidt number
T $\in$	- -	Temperature Small reference parameter << 1
Gr	-	Free convection parameter due to temperature

Gm	-	Free convection parameter due to concentration
m	-	Hall parameter
A	-	Suction parameter
N	-	A constant exponential index
D	-	Molar diffusivity
β	-	Coefficient of volumetric thermal expansion of the fluid
β^*	-	Volumetric coefficient of expansion with concentration
Tm KT	- -	Mean fluid temperature Thermal diffusion ratio
σ	-	Electrical conductivity
ω_e	-	Eectron frequency
τ_e	-	Eectron collision time
ne	-	Number density of the electron
Pe	-	Electron pressure
So	-	Soret number
Du	-	Dufour number
k_r^2		Rate of chemical reaction
a_R		Rosseland radiation absorbtivity
Dm		Mass diffusion coefficient

4. Results and discussion

In order to describe the physics of the problem, the numerical calculations for velocity, temperature, concentration, Skin-friction coefficient, rate of heat and mass transfer across the boundary layer for various values of flow parameters such as Hall parameter, Chemical reaction parameter, Grashof number, Modified Grashof number, Eckert number, Magnetic Parameter, Radiation parameter, Prandtl number, Schmidt number, Soret and Dufour numbers in the presence of heat source have been carried out. To be realistic, the values of Prandtl number (Pr) are chosen to be Pr = 0.71 and Pr = 7.0, which represent air and water at temperature 20◦C and one atmosphere pressure, respectively.

The effects of Soret (So) and Dufour (Du) on velocity field u is shown in the figures (1) and (2) respectively. The definition of the Soret number So is the effect of the temperature gradients to the inducing significant mass diffusion while the Dufour number defines the contribution of the concentration gradients to the thermal energy flux in the flow. It is observed that the velocity of the fluid increases as the value of Soret and Dufour numbers increases.

Effects of the Soret and Dufour on Cross flow velocity field w is presented in figures (8) and (9) respectively. It is observed that the cross flow velocity w increases with the increasing values of So and Du. Further from figures (11) and (16), it is also observed that an increase in So and Du leads to an increase in the concentration and temperature of the fluid respectively. Figures (3) and (6) show the effect of Hall parameter (m) on velocity field's u and w respectively, in the presence of heat source. It is seen from the figures that main flow velocities u increases as the value of *m* increases whereas secondary velocity w decreases on increasing value of *m*. Furthermore, it is noted that both the velocities u and w increase in the presence of heat source as the internal heat generation is to increase the rate of heat transport to the fluid.

Figure (4) shows the effects of the Thermal Grashof number Gr, Solutal Grashof number Gm and magnetic parameter M on velocity field u while figure (7) reveals the effect magnetic parameter M on cross flow velocity w. It is observed that an increase in magnetic parameter M leads to an increase in the secondary velocity w. it is seen from figure (4) that an increase in Gr and Gm leads to an increase in the velocity u. This is due to the fact that buoyancy force has the tendency to increase the velocity profile. Further, it is interesting to note that the increasing value of magnetic parameter is to reduce the velocity of the flow. This is due to the physical fact that the introduction of magnetic field normal to the fluid flow has a tendency to give rise to a resistive-type force called the Lorentz force, which acts against the fluid flow and hence results in reducing the velocity profile due to this type of magnetic pull of Lorentz force.

The effect of heat source parameter on temperature distribution is shown in figure (13). It is evident that the temperature increases with the increasing values of heat source parameter. This result qualitatively agrees with the physical fact that heat generation is to increase the rate of heat transport to the fluid there by increasing the temperature of the fluid. Figure (12) is drawn for various values of Pr on temperature field in the presence of heat source. A comparative study of the graph reveals that increasing values of Prandtl number Pr, decreases the temperature of the fluid as the higher Pandtl number fluid has relatively lower thermal conductivity. It is a good agreement with physical fact that an increase in Pr leads to decrease in the thermal boundary layer thickness.

