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C-Bochner Curvature Tensor in Almost *C(α)* Manifolds

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Abstract: The object of the present paper is to study C-Bochner curvature tensor in almost $C(\alpha)$ manifolds which satisfy the semi-symmetric conditions $B \cdot S = 0$, $B \cdot R = 0$ and pseudosymmetric conditions $B \cdot S = LsQ(g, S)$ and $B \cdot R = L_R Q(g, R)$, where B is the C-Bochner curvature tensor, S is Ricci tensor.

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1. Introduction

A Riemannian manifold M is called locally symmetric if its curvature tensor R is parallel, i.e., $\nabla R = 0$, where ∇ denotes the Levi-Civita connection. As a generalization of locally symmetric manifolds the notion of semisymmetric manifolds was defined by

$$\left(R(X, Y) \cdot R\right)(U, V)W = 0, \quad X, Y, U, V, W \in TM$$

and studied by many authors[1-3]. Z.I. Szabo[4] gave a full intrinsic classification of these spaces. R. Deszcz[5,6] weakened the notion of semi symmetry and introduced the notion of pseudo symmetric manifolds by

$$(1.1)\qquad (R(X,Y)\cdot R)(U,V)W = L_R\left[((X\Lambda Y)\cdot R)(U,V)W\right],$$

where L_R is smooth function on M and $X\Lambda Y$ is an endomorphism defined by

$$(1.2)\qquad (X\Lambda Y)Z = g(Y,Z)X - g(X,Z)Y .$$

In his study on Betti numbers of Kaehler manifolds, S. Bochner introduced a tensor which plays similar role of the Weyl tensor on Riemannian manifolds. Thus a conformally flat manifold is an extension of a real space form. So a Bochner flat Kaehler manifold has to be an extension in the same sense of a complex space form. By Putting additional structures to Kaehler manifolds one can also study classes of odd dimensional manifolds and in particular Sasakian, co-Kaehlerian /cosymplectic and Kenmotsu manifolds.

D. Janssens and L.Vanhecke[7] using decomposition theory defined Bochner curvature for a class of almost contact metric manifolds known as C-Bochner curvature tensor. The elements of this class are called as almost $C(\alpha)$-manifolds. These are defined by the conditions on the Riemannian curvature.

These manifolds are extensively studied by[8-14] and obtained results on conformally flat, on Ricci tensor and quasi-conformal curvature tensor, on quasi conformally flat spaces, on the Conhormonic and Concircular curvature tensors, on different curvature tensors, on W_2-curvature tensors of $C(\alpha)$ manifolds respectively. In the present paper we extend to C-Bochner curvature tensor, section 2 contains preliminaries. In section 3 we study flat C-Bochner curvature tensor in almost $C(\alpha)$ manifold. In section 4 we study almost $C(\alpha)$ manifolds satisfying $B\cdot S=0$. In section 5 we study almost $C(\alpha)$ manifolds satisfying $B\cdot R=0$. In section 6 we study almost $C(\alpha)$-manifolds satisfying $B\cdot S = LsQ(g,S)$. In section 7 we study almost $C(\alpha)$ manifolds satisfying $B\cdot R = L_R Q(g,R)$.

2. Preliminaries

Let M be $(2n+1)$ dimensional connected differentiable manifold endowed with an almost contact metric structure (φ, ξ, η, g) where φ is a tensor field of type $(1,1)$, ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that

(2.1) $$\varphi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1,$$

(2.2) $$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \quad X, Y \in TM,$$

(2.3) $$\varphi\xi = 0, \quad \eta(\varphi X) = 0,$$

(2.4) $$g(\varphi X, X) = 0,$$

(2.5) $$(\nabla_X \varphi)(Y) = g(X, Y)\xi - \eta(Y).$$

If an almost contact Riemannian manifold M satisfies the condition

(2.6) $$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$$

for some smooth functions a and b on M, then M is said to be an η-Einstein manifold. If, in particular, $a = 0$ then this manifold will be called a special type of η-Einstein manifold.

