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On Half Soft β Open Sets

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Abstract: The authors in this paper aim to introduce and study the concepts of half-soft β open sets. Furthermore, the properties of half-soft β separated and half-soft β connected sets are introduced and discussed with suitable examples. Finally, the half-soft β continuous function is defined and studied.

Keywords: soft sets, half soft β separated sets, half soft β connected sets, half soft β continuity.

1. Introduction

Complex problems in many other disciplines, like economics, sociology, engineering, and medicine, could not be resolved utilizing vague approaches. Traditional approaches, such as problem modeling, proved unsuccessful in solving the issues. Different kinds of uncertainty existed that couldn't be resolved using these conventional techniques.

To address this problem, Zadeh[1] in the year 1965 proposed the Fuzzy Set Theory to represent and process the concepts which are vague. The idea of fuzzy set theory was to give fuzzy membership which would assign a membership grade from 0 to 1 to each object.

Along with fuzzy set theory, other theories like rough set theory[2], vague set theory[3] and others have been used for almost three decades to solve these problems. But Molodtsov[4] in 1999 described that all these theories which were considered to be the tools to deal with uncertainties had some difficulties on their own. The difficulty was that they were inadequate to use the parametrization tool.

Molodtsov[4] therefore proposed soft set theory which is a whole new approach to model uncertainties. Soft set theory has been successfully applied in many areas, including game theory, Riemann integration, Perron integration, etc. As mentioned in his paper, this theory did not suffer from the limitations of parametrization. Research in this area has advanced significantly.

In 2002, soft sets were used to determine decision-making problems by Maji *et al*.[5] who also did a detailed study on soft sets. In 2011, a topology was defined over a family of soft sets by Shabir and Naz[6], called the soft topology. They also defined some basic ideas of soft topological spaces like soft open sets, soft closed sets, soft closure, soft neighborhood of a point, soft normal space, and soft regular space and mentioned several of their properties. The study on soft topological spaces was further continued by researchers like Zorlutuna *et al*.[7], Aygunoglu and Aygun[8] and Hussain et al. contributed several significant findings to the study of soft topological spaces. Later A Kharral *et al*.[9] made a study on different properties of the images of soft sets and also the inverse images of soft sets while defining the mapping on soft classes giving suitable examples.

It was Chen[10] who initiated to investigation and study of the weaker forms of soft open sets and their properties. He contributed a number of significant findings to the study of soft semi-open sets in a soft topological space.

In recent years many weaker forms of soft open sets have been studied by a lot of researchers in soft topological space like Arockiarani and Arokialancy[11] who defined soft β-open sets and Akdag and Ozkan[12,13] who came up with soft α-open sets and continued the study on soft β-open sets. Further, studies were made in soft topological space by Benchalli *et al*.[14-16]. They investigated and studied soft β-separation axioms, soft β-compactness, soft β-connected spaces. In addition to this more properties were discussed and explored to the concepts of soft β-first countable and soft β-second countable spaces with suitable examples.

In this article, we define the idea of half-soft β-separated sets and go over some of its characteristics. In addition, we describe and explore the characteristics of half-soft β-connected sets. The definition of a half-soft continuous function is defined and discussed in detail with suitable examples.

2. Preliminaries

In this section, some basic definitions and findings that we will utilize throughout the paper are provided. Throughout the entire paper, we will consider (X, τ, E) to be soft topological space.

Definition 2.1:[8] Let the initial universe be X and E be the set of parameters. We shall denote $P(X)$ to be the power set of X. If we have a non empty subset A of E, then the pair (F, A) is called a soft set over X where F is a mapping $F: A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of X. For $\epsilon \in A, F(\epsilon)$ can be considered as a set of ϵ-approximate elements of soft set (F, A). Clearly a soft set is not a set.

Definition 2.2:[8] If τ is the collection of soft sets over X, then τ is a soft topology on X if the following conditions hold:

1. X, ϕ belong to τ.
2. Union of any number of soft sets in τ belongs to τ.
3. Intersection of any two sets in τ belongs to τ.

Definition 2.3:[13] Let (X, τ, E) be a soft topological space over X and (F, E) be the soft set over X. Then

1. Soft interior of (F, E) is the soft set int $(F, E) = \cup\{(O, E) : (O, E)$ which is soft open and $(O, E) \subset (F, E)\}$.
2. Soft closure of (F, E) is the soft set cl$(F, E) = \cap\{(G, E) : (G, E)$ which is soft open and $(F, E) \subset (G, E)\}$.

Clearly cl(F, E) is the smallest soft closed set over X which contains (F, E) and int(F, E) is the largest soft open set over X which is contained in (F, E).

Definition 2.4:[13] A soft set (A, E) of a soft topological space (X, τ, E) is a soft α- open set if

$$(A, E) \subset int\ (cl\ (int\ (A, E))).$$

The compliment of soft α-open set is soft α-closed set.

Definition 2.5:[11] A soft set (A, E) of a soft topological space (X, τ, E) is a soft β- open set if

$$(A, E) \subset cl\ (int\ (cl\ (A, E))).$$

The compliment of soft β-open set is soft β-closed set.

Definition 2.6:[16] Let (X, τ, E) be a soft topological space and (F_1, E) and (F_2, E) be two soft β-open sets over X. Then these soft β-open sets are said to be soft β-separated if

$$(F1, E) \cap s\beta\ cl(F2, E) = \phi \text{ and } s\beta\ cl(F1, E) \cap (F2, E) = \phi$$

Definition 2.7:[16] Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is soft β- connected if there does not exist a pair (F_1, E) and (F_2, E) of non-empty disjoint open subsets of (X, τ, E) such that $(F_1, E) \cup (F_2, E) = X$. Otherwise (X, τ, E) is said to be soft β-disconnected. In this case, the pair (F_1, E) and (F_1, E) are called soft β-disconnection of X.

3. Main Results

Definition 3.1: Two null soft sets F_e and G_e of a soft topological space (X, τ, E) are said to be soft half $\beta -$ separated sets if

$$F_e \cap S\beta cl(G_e) = \phi \quad or \quad S\beta cl(F_e) \cap G_e = \phi.$$

Obviously, from the fact that $S\beta cl(F_e) \subseteq Scl(F, E)$, for each set $F_e \in SS(x, \tau_e)$. Thus every soft half separated set is soft half β separated. But the converse may not be true from the following example.

Let $Y = \{k_1, k_2, k_3, k_4\}, E = \{e_1, e_2\}$

where

$$G_1(E) = \{G_1(e_1) = \{k_1\}, G_1(e_2) = \{k_1\}\}$$

$$G_2(E) = \{G_2(e_1) = \{k_2\}, G_2(e_2) = \{k_2\}\}$$

$$G_3(E) = \{G_3(e_1) = \{k_1 k_2\}, G_3(e_2) = \{k_1 k_2\}\}$$

The soft sets L_e and M_e which is defined by

$$L_e = \{L(e_1) = \{k_1\}, L(e_2) = \{k_1\}\}$$

$$M_e = \{M(e_1) = \{k_3, k_4\}, M(e_2) = \{k_3, k_4\}\}$$

are half soft β- separated but not soft half separated.

Definition 3.2: A soft set G_e of a soft topological space (X, τ, E) is said to be soft half β-clopen if it is both half soft β-open set and half soft β-closed set.

Proposition 3.3:

1. Every two half soft β- separated sets are always soft disjoint.
2. Each two disjoint soft sets in which both of them are either half soft β open sets or half soft β closed sets are half soft β-separated.

Theorem 3.4: *Let F_e and G_e be a non-null soft sets of the topological space (X, τ, E). Then the following statements hold*:

1. *If Fe and Ge are half soft β-separated and He ⊆ Fe and Ie ⊆ Ge then He and Ie are half soft β-separated sets.*
2. *If Fe and Ge are half soft β- open sets, Me = Fe ∩ (X - Ge) and Ne = Ge ∩ (X - Fe), then Me and Ne are half soft β-separated sets.*