Figures (5), (10) and (14) show effect of radiation parameter NR on temperature, main and secondary flow velocities respectively. It is observed that the temperature, main and secondary flow velocities of the fluid increase as the value of NR increases. This is due to the mathematical fact that an increase in the value of radiation parameter, $NR = 16\sigma^{*}T_{\infty}^{3}\big/3k\,a_R$, forgiven k and T_{∞} , leads to decrease in the Roseland radiation absorbtivity (a_R). But from equations (8) and (11) it is interesting to note that as a_R decreases the divergence of the radiation heat flux ($\partial q_r / \partial y^{*}$) increases, it means that the rate of radiative heat transferred to the fluid increases, consequently the temperature and main and secondary flow velocities of their particles increases.

The variation of the temperature field along the coordinate η is shown in figures (15) for various values of Ec. The analysis of figure (15) reveals that temperature of the fluid increases for increasing values of Eckert number (Ec). This is physically true due to the fact that increasing value of Ec grows the viscous dissipation heating within in the system in such way that temperature of the fluid increases with increase in Ec. Comparison of the curves in the figure (17) reveals that an increase in Ch leads decrease in the concentration distribution.

Skin-friction coefficient, Nusselt and Sherwood numbers are presented in tables (1), (2) and (3) respectively, both in presence and absence of Soret and Dufour effects. It is observed that Skin–friction, Nusselt and Sherwood numbers increase in the presence of Soret and Dufour. It is also observed that Skin-friction increases with the increasing values of m,NR,Ec, Gr and Gm but, it decreases as M and Pr increases. Further, it is interesting to note that an increase in Ec, m and S leads to an increase in the Nusselt number but an increase in the Sc and Ch decreases the Sherwood number.

In order to access the validity of the present numerical scheme, the present results are compared with previous published data[33] for Skin-friction, rate of heat and mass transfer in the absence of Soret and Dufour parameters. The results of the validation of the present work agree significantly.

5. Conclusions

Effects of Hall current, Soret and Dufour variations on MHD unsteady laminar boundary layer flow of a radiating and chemically reacting fluid along a semi-infinite vertical plate, by the presence of heat source with viscous dissipation are analysed. From this study the following conclusions are drawn.

(1) The effect of increasing values of Hall parameter m results in increasing both the velocity profiles u and w.
(2) The magnetic parameter (M) reduces the main flow velocity (u) at all points of the flow field due to the magnetic pull of the Lorentz force.
(3) The temperature, velocity, Skin–friction and Nusselt number increase in the presence heat source.
(4) For increasing values of Soret and Dufour parameters, there is a considerable enhancement in main and a secondary flow velocity of the fluid is observed. Dufour effects greatly influence the temperature profile in the thermal boundary layer.
(5) Temperature, primary and secondary velocities of the fluid flow increase as radiation parameter increases. This due to the fact that the effect of increasing values of radiation is to increase the rate of radiative heat transfer to the fluid.
(6) Skin–friction, Nusselt and Sherwood numbers increase in the presence of Soret and Dufour effects.
(7) The results of the validation of this work in the absence of Soret and Dufour parameter agree significantly with previous work[33].

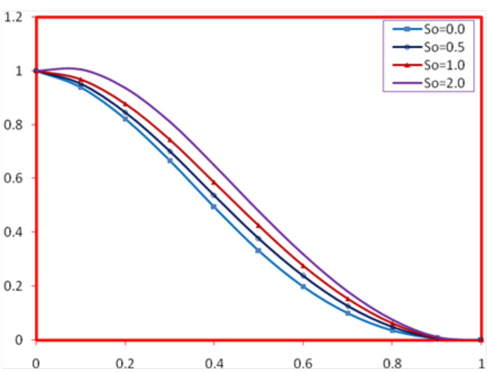


Figure 1: Velocity field u for various values of So
(Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