Definition 2.1: *An almost contact manifold is called an almost $C(\alpha)$ manifold if the Riemannian curvature R satisfies the following relation*[7]

(2.7) $$R(X, Y)Z = R(\varphi X, \varphi Y)Z - \alpha\big[g(Y, Z)X - g(X, Z)Y - g(\varphi Y, Z)\varphi X + g(\varphi X, Z)\varphi Y\big],$$

where, $X, Y, Z \in TM$ and α is a real number.

Remark 2.1: *A $C(1)$-curvature tensor is a Sasakian curvature tensor, a $C(0)$-curvature tensor is a co-Kahler or CK-curvature tensor and a*

$C(-1)$ *curvature tensor is a Kenmotsu curvature tensor.*

Example 1.1: For 3-Dimensional almost $C(\alpha)$ manifold. We consider a 3-dimensional manifold $M=\{(x, y, z)\in R^3; z\neq 0\}$, where (x, y, z) are the standard co-ordinates in R^3. Let $\{E_1, E_2, E_3\}$ be linearly independent global frame field on M given by $E_1 = z\left(\frac{\partial}{\partial x}+y\frac{\partial}{\partial z}\right)$, $E_2 = z\frac{\partial}{\partial y}$, $E_3=\frac{\partial}{\partial z}$.

Let g be the Riemannian metric defined by

$$g(E_1, E_2)=g(E_2, E_3)=g(E_1, E_3)=0,$$

$$g(E_1, E_1)=g(E_2, E_2)=g(E_3, E_3)=1,$$

where g is given by

$$g=\frac{1}{z^2}\left[\left(1-y^2z^2\right)dx\otimes dx+dy\otimes dy+dz\otimes dz\right].$$

The (φ, ξ, η) is given by $\eta=dz-ydx$, $\xi=E_3=\frac{\partial}{\partial z}$, $\varphi E_1=-E_2$, $\varphi E_2=E_1$, $\varphi E_3=0$. The linearity property of φ and g yields that $\eta(E_e)=1$, $\varphi^2U=-U+\eta(U)E_3$, $g(\varphi U, \varphi W)=g(U,W)-\eta(U)\eta(W)$, for any vector fields U, W on M. By the definition of Lie bracket, we have

$$[E_1, E_2]=yE_2-z^2E_3,$$

$$[E_1, E_3]=-\frac{1}{z}E_1,\ [E_2, E_3]=-\frac{1}{z}E_2$$

Let ∇ be the Levi-Civita connection with respect to the above metric g by the Koszula formula.

$$2g(\nabla_X Y, Z)=X\left(g(Y, Z)\right)+Y\left(g(Z, X)\right)-Z\left(g(X, Y)\right)$$
$$-g\left(X,[Y, Z]\right)-g\left(Y,[X, Z]\right)+g\left(Z,[X, Y]\right)$$

Then,

$$\nabla_{E_1} E_3 = -\frac{1}{z} E_1 + \frac{1}{2} z^2 E_2,$$

$$\nabla_{E_2} E_3 = -\frac{1}{z} E_2 - \frac{1}{2} z^2 E_1, \ \nabla_{E_3} E_3 = 0,$$

$$\nabla_{E_2} E_2 = yE_1 + \frac{1}{2} E_3, \ \nabla_{E_1} E_2 = -\frac{1}{2} z^2 E_3,$$

$$\nabla_{E_2} E_1 = \frac{1}{2} z^2 E_3 - yE_2, \ \ \nabla_{E_1} E_1 = \frac{1}{z} E_3,$$

$$\nabla_{E_3} E_2 = -\frac{1}{2} z^2 E_1, \ \ \nabla_{E_3} E_1 = \frac{1}{2} z^2 E_2.$$

The tangent vectors X, Y and Z to M are expressed as linear combination of E_1, E_2, E_3 that is, $X = \sum_{i=1}^{3} a_i E_i$, $Y = \sum_{i=1}^{3} b_i E_i$ and $Z = \sum_{i=1}^{3} c_i E_i$, where a_i, b_i and c_i are scalars. Clearly (φ, ξ, η, g) and X, Y, Z satisfying (2.7). Thus M is a almost $C(\alpha)$-manifold.