Proof: Let *Fe* and *Ge* be half soft *β*- open sets. Then, (*X* -*Fe*) and (*X* -*Ge*) are half soft *β*-closed soft sets. Assume that *Me* = *Fe* ∩ (*X* - *Ge*) and *Ne* = *Ge* ∩ (*X* - *Fe*), then *Me* ⊆ (*X* - *Ge*) and *Ne* ⊆ (*X* - *Fe*).Hence *Shβ cl*(*Me*) ⊆ (*X* - *Ge*) ⊆ (*X* - *Ne*) or *Shβ cl*(*Ne*) ⊆ (*X* - *Fe*) ⊆ (*X* - *Me*). Consequently, *Shβ cl*(*Me*) ∩ *Ne* = ϕ or *Shβ cl*(*Ne*) ∩ *Me* = ϕ. Therefore, *Me* and *Ne* are half soft *β* separated sets.

Since $He \subsetneq Fe$ then $Sh\beta\ cl(He) \subsetneq Sh\beta\ cl(Fe)$. Hence $Ie \cap Sh\beta\ cl(He) \subsetneq Ge \cap Sh\beta\ cl(Fe) = \phi$. Similarly, $He \cap Sh\beta\ cl(Ie) = \phi$ or $Sh\beta\ cl(He) \cap Ie = \phi$. Thus He and Ie are half soft β- separated sets.

Theorem 3.5: *Any two soft sets F_e and G_e of a soft topological space (X, δ, E) are half soft β-separated sets if and only if there exists half soft β-open sets L_e and F_e such that $F_e \subseteq L_e$, $G_e \subseteq M_e$ and $F_e \cap M_e = \phi$, $G_e \cap L_e = \phi$.*

Proof: Let (X, δ, E) be a soft topological space. Let us consider F_e and G_e be half soft β-separated sets. Then $F_e \cap Hs\beta Cl(G_e) = \phi$ or $Hs\beta\ cl(F_e) \cap G_e = \phi$. Let $L_e = X - hs\beta\ cl(F_e)$ and $M_e = X - hs\beta\ cl(G_e)$. Then L_e and M_e are half soft β-open sets such that $F_e \subseteq L_e$; $G_e \subseteq M_e$ and $G_e \cap L_e = \phi$ and $F_e \cap M_e = \phi$. On the other side, let L_e, M_e be half soft β-open sets such that $F_e \subseteq L_e$ and $G_e \subseteq M_e$ and $G_e \cap L_e = \phi$ and $F_e \cap M_e = \phi$. Since $(X - M_e)$ and $(X - L_e)$ are half soft β-closed sets, then $hs\beta cl(F_e) \subseteq (X - M_e) \subseteq (X - G_e)$ or $hs\beta cl(G_e) \subseteq (X - L_e) \subseteq (X - F_e)$. Thus $hs\beta cl(F_e) \cap G_e = \phi$ or $hs\beta cl(G_e) \cap F_e = \phi$. This implies that F_e and G_e are half soft β-separated sets.

Definition 3.6: Let (X, δ, E) be a soft topological space. Two soft subsets F_e and G_e of space δ are said to be half soft β-separated if and only if $F_e \cap s\beta cl(G_e) = \phi$ or $s\beta cl(F_e) \cap G_e = \phi$.

Definition 3.7: Let (X, δ, E) be a soft topological space. A soft subset F_e of a soft space δ is said to be half soft β-connected, if F_e is not the union of two soft non-empty half β- separated sets in δ.

Definition 3.8: A soft subset F_e of a space δ is said to be half soft β-connected , if F_e is not the soft union of two non-empty half β-separated sets in the soft topological space (X, δ, E).

Theorem 3.9: *Let (X, δ, E) be a soft topological space and is said to be soft half connected if and only if it cannot be expressed as the disjoint union of non-empty soft open* (*soft β-open*) *set and a non-empty soft closed* (*soft β-closed*) *set.*