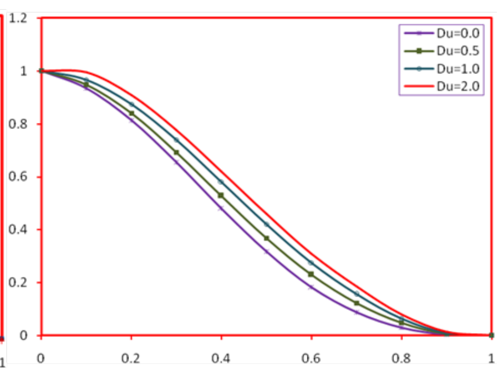
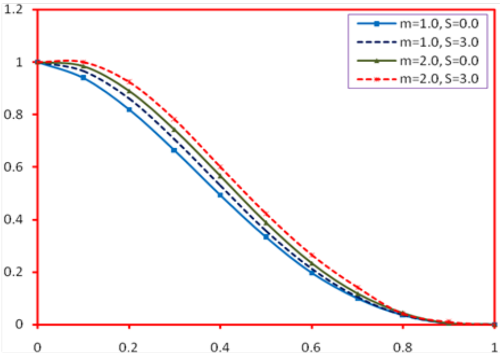


Figure 2: Velocity field u for various values of Du
(Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, So=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)



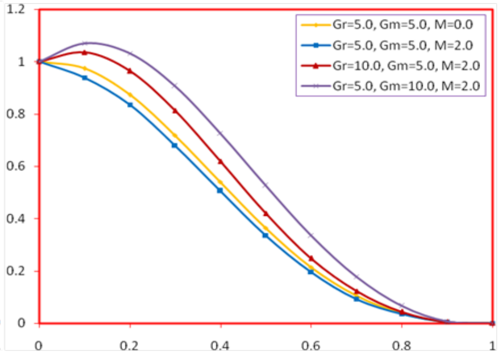
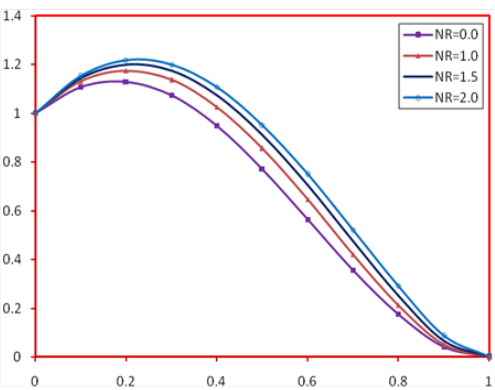


Figure 3: Velocity field u for various values of m in the presence of heat source (Gr=5.0, Gm=5.0, M=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, So=1.0, Sc=0.22, A=0.3 and ε =0.01)

Figure 4: Velocity field u for different values of Gr, Gm and M (m=1.0, S=1.0, So=1.0, Du=1.0,Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)



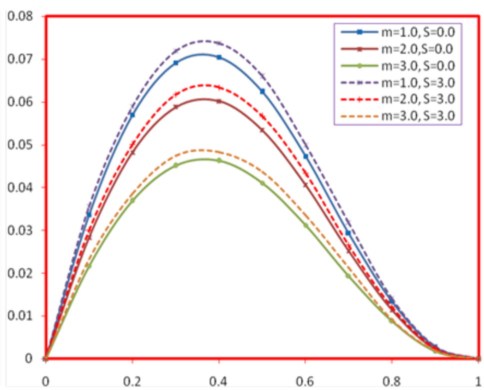


Figure 5: Velocity field u for different values of NR in the presence of heat source (Gr=5.0, Gm=5.0, M=1.0, So=1.0, Du=1.0, Pr=0.71, Ec=0.5, S=1.0, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

Figure 6: Velocity field w for different values of m in the presence of heat source (Gr=5.0, Gm=5.0, M=1.0, So=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

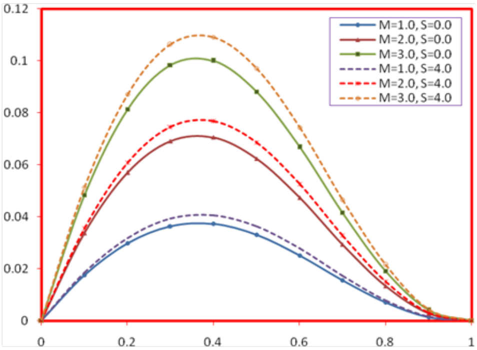


Figure 7: Velocity field w for different values of M in the presence of heat source (Gr=5.0, Gm=5.0, m=1.0, So=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

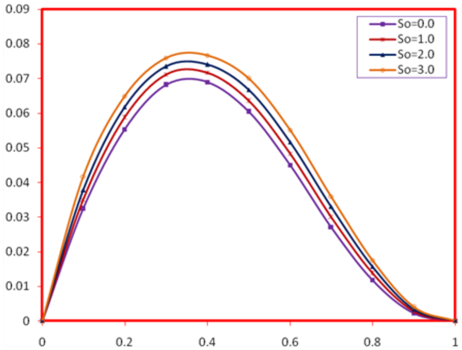


Figure 8: Velocity field w for different values of So (Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

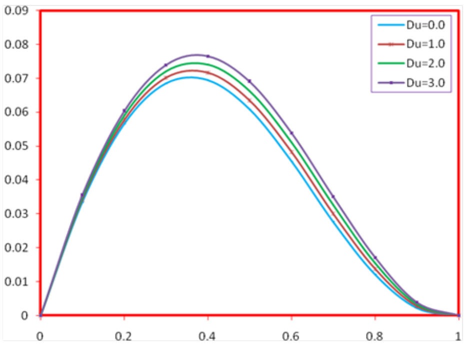


Figure 9: Velocity field w for different values of Du (Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, So=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01) (Gr=5.0, Gm=5.0, m=1.0, M=1.0, So=1.0, Pr=0.71, Sc=0.22, A=0.3 and ε =0.01)

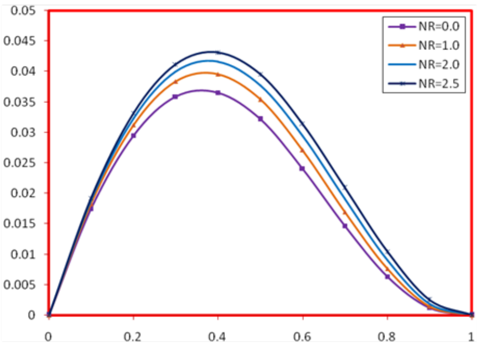


Figure 10: Velocity field w for different values of NR (Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, So=1.0, Pr=0.71, Ec=0.5, Du=1.0, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01) (Gr=5.0, Gm=5.0, m=1.0, M=1.0, So=1.0, Pr=0.71, Sc=0.22, A=0.3 and ε =0.01)

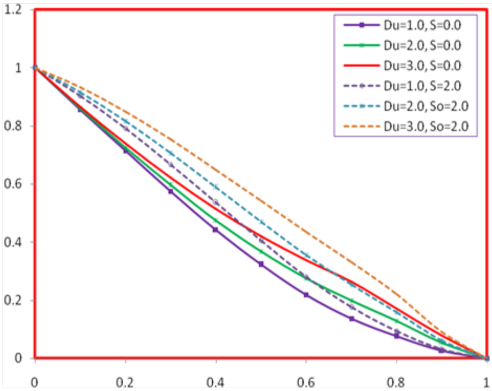


Figure 11: Temperature field **θ** for different values of Du in the presence of heat source (Gr=5.0, Gm=5.0, m=1.0, M=1.0, So=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

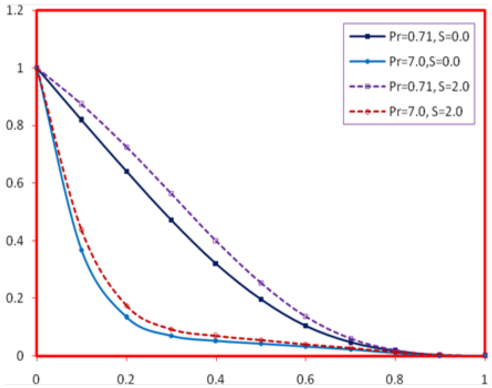


Figure 12: Temperature field **θ** for different values of Pr in the presence of heat source (Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01) (So=1.0, Du=1.0, Sc=0.22, A=0.3 and ε =0.01)