From (2.7) we have the following

(2.8) $$R(X,Y)\xi = R(\varphi X, \varphi Y)\xi - \alpha\left[\eta(Y)X - \eta(X)Y\right],$$

(2.9) $$R(\xi, Y)Z = -\alpha\left[g(Y,Z)\xi - \eta(Z)Y\right],$$

(2.10) $$R(\xi, Y)\xi = -\alpha\left[\eta(Y)\xi - Y\right],$$

(2.11) $$R(\xi, \xi)Z = 0.$$

On an almost $C(\alpha)$ manifold, we also have[10]

(2.12) $$QX = AX + B\eta(X)\xi,$$

where

(2.13) $$A = -\alpha(2n-1), \quad B = -\alpha$$

where Q is the Ricci operator, i.,e $g(QX, Y) = S(X, Y)$ for all vector fields on the tangent space of M.

$$\eta(QX) = (A+B)\eta(X), \tag{2.14}$$

$$S(X, Y) = Ag(X, Y) + B\eta(X)\eta(Y), \tag{2.15}$$

$$r = -4n^2\alpha, \tag{2.16}$$

$$S(X, \xi) = (A+B)\eta(X), \tag{2.17}$$

$$S(\xi, \xi) = A+B, \tag{2.18}$$

$$g(QX, Y) = S(X, Y). \tag{2.19}$$

The C-Bochner curvature tensor is given by[15]

$$\begin{aligned} B(X, Y)Z &= R(X, Y)Z + \frac{1}{2n+4}\Big[g(X, Z)QY - S(Y, Z)X \\ &-g(Y, Z)QX + S(X, Z)Y + g(\varphi X, Z)Q\varphi Y - S(\varphi Y, Z)\varphi X \\ &-g(\varphi Y, Z)Q\varphi X + S(\varphi X, Z)\varphi Y + 2S(\varphi X, Y)\varphi Z \\ &+2g(\varphi X, Y)Q\varphi Z + \eta(Y)\eta(Z)QX - \eta(Y)S(X, Z)\xi \\ &+\eta(X)S(Y, Z)\xi - \eta(X)\eta(Z)QY\Big] - \frac{D+2n}{2n+4}\Big[g(\varphi X, Z)\varphi Y \\ &-g(\varphi Y, Z)\varphi X + 2g(\varphi X, Y)\varphi Z\Big] + \frac{D}{2n+4}\Big[\eta(Y)g(X, Z)\xi \\ &-\eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y - \eta(X)g(Y, Z)\xi\Big] \\ &-\frac{D-4}{2n+4}\Big[g(X, Z)Y - g(Y, Z)X\Big], \end{aligned} \tag{2.20}$$

where $D = \dfrac{r+2n}{2n+2}$ and r is the Scalar Curvature.

In View of (2.7) we get from above,

(2.21)
$$\begin{aligned} B(X,Y)Z &= R(\varphi X,\varphi Y)Z-\alpha\big[g(Y,Z)X-g(X,Z)Y \\ &-g(\varphi Y,Z)\varphi X+g(\varphi X,Z)\varphi Y\big]+\frac{1}{2n+4}\big[g(X,Z)QY \\ &-S(Y,Z)X-g(Y,Z)QX+S(X,Z)Y+g(\varphi X,Z)Q\varphi Y \\ &-S(\varphi Y,Z)\varphi X-g(\varphi Y,Z)Q\varphi X+S(\varphi X,Z)\varphi Y \\ &+2S(\varphi X,Y)\varphi Z+2g(\varphi X,Y)Q\varphi Z+\eta(Y)\eta(Z)QX \\ &-\eta(Y)S(X,Z)\xi+\eta(X)S(Y,Z)\xi-\eta(X)\eta(Z)QY\big] \\ &-\frac{D+2n}{2n+4}\big[g(\varphi X,Z)\varphi Y-g(\varphi Y,Z)\varphi X+2g(\varphi X,Y)\varphi Z\big] \\ &+\frac{D}{2n+4}\big[\eta(Y)g(X,Z)\xi-\eta(Y)\eta(Z)X+\eta(X)\eta(Z)Y \\ &-\eta(X)g(Y,Z)\xi\big]-\frac{D-4}{2n+4}\big[g(X,Z)Y-g(Y,Z)X\big]. \end{aligned}$$