Proof: Let (X, δ, E) be a soft topological space. Assume that $X = F_e \cap G_e = \phi$, where $F_e \cap G_e = \phi$, F_e is a non-empty soft open set and G_e is a non-empty soft closed set in δ. Since G_e is a soft closed set in δ, $F_e \cap cl(G_e) = \phi$ and therefore F_e and G_e are half soft separated. Thus (X, δ, E) is not a half soft connected space, which gives the contradiction. On the other hand, we suppose that (X, δ, E) is not a half soft connected (half soft β-connected) space. Then there exists non-empty half soft separated sets (half soft β-separated sets) L_e and M_e such that $X = L_e \cup M_e$. Let $L_e \cap scl(M_e) = \phi$. We set $F_e = X - scl(M_e)$ and $G_e = scl(M_e)$.. Then $F_e \cup G_e = X$ and $F_e \cup G_e = \phi$. Also F_e is a non-empty soft open set (soft β-open) and G_e is a non-empty soft closed (soft β-closed) set.

Theorem 3.10: *Let (X, δ, E) be a soft topological space. If F_e is a half soft connected (soft β-connected) subset of δ and L_e and M_e are half soft separated (half soft β-separated) subsets of δ with $F_e \subseteq (L_e \cup M_e)$, then either $F_e \subseteq L_e$* or $F_e \subseteq M_e$.

Proof: Let (X, δ, E) be a soft topological space and given that F_e is a half soft connected (half softβ-connected) set and $F_e \subseteq (L_e \cup M_e)$. Given that L_e and M_e are half soft separated $M_e \cap scl(L_e) = \phi$ or $scl(M_e) \cap L_e = \phi$. Let us consider $M_e \cap scl(L_e) = \phi$. Since $F_e \cong (F_e \cap L_e) \cup (F_e \cap M_e)$, then $(F_e \cap M_e) \cup scl(F_e \cap L_e) \subseteq M_e \cap scl(L_e) = \phi$. If we consider $F_e \cap L_e$ and $F_e \cap M_e$ are non soft empty then F_e is not soft half connected, which is a contradiction. Thus either $F_e \cap L_e = \phi$ or $F_e \cap M_e = \phi$, which implies that $F_e \subseteq L_e$ or $F_e \subseteq M_e$.

Theorem 3.11: *If F_e and G_e are soft half connected (soft half β-connected) sets of a space X and F_e and G_e are not soft half separated, then $F_e \cup G_e$ is soft half connected (soft half β-connected).*

Definition 3.12: Let (X, δ, E) and (Y, δ, E) be soft topological spaces and $f: X \to Y$ be a mapping then

(i) f is called soft half continuous if $f^{-1}(F_e)$ is half soft closed set over X for every half soft closed set (F_e) over Y.
(ii) soft half closed if the image under f of each half closed in X is half closed in Y.
(iii) soft half irresolute if for each point $x \in X$ and each half open set F_e of Y containing $f(x)$, there exists a half open set G_e of X containing e_x such that $f(G_e) \subseteq F_e$.

Theorem 3.13.: *The soft continuous image of a soft half connected space is soft half connected.*

Proof: Let $f: X \to Y$ be a continuous function and X be a soft half connected space. Suppose that $f(X)$ is not soft half connected subset of Y. Then there exists soft half separated sets F_e and G_e in Y such that $f(X) = F_e \cup G_e$. Since F_e and G_e are soft half separated, $scl(F_e) \cap G_e = \phi$ or $F_e \cap scl(G_e) = \phi$. Since f is soft continuous, $scl(f^{-1}(F_e) \cap f^{-1}(G_e)) \subseteq f^{-1}(scl(F_e) \cap f^{-1}(G_e)) = f^{-1}(scl(F_e) \cap G_e) = \phi$ or $f^{-1}(F_e) \cap scl(f^{-1}(G_e)) \subseteq f^{-1}(G_e) \cap f^{-1}(scl(G_e)) = f^{-1}(F_e \cap scl(G_e)) = \phi$. Since $F_e \neq G_e$, there exists a soft point $e_x \in X$ such that $f(e_x) \in F_e$ and hence $f^{-1}(F_e) \neq \phi$. Similarly $f^{-1}(G_e) \neq \phi$. Therefore $f^{-1}(F_e)$ and $f^{-1}(G_e)$ are soft half separated sets such that $X = f^{-1}(F_e) \cup f^{-1}(G_e)$. Therefore X is not a soft half connected space, which is a contradiction, which completes the proof.

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