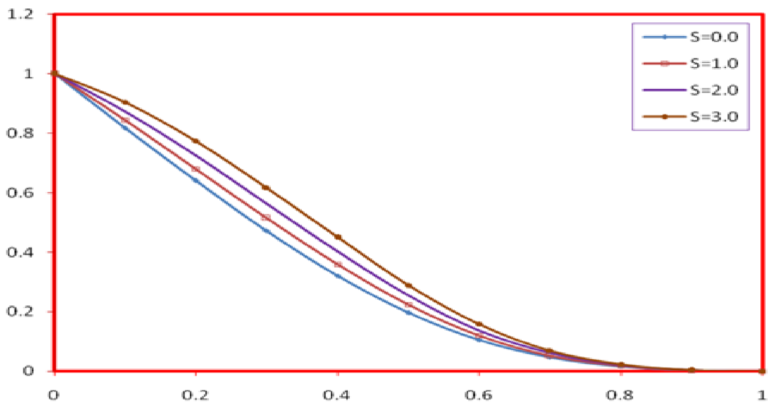


Figure 13: Temperature field **θ** for different values of S (Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

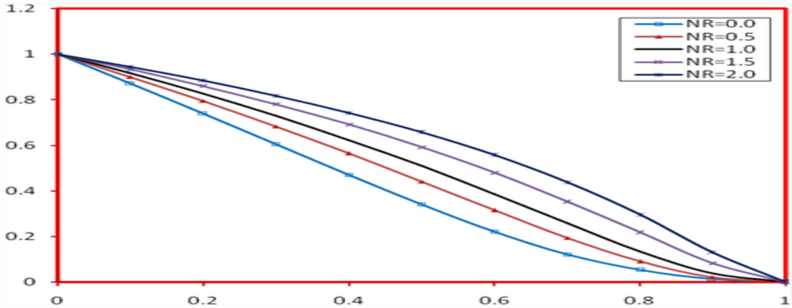


Figure 14: Temperature field **θ** for different values of NR (Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Pr=0.71, Ec=0.5, S=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

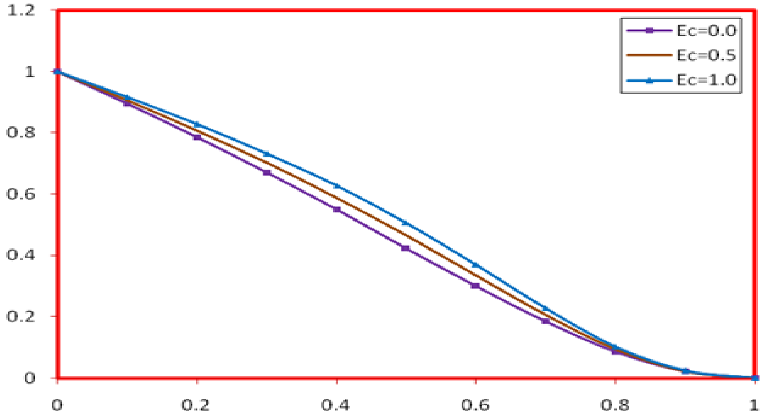


Figure 15: Temperature field **θ** for different values of Ec (Gr=5.0, Gm=5.0, m=1.0, M=1.0, Du=1.0, So=1.0, Pr=0.71, NR=0.5, S=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

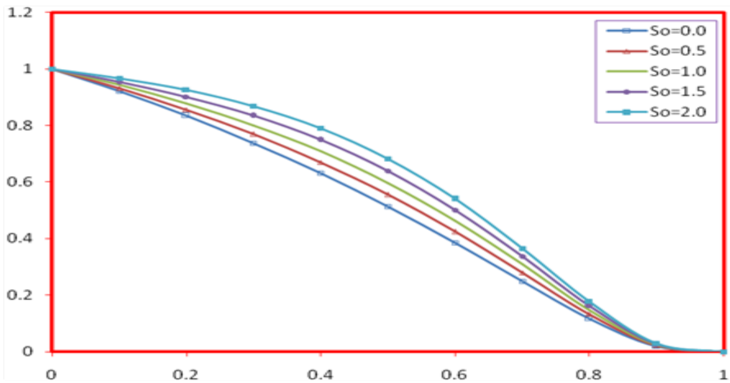


Figure 16: Concentration field Φ for different values of So (Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Ch=0.5, Sc=0.22, A=0.3 and ε =0.01)

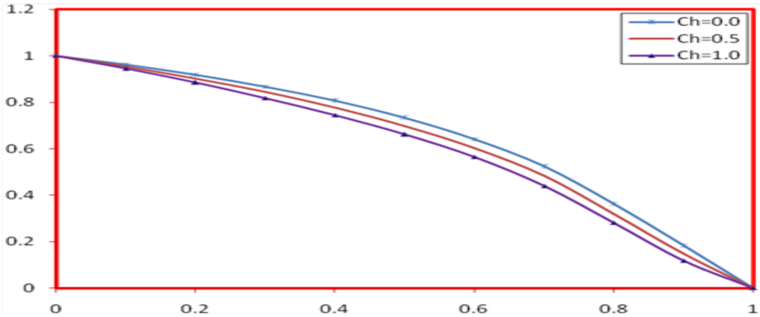


Figure 17: Concentration field Φ for different values of Ch (Gr=5.0, Gm=5.0, m=1.0, M=1.0, S=0.5,So=1.0, Du=1.0, Pr=0.71, Ec=0.5, NR=0.5, Sc=0.22, A=0.3 and ε =0.01)

Table 1 Effects of Gr, Gm, Pr, Sc, Ch, NR , M, m, S and Ec on Skin-Friction coefficient

	Gm	Pr	Sc	Ch	*NR*	M	m	S	Ec	τ So=0.0,Du=0.0 Previous [33]	τ So=1.0, Du=1.0 Present
5.0	5.0	0.71	0.22	0.5	0.0	1.0	0.2	0.5	0.1	0.77630	0.884784
5.0	5.0	0.71	0.22	0.5	1.0	1.0	0.2	0.5	0.1	1.04368	1.122814
5.0	5.0	0.71	0.22	0.5	1.0	2.0	0.2	0.5	0.1	0.73225	0.805658
5.0	5.0	0.71	0.22	0.5	1.0	1.0	1.0	0.5	0.1	1.19806	1.280061
5.0	5.0	0.71	0.22	0.5	1.0	1.0	0.2	2.0	0.1	1.10991	1.188220
5.0	5.0	0.71	0.22	0.5	1.0	1.0	0.2	-2.0	0.1	0.947146	1.027913
5.0	5.0	0.71	0.22	0.5	1.0	1.0	0.2	0.5	0.3	1.058478	1.136297
10.0	5.0	0.71	0.22	0.5	1.0	1.0	0.2	0.5	0.1	2.446682	2.562169
5.0	10.0	0.71	0.22	0.5	1.0	1.0	0.2	0.5	0.1	2.684114	2.805886
5.0	5.0	7.0	0.22	0.5	1.0	1.0	0.2	0.5	0.1	0.196928	0.358361

Table 2 Effects of M, m, NR, Ec and S on Nusselt number (Gr=5.0, Gm=5.0, Pr=0.71, Sc=0.22, Ch=0.5)

M	M	NR	Ec	S	Nu So=0.0, Du=0.0 Previous[33]	Nu So=1.0, Du=1.0 Present
1.0	0.2	0.0	0.1	0.0	-1.4496	-1.392160
1.0	0.2	1.0	0.1	0.0	-0.9656	--0.923174
1.0	0.2	1.0	0.1	1.0	-0.8220	-0.772910
1.0	0.2	1.0	0.1	-1.0	-1.0992	-1.06268
1.0	1.0	1.0	0.1	1.0	-0.8215	-0.772384
2.0	0.2	1.0	0.1	1.0	-0.8227	-0.773709
1.0	0.2	1.0	0.3	1.0	-0.7898	--0.741413

Table 3 Effects of Sc and Ch on Sherwood number (Gr=5.0, Gm=5.0, Pr=0.71, NR=0.5, M=1.0, m=0.5, S=0.5 and Ec=0.1)

Sc	Ch	Sh So=0.0,Du=0.0,(Previous [33])	Sh So=1.0, Du=1.0 (Present)
0.22	0.0	-0.514	-0.461602
0.22	1.0	- 0.6261	-0.577731
0.60	1.0	- 0.9760	-0.856968
0.94	1.0	-1.1085	-1.006822

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