We also have the following from the above,

(2.22)
$$B(X,Y)\xi=R(\varphi X,\varphi Y)\xi+\frac{2(\alpha+1)}{(n+2)}\big[\eta(X)Y-\eta(Y)X\big],$$

(2.23)
$$B(\xi,Y)Z=\frac{2(\alpha+1)}{n+2}\big[\eta(Z)Y-g(Y,Z)\xi\big],$$

(2.24)
$$B(X,\xi)Z=\frac{2(\alpha+1)}{n+2}\big[g(X,Z)\xi-\eta(Z)X\big],$$

(2.25)
$$B(\xi,Y)\xi=\frac{2(\alpha+1)}{n+2}\big[Y-\eta(Y)\xi\big],$$

(2.26)
$$B(\xi,\xi)Z=0,$$

(2.27)
$$\eta\big(B(\xi,Y)Z\big)=\frac{2(\alpha+1)}{n+2}\big[\eta(Z)\eta(Y)-g(Y,Z)\big],$$

(2.28)
$$\eta\big(B(X,\xi)Z\big)=\frac{2(\alpha+1)}{n+2}\big[g(X,Z)-\eta(X)\eta(Z)\big]$$

(2.29) $$\eta\left(B(\xi, Y)\xi\right) = 0 .$$

3. Flat C-Bochner Curvature Tensor in Almost $C(\alpha)$-Manifold

Let us consider an almost $C(\alpha)$-manifold which has flat C-Bochner curvature tensor i.,e $B = 0$, then we get from (2.21)

(3.1)
$$\begin{aligned} 0 &= R(\varphi X, \varphi Y)Z - \alpha\left[g(Y, Z)X - g(X, Z)Y\right.\\ &\left.-g(\varphi Y, Z)\varphi X + g(\varphi X, Z)\varphi Y\right] + \frac{1}{2n+4}\left[g(X, Z)QY\right.\\ &-S(Y, Z)X - g(Y, Z)QX + S(X, Z)Y + g(\varphi X, Z)Q\varphi Y\\ &-S(\varphi Y, Z)\varphi X - g(\varphi Y, Z)Q\varphi X + S(\varphi X, Z)\varphi Y\\ &+2S(\varphi X, Y)\varphi Z + 2g(\varphi X, Y)Q\varphi Z + \eta(Y)\eta(Z)QX\\ &\left.-\eta(Y)S(X, Z)\xi + \eta(X)S(Y, Z)\xi - \eta(X)\eta(Z)QY\right]\\ &-\frac{D+2n}{2n+4}\left[g(\varphi X, Z)\varphi Y - g(\varphi Y, Z)\varphi X + 2g(\varphi X, Y)\varphi Z\right]\\ &+\frac{D}{2n+4}\left[\eta(Y)g(X, Z)\xi - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y\right.\\ &\left.-\eta(X)g(Y, Z)\xi\right] - \frac{D-4}{2n+4}\left[g(X, Z)Y - g(Y, Z)X\right]. \end{aligned}$$

Taking inner product of (3.1) with W

(3.2)
$$\begin{aligned} 0 &= g\left(R(\varphi X, \varphi Y)Z, W\right) - \alpha\left[g(Y, Z)g(X, W) - g(X, Z)g(Y, W)\right.\\ &\left.-g(\varphi Y, Z)g(\varphi X, W) + g(\varphi X, Z)g(\varphi Y, W)\right] + \frac{1}{2n+4}\left[g(X, Z)\right.\\ &g(QY, W) - S(Y, Z)g(X, W) - g(Y, Z)g(QX, W) + S(X, Z)\\ &g(Y, W) + g(\varphi X, Z)S(\varphi Y, W) - S(\varphi Y, Z)g(\varphi X, W) - g(\varphi Y, Z)\\ &g(\varphi X, W) + S(\varphi X, Z)g(\varphi Y, W) + 2S(\varphi X, Y)g(\varphi Z, W)\\ &+2g(\varphi X, Y)S(\varphi Z, W) + \eta(Y)\eta(Z)S(X, W) - \eta(Y)\eta(W) \end{aligned}$$

$$S(X,Z)+\eta(X)\eta(W)S(Y,Z)-\eta(X)\eta(Z)S(Y,W)\Big]-\frac{D+2n}{2n+4}$$

$$\Big[g(\varphi X,Z)g(\varphi Y,W)-g(\varphi Y,Z)g(\varphi X,W)+2g(\varphi X,Y)g(\varphi Z,W)\Big]$$

$$+\frac{D}{2n+4}\Big[\eta(Y)\eta(W)g(X,Z)-\eta(Y)\eta(Z)g(X,W)$$

$$+\eta(X)\eta(Z)g(Y,W)-\eta(X)\eta(W)g(Y,Z)\Big]$$

$$-\frac{D-4}{2n+4}\Big[g(X,Z)g(Y,W)-g(Y,Z)g(X,W)\Big].$$

Put $Y=Z=e_i$ in (3.2) and taking summation $i=1,2,.........n$ we get,

(3.3)
$$0=S(\varphi X,W)-\alpha\Big[(n-1)g(X,W)-g(\varphi X,\varphi W)\Big]$$

$$+\frac{1}{2n+4}\Big[-(n-3)S(X,W)-rg(X,W)-6S(\varphi X,\varphi W)$$

$$+\big(r-2(A+B)\big)\eta(X)\eta(W)\Big]+\frac{3(D+2n)}{2n+4}\Big[g(\varphi X,\varphi W)\Big]$$

$$+\frac{D}{2n+4}\Big[-(n-2)\eta(X)\eta(W)-g(X,W)\Big]$$

$$-\frac{D-4}{2n+4}\Big[-(n-1)g(X,W)\Big].$$

Using (2.2) and (2.15) in (3.3) we get

(3.4)
$$0=S(\varphi X,W)-\alpha\Big[(n-2)g(X,W)-\eta(X)\eta(W)\Big]$$

$$+\frac{1}{2n+4}\Big[-(n-3)S(X,W)-(r+6A)g(X,W)$$

$$+\big(6A+r-2(A+B)\big)\eta(X)\eta(W)\Big]+\frac{3(D+2n)}{2n+4}$$

$$\Big[g(X,W)-\eta(X)\eta(W)\Big]+\frac{D}{2n+4}\Big[-(n-2)\eta(X)$$

$$\times\eta(W)-g(X,W)\Big]-\frac{D-4}{2n+4}\Big[-(n-1)g(X,W)\Big].$$

Interchanging X and W

(3.5)
$$0=S(\varphi W, X)-\alpha\left[(n-2)g(W, X)-\eta(W)\eta(X)\right]$$
$$+\frac{1}{2n+4}\left[-(n-3)S(W, X)-(r+6A)g(W, X)\right.$$
$$\left.+(6A+r-2(A+B))\eta(W)\eta(X)\right]+\frac{3(D+2n)}{2n+4}$$
$$\left[g(W, X)-\eta(W)\eta(X)\right]+\frac{D}{2n+4}\left[-(n-2)\eta(W)\right.$$
$$\left.\times\eta(X)-g(W, X)\right]-\frac{D-4}{2n+4}\left[-(n-1)g(W, X)\right].$$

Adding (3.4) and (3.5) and using the values of A and B we get

(3.6)
$$S(X,W)=\frac{1}{n-3}\left[-2\alpha\left(3n^2+6n-7\right)+2(D+3n)\right.$$
$$\left.(n-1)(D-4)\right]g(X,W)+\frac{1}{n-3}\left[-2\alpha\left(2n^2+5n-1\right)\right.$$
$$\left.-3(D+2n)-D(n-2)\right]\eta(X)\eta(W).$$

Theorem 3.1: *A C-Bochnerly flat almost $C(\alpha)$ manifold is η-Einstein manifold.*

4. Almost $C(\alpha)$ manifolds satisfying $B\cdot S=0$

Let us consider an almost $C(\alpha)$ manifold M with $B\cdot S=0$. Then we get

$$(B(X,Y)\cdot S)(U,V)=0,$$

(4.1)
$$S(B(X,Y)U,V)+S(U, B(X,Y)V)=0.$$

Putting $V=\xi$ in (4.1), we get

$$S(B(X,Y)U, \xi)+S(U, B(X,Y)\xi)=0.$$

Using (2.22) we get from above

(4.2)
$$S\left(B(X,Y)U,\xi\right)+S\left(U,B(\varphi X,\varphi Y)\xi\right)+\frac{2(\alpha+1)}{n+2}$$
$$\left[\eta(X)S(Y,U)-\eta(Y)S(X,U)\right]=0.$$

Putting $X=\xi$ in (4.2) we get

(4.3)
$$S\left(B(\xi,Y)U,\xi\right)+\frac{2(\alpha+1)}{n+2}\left[S(Y,U)-(A+B)\eta(Y)\eta(U)\right]=0.$$

Using (2.23) in (4.3) and in view of (2.17) and (2.18) we get

(4.4)
$$\frac{2(\alpha+1)}{n+2}\left[S(Y,U)-(A+B)g(Y,U)\right]=0.$$

Therefore either $\alpha=-1$ or $S(Y,U)=(A+B)g(Y,U)$.
Thus we can state the following

Theorem 4.1: *Every almost* $C(\alpha)$ *manifold satisfying* $B\cdot S=0$ *is an Einstein manifold Provided* $\alpha=-1$.

Remark 4.4: *If* $\alpha=-1$, *then almost* $C(\alpha)$ *manifold is not an Einstien manifold and it reduces to Kenmotsu structure.*

5. Almost $C(\alpha)$ Manifolds Satisfying $B\cdot R=0$

Let us consider an almost $C(\alpha)$ manifold M with $B\cdot R=0$. Then we get

$$\left(B(X,Y)\cdot R\right)(U,V)Z=0,$$

(5.1)
$$B(X,Y)R(U,V)Z-R\left(B(X,Y)U,V\right)Z$$
$$-R\left(U,B(X,Y)V\right)Z-R(U,V)B(X,Y)Z=0.$$

Putting $Y=U=\xi$ in (5.1) and using (2.9) and (2.22) we get

(5.2)
$$\frac{2(\alpha+1)}{n+2}\left[R(X,V)Z-\alpha\left[g(X,Z)V-g(V,Z)X\right]\right]=0.$$

(5.3) Either $\alpha=-1$ or $R(X,V)Z=\alpha\left[g(X,Z)V-g(V,Z)X\right]$.

Taking inner product of (5.3) with W, we get

$$R(X,V,Z,W)Z=\alpha\left[g(X,Z)g(V,W)-g(V,Z)g(X,W)\right]. \tag{5.4}$$

Putting $V=Z=e_i$ in (5.4) and taking summation $i=1,2,......n$ we get

$$S(X,W)=\alpha\left[g(X,W)-(2n+1)g(X,W)\right],$$

$$S(X,W)=-2\mathrm{n}\alpha\mathrm{g}(X,W). \tag{5.5}$$

Thus we are in a position to state the following

Theorem 5.1: *Every almost $C(\alpha)$ manifold satisfying $B\cdot R=0$ is an Einstein manifold provided $\alpha\neq-1$.*

Remark 5.1: *If $\alpha=-1$, then almost $C(\alpha)$ manifold is not an Einstien manifold and it reduces to Kenmotsu structure.*

6. Almost $C(\alpha)$ manifolds satisfying $B\cdot S=L_S Q(g,S)$

Let us consider an almost $C(\alpha)$ manifold M with $B\cdot S=L_S Q(g,S)$, then we have

$$(B(X,Y)\cdot S)(U,V)=L_S((X\Lambda Y)\cdot S)(U,V),$$

$$S(B(X,Y)U,V)+S(U,B(X,Y)V)$$
$$=L_S\left[(S(X\Lambda Y)U,V)+S(U,(X\Lambda Y)V)\right]. \tag{6.1}$$

Putting $V=\xi$ in (6.1)

$$S(B(X,Y)U,\xi)+S(U,B(X,Y)\xi)$$
$$=L_S\left[(S(X\Lambda Y)U,\xi)+S(U,(X\Lambda Y)\xi)\right]. \tag{6.2}$$

Putting $X=\xi$ in (6.2) and in view of (2.17), (2.18) and (2.22) we get,

$$\frac{2(\alpha+1)}{n+2}\left[S(Y,U)-(A+B)g(Y,U)\right]$$
$$=-L_S\left[S(Y,U)-(A+B)g(Y,U)\right],$$

(6.3) $$\left[L_S+\frac{2(\alpha+1)}{n+2}\right]\left[S(Y,U)-(A+B)g(Y,U)\right]=0$$

Therefore either $L_S=\frac{-2(\alpha+1)}{n+2}$ or $S(Y,U)=(A+B)g(Y,U)$.
Thus we are in a position to state the following

Theorem 6.1: *Every almost* $C(\alpha)$ *manifold satisfying* $B\cdot S=L_S Q(g,S)$ *is an Einstein manifold provided* $L_S\neq\frac{-2(\alpha+1)}{n+2}$

Remark 6.1: *If* $L_S=\frac{-2(\alpha+1)}{n+2}$, *then almost* $C(\alpha)$ *manifold is not an Einstien manifold and it reduces to Kenmotsu structure.*

7. Almost $C(\alpha)$ Manifolds Satisfying $B\cdot R=L_R Q(g,R)$

Let us consider an almost $C(\alpha)$ manifold with $B\cdot S=L_S Q(g,S)$, then we have

$$\big(B(X,Y)\cdot R\big)(U,V)Z=L_R\big((X\Lambda Y)\cdot R\big)(U,V)Z,$$

(7.1) $$\big(B(X,Y)\cdot R\big)(U,V)Z=L_R\Big[(X\Lambda Y)R(U,V)Z-R\big((X\Lambda Y)U,V\big)Z$$
$$-R\big(U,(X\Lambda Y)V\big)Z-R(U,V)(X\Lambda Y)Z\Big].$$

Putting $Y=U=\xi$ In (7.1) and using (2.9), (2.11) and (2.22) we get

$$\frac{2(\alpha+1)}{n+2}\left[R(X,V)Z+\alpha\big(g(V,Z)X-g(X,Z)V\big)\right]$$

$$=-L_R\left[R(X,V)Z+\alpha\left(g(V,Z)X-g(X,Z)V\right)\right],$$

(7.2) $$\left[L_R+\frac{2(\alpha+1)}{n+2}\right]\left[R(X,V)Z+\alpha\left(g(V,Z)X-g(X,Z)V\right)\right]=0,$$

(7.3) Either $L_R=\frac{-2(\alpha+1)}{n+2}$ or $R(X,V)Z=\alpha\left[g(V,Z)X-g(X,Z)V\right]$.

Taking inner product of (7.3) with W , we get

(7.4) $$R(X,V,Z,W)=\alpha\left[g(X,Z)g(V,W)-g(V,Z)g(X,W)\right].$$

Putting $V=Z=e_i$ in (7.4) and taking summation $i=1,2,......,n$ we get

$$S(X,W)=\alpha\left[g(X,W)-(2n+1)g(X,W)\right],$$

(7.5) $$S(X,W)=-2nag(X,W)$$

Theorem 7.1: *Every almost* $C(\alpha)$ *manifold satisfying* $B\cdot S=L_S Q(g,S)$ *is an Einstein manifold provided* $L_R\neq\frac{-2(\alpha+1)}{n+2}$.

Remark 7.1: *If* $L_R=\frac{-2(\alpha+1)}{n+2}$, *then almost* $C(\alpha)$ *manifold is not an Einstien manifold and it reduces to Kenmotsu structure.*

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