

Propagation of the Swine Influenza in the Cervical Esophagus: A Mathematical Paradigm

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Abstract: Influenza is a viral infection that affects human respiratory system: lungs, throat, nose etc. Influenza virus spreads from person to person by sneezing, coughing as well as the fluid medium. Further, the virus is carried to various parts of the human body to infect tissues during swallowing, breathing, drinking. In the present model, we study the diffusion of H1N1 virus in the viscous fluid medium via saliva which will help to predict the spread of the virus particles in the esophagus. According to that, a mathematical model is developed to simulate the nature of Swine Influenza virus through the saliva driven by the natural pumping mechanism. The BBO equation is considered for simulation of progression of virus particles in cervical esophagus. The impacts of fluid (saliva viscosity), and virus particle diameter have been analyzed and addressed the significance of various pertinent physical parameters.

Keywords: Influenza virus; Swine Influenza; peristaltic motion; BBO equation.

1. Introduction

The flu usually goes away on its own for most of living organisms and affects the internal and external parts of the body. It causes a variety of symptoms, some of which are severe enough to require hospitalization or even result in death. Typically, there are four types of Influenza viruses found i.e. A, B, C, and D. Influenza A is the only virus that can cause a pandemic which is a global disease outbreak. Influenza B can cause a seasonal epidemics which only affects humans. While Influenza B & C viruses lead to mild illnesses¹⁻³. Swine influenza (H1N1) is one of the types of influenza A virus that causes the seasonal flu. Ram *et al.*⁴ have studied the transmission of Swine influenza virus in the esophagus with the transverse magnetic field.

They showed that a magnetic field reduces the motion of the Swine influenza virus particles. Further many researchers have studied the transmission of the influenza virus and showed how virus transmission can be controlled⁵⁻⁷.

Analysts regularly examine how the Influenza virus changes and evolves in nature. In this manner, it is found that Swine Influenza virus particles are spherical in shape with diameters from 80-120nm⁸. Complex flow phenomena are involved in the behaviour of virus particles within the human body. To understand how the Influenza virus spreads throughout the gastrointestinal system, physical interpretation of virus propagation inside the esophagus is crucial. In this way, using the Basset-Boussinesq-Oseen (BBO) equation, Maxey and Riley⁹ have given the expression for the motion of particles inside the non-uniform fluid flow. BBO equation describes the motion of nano-size spherical particles in non-uniform unsteady flow which is the sum of virtual mass, gravity, buoyancy, viscous, and Basset forces. Based on the BBO equation, Tripathi *et al.*¹⁰ studied the transmission of virus particles through the blood flow in the vessels. They showed how the thermal effect controls the transmission of the coronavirus. Islam *et al.*¹¹ discussed how far SARS-CoV-2 can go through the respiratory system. Smith *et al.*¹² demonstrated that severely infected individuals with high viral loads in their saliva and super spreaders who release a lot of aerosols are likely to be much more deadly transmitters of infection. However, no literature examined Swine influenza virus transmission through viscous fluid in the esophagus with peristaltic pumping considering the effect of virus diameter, virus density, and fluid viscosity.

Keeping in view the gap in the literature on virus movement in a viscous medium, a mathematical model is developed to investigate virus propagation through the saliva driven by the peristaltic pumping. Peristaltic pumping is a unique form of pumping that makes it simple to move a variety of complex rheological fluids from one place to another. The mechanism includes involuntary periodic contraction followed by relaxation or expansion of the ducts. This pumping can be observed in food transport in the digestive tract, blood flow in vessels, urine flow from the kidney to the bladder, etc. On peristaltic flow, some important investigations¹³⁻¹⁷ have been reported. Expressions are derived for virus particle axial and transverse velocities to see the propagation along the saliva movement. This study provides observation, which will aid in understanding how the influenza virus transmits through fluid flow in the digestive system.

2. Problem Formulation

2.1 Peristaltic movement of saliva through the cervical esophagus

In this analysis, a two-dimensional incompressible viscous fluid flow is considered. The viscous fluid flow is governed by the peristaltic mechanism. The governing equations for the same are considered as:

$$(1) \quad \begin{cases} \nabla \cdot \vec{V}_f = 0 \\ \rho \left(\frac{\partial \vec{V}_f}{\partial \bar{t}} + (\vec{V}_f \cdot \nabla) \vec{V}_f \right) = -\nabla \bar{p} + \mu (\nabla^2 \vec{V}_f) \end{cases},$$

where ρ , μ , $\bar{t}$, and $\bar{p}$ symbolize respectively saliva density, viscosity, time, and pressure. $\vec{V}_f = (\bar{u}_f, \bar{v}_f, 0)$ denotes the velocity vector of saliva. We assume that the fluid (saliva) flow is governed due to the rhythmic propagation of the walls (i.e., peristaltic pumping) as shown in Figure-1, and walls propagation is mathematically formulated as:

$$(2) \quad \bar{h}(\bar{x}, \bar{t}) = a_0 + a \cos^2 \frac{\pi}{\lambda} (\bar{x} - c\bar{t}),$$

where $\bar{h}$ and a represent respectively transverse vibration of the walls and amplitude of the wave. c, λ denote the wave velocity, and wavelength of the cervical esophagus (channel) respectively. $R_w = a + a_0$ is the half-width of the channel.

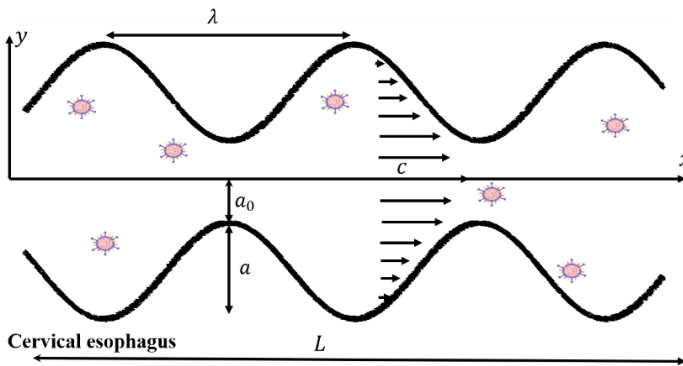


Fig. 1. Schematic representation of the virus transport in peristaltic flow medium.

Introducing scaling parameters; $x = \frac{\pi \bar{x}}{\lambda}$, $y = \frac{\bar{y}}{R_w}$, $t = \frac{c\pi \bar{t}}{\lambda}$, $u_f = \frac{\bar{u}_f}{c}$, $v_f = \frac{\bar{v}_f}{c\varepsilon}$, $h = \frac{\bar{h}}{R_w}$, $\phi = \frac{a}{a_0}$, $p = \frac{\bar{p}\varepsilon R_w}{\mu c}$, where x , y , t , u_f , v_f , p , stand respectively for dimensionless counterparts of the dimensional parameters. $\varepsilon = \pi \frac{R_w}{\lambda}$ is the wave number, $Re = (\frac{\rho c R_w}{\mu})$ is a Reynolds number, and $\nabla^2 = \varepsilon^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is considered as the Laplacian operator. The flow transport under the lubrication theory (low Reynolds number ($Re \in [0, \varepsilon]$) and large wavelength number $\varepsilon \ll 1$) where the viscous effect is dominating. Neglecting the term of $Re \in [0, \varepsilon]$, we obtained the governing equations in the following forms:

$$(3) \quad \frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0, \quad \frac{\partial p}{\partial x} = \frac{\partial^2 u_f}{\partial y^2}, \quad \frac{\partial p}{\partial y} = 0.$$

The following boundary conditions are considered as:

no-slip boundary condition at the wall $y = h$, i.e. $u_f|_{y=h} = 0$, fluid stress condition at the center $y = 0$, i.e. $\frac{\partial u_f}{\partial y}|_{y=0} = 0$, rate of change of impermeable

wall, i.e. $v_f|_{y=h} = \frac{\partial h}{\partial t}$, regular condition as $v_f|_{y=0} = 0$ and pressure at inlet and outlet are $p|_{y=0} = p(0, t)$ and $p|_{y=L} = p(L, t)$.

The axial velocity, transverse velocity and pressure gradient are derived respectively as:

$$(4) \quad u_f = \frac{1}{2} \frac{\partial p}{\partial x} (y^2 - h^2),$$

$$(5) \quad v_f = \frac{y}{6} \left\{ 6h \frac{\partial h}{\partial x} \frac{\partial p}{\partial x} - \frac{\partial^2 p}{\partial x^2} (y^2 - 3h^2) \right\},$$

$$(6) \quad \frac{\partial p}{\partial x} = \frac{1}{h^3} \left[G(t) + 3 \int_0^x \frac{\partial h}{\partial t} ds \right],$$

where $G(t)$ is the arbitrary function of t and computed as:

$$(7) \quad G(t) = \frac{\Delta p_L - 3 \int_0^L \frac{1}{h^3} \int_0^x \frac{\partial h}{\partial t} ds}{\int_0^L h^{-3} dx}.$$

The pressure difference along the channel length can be computed as:

$$(8) \quad \Delta p_L = p(L, t) - p(0, t) = \int_0^L \frac{\partial p}{\partial x} dx.$$

The stream function can be defined as $u_f = \frac{\partial \psi}{\partial y}$, and $v_f = -\frac{\partial \psi}{\partial x}$ and derived as:

$$(9) \quad \psi = \frac{\partial p}{\partial x} \left(\frac{y^3}{6} - \frac{h^2 y}{2} + \frac{h^3}{3} \right).$$

2.2. Virus transport in cervical esophagus

The viscous fluid flow of the nano-size virus transport is described by the Basset-Boussinesq-Oseen (BBO) equation at a low Reynolds number (Jiménez-Lozano *et al.*¹⁹). The Stokes' drag force in relation to the BBO equation to see the motion of virus transport in a non-uniform flow is considered. The equation of motion of particles (viruses) in two-dimensional form is $\frac{\pi}{6} d_p^3 \rho_p \frac{d\vec{V}_p}{dt} = -3\pi d_p \mu \xi \{\vec{V}_p - \vec{V}_f\}$. Here $\vec{V}_p = \{u_p, v_p\}$ is the particle velocity vector, $\vec{g}$ is the gravity vector, ρ_p is the density of the virus, $r = \frac{d_p}{2}$ is the particle radius and d_p is the particle diameter.

The non-dimensional velocity components are defined as $u_p = \frac{\bar{u}_p}{c}$ and $v_p = \frac{\bar{v}_p}{c}$. The fluid characteristic time is defined as $\tau_c = \frac{L}{\pi c}$, the fluid characteristic time is defined as $\tau_p = \frac{\rho_p d_p^2}{18\mu}$. The nondimensional BBO equation with its force terms per particle mass is given by

$$(10) \quad \frac{dv_p}{dt} = \frac{\xi}{S_N} \left(\frac{2S}{2S+1} \right) (V_f - V_p) = \beta_1 (V_f - V_p),$$

where $S = \rho_p / \rho_f$ is the density ratio, $S_N = \frac{\tau_p}{\tau_c}$ is the Stokes number. For the spherical shape, the drag force and Stokes drag are the same however drag force is defined as: $F_{drag} = 3\pi d_p \mu \xi (V_f - V_p)$, where ξ is unity for the sphere. The axial and transverse components of the particle velocity vector are expressed as:

$$(11) \quad \frac{du_p}{dt} = \beta_1 (u_f - u_p) \text{ and } \frac{dv_p}{dt} = \beta_1 (v_f - v_p)$$

Thus, the solutions of equation (11) are obtained subjected to the boundary condition $u_p|_{t=0} = 0$, and $v_p|_{t=0} = 0$:

$$(12) \quad u_p = e^{-\beta_1 t} \int_0^t e^{\beta_1 t} \beta_1 u_f dt \text{ and } v_p = e^{-\beta_1 t} \int_0^t e^{\beta_1 t} \beta_1 v_f dt.$$

3. Numerical results and discussion

To analyse the behaviour of Swine Influenza (H1N1 virus)⁴ in the region of the cervical esophagus, we computed the axial and transverse particle velocities of the H1N1 virus for two different values of fluid viscosity based on the human saliva range in $0.89 - 2.9 \times 10^{-3}$. Then we discussed the variation in the normalized axial particle velocity with time for a better understanding of the virus propagation in the viscous fluid medium. At last, the influence of the virus particle diameter on the velocity profile is discussed. Based on the Sharp *et al.*⁸ study of the swine influenza virus, the diameter of the H1N1 virus is taken in the range of 80-120 nm, and the density $\rho_p = 1100 \text{ kg/m}^3$ of the H1N1 is retained for the more accurate result of the present analysis. From the literature survey¹⁸, it is found that the length of the cervical esophagus is about 5-6 cm long with a diameter in the range of 1.4-1.5 cm, where saliva is transported into the esophagus through peristaltic motion^{4,20} whose viscosity varies from $0.89-1.29 \times 10^{-3}$. Based on that parametric values, the particle relaxation time $\tau_p = \frac{\rho_p d_p^2}{18\mu}$ and the fluid characteristic time $\tau_c = \frac{L}{\pi c}$ is calculated. Further, the effect of stokes' drag

force for the H1N1 virus on the viscous fluid medium is calculated by using the relation between particle relaxation time and fluid characteristic time as $(S_N = \frac{\tau_p}{\tau_c})$. And the density ratio $(S = \rho_p/\rho)$ of the particle to fluid is about 1.104. For the computational simulation of the virus particle in the cervical esophagus, the parametric value of the $\beta_1 = \frac{\xi}{S_N} \left(\frac{2S}{2S+1} \right)$ is calculated for the $\mu = 0.89 \times 10^{-3}$.

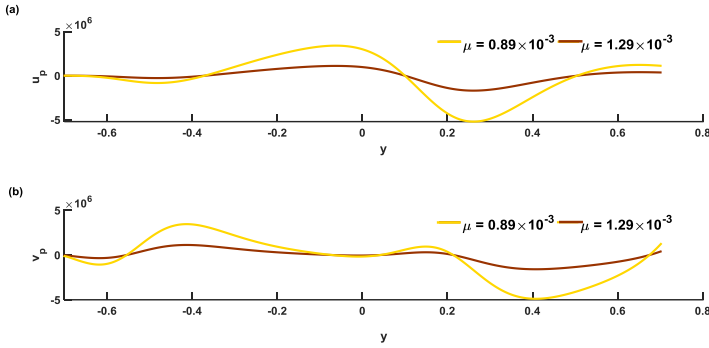


Fig. 2. Effect of fluid viscosity on (a) axial virion velocity (b) transverse virion velocity for Swine Influenza.

Figures 2 (a & b) show the profiles of axial and transverse velocities of the H1N1 virus for different values of fluid viscosity. From figure 1a, it is found that the fluctuation in the axial particle velocity is more for the $\mu = 0.89 \times 10^{-3}$ than 1.29×10^{-3} . It means that the less viscosity is responsible for the increment in the axial particle velocity as shown in Fig 2a. Similarly, transverse particle velocity is more for the small viscosity as compared to the high viscosity which is depicted in Fig 2b. From this result, it is concluded that during the drinking process, the thickness of the viscosity is reduced which is highly spreading the transmission of the H1N1 virus in the cervical esophagus.

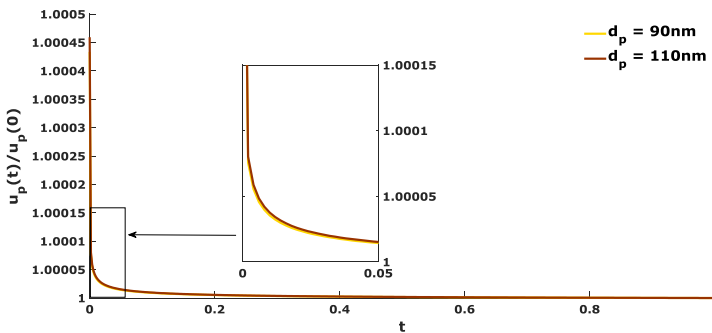


Fig. 3. Normalized axial velocity with respect to time for different diameter of H1N1 virus.

Figure-3 shows the variation in normalized axial virus velocity with time space for different virus diameters. It is seen that the normalized axial virus velocity decreases in the initial time period to maintain the flow and further behave asymptotically. Also, there is negligible change is observed in the normalized axial velocity for both values of virus particle diameter, i.e. $d_p = 90nm$ and $110nm$. Since the density of the virus particle does not vary due to the virus diameter that means the flow behaviour is almost similar at any position over time.

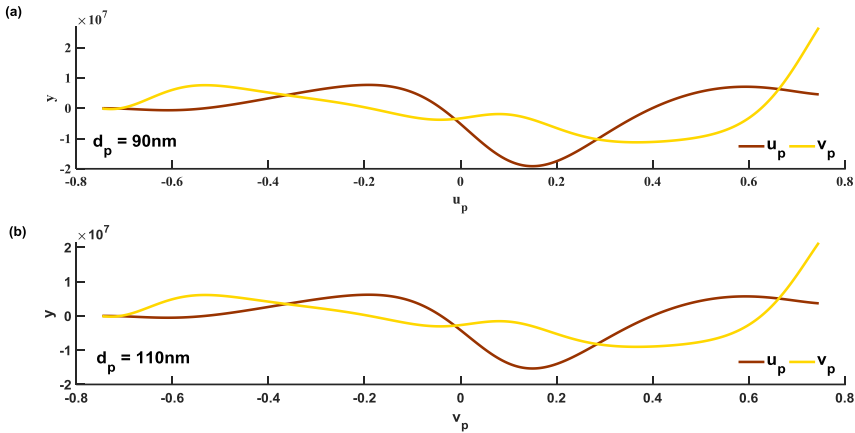


Fig. 4. Axial and transverse virion velocities for (a) $d_p = 90nm$ (b) $d_p = 110nm$ at fixed value of $\mu = 0.89 \times 10^{-3}$.

Figure-4 shows the axial and transverse velocities of Swine Influenza for different values of virus diameter. From the figure, it is observed that the magnitude of both velocities is maximum for the $d_p = 90nm$ as shown in Fig. 4a. It means that the small virus particle rapidly propagates as compared to the large particle $d_p = 110nm$ as depicted in Fig. 4b. However, the flow pattern is almost similar for both diameters.

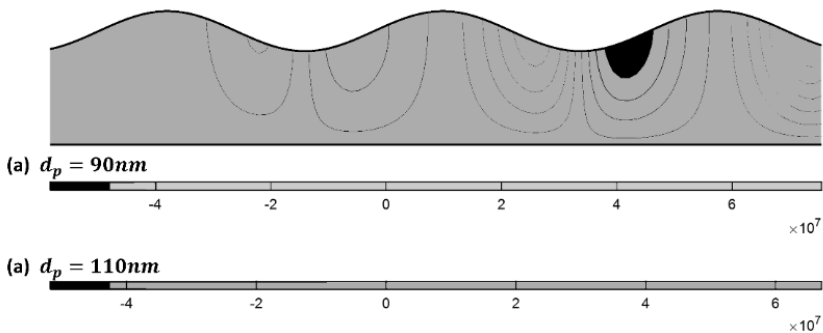


Fig. 5. Contour plots of Swine Influenza virus for (a) $d_p = 90nm$ and (b) $d_p = 110nm$.

In Fig. 5, the contour of the axial velocity is sketched which gives a better understanding of virus transport phenomena associated with the physiological system. This figure shows that virus particle streamlines are more predominant close to the contraction region of the cervical esophagus. It is noted that the maximum magnitude is attained near the contraction region.

4. Conclusions

Based on the simulated results, it is interesting to mention that the Stokes drag force acting on the spherical particles of the H1N1 virus during the motion, plays a significant role in the transmission of the virus infections through Saliva. It is further concluded that the virus particles may be circulated in the fluid flow medium where the flow is governed by rhythmic propagation and more circulations of the viruses are reported in the contracted region as compared to the relaxed region. During the drinking process, the viscosity of the saliva is less causing the rapid movement of the H1N1 virus into the esophagus, resulting in the infection may be transported to the kidney or liver. It is further concluded that the small size of the virus particle is rapidly propagated as compared to the large virus particle. This study will help biomedical engineers to develop some technologies for controlling virus transmission in the physiological transport phenomena.

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References

1. Edward C. Hutchinson; Influenza virus, *Trends in Microbiology*, 26-9 (2018), 809-810.
2. Ronan Le Goffic, Viviane Balloy, Micheline Lagranderie, Lena Alexopoulou, Nicolas Escriou, Richard Flavell, Michel Chignard and Mustapha Si-Tahar; Detrimental contribution of the Toll-like receptor (TLR) 3 to influenza A virus-induced acute pneumonia, *PLoS pathogens*, 2-6 (2006), e53.
3. Christophe Paget, and François Trottein; Mechanisms of bacterial superinfection post-influenza: a role for unconventional T cells, *Frontiers in immunology*, 10 (2019), 336.
4. Ram, Daya, D. S. Bhandari, Dharmendra Tripathi and Kushal Sharma; Propagation of H1N1 virus through saliva movement in oesophagus: a mathematical model, *The European Physical Journal Plus*, 137-7 (2022), 866.
5. O. Sharomi, C. N. Podder, A. B. Gumel, S. M. Mahmud and Rubinstein; Modelling the transmission dynamics and control of the novel 2009 swine influenza (H1N1) pandemic, *Bulletin of mathematical biology*, 73-3 (2011), 515-548.

6. Suzanne L. Epstein; Control of influenza virus infection by immunity to conserved viral features, *Expert review of anti-infective therapy*, 1-4 (2003), 627-638.
7. Soyoung Kim, Jonggul Lee, and Eunok Jung; Mathematical model of transmission dynamics and optimal control strategies for 2009 A/H1N1 influenza in the Republic of Korea, *Journal of theoretical biology*, 412 (2017), 74-85.
8. D. G. Sharp, A. R. Taylor, I. W. McLean Jr, D. Beard and J. W. Beard; Densities and sizes of the influenza viruses A (PR8 strain) and B (Lee strain) and the swine influenza virus, *Journal of Biological Chemistry*, 159-1 (1945), 29-44.
9. Martin R. Maxey and James J. Riley; Equation of motion for a small rigid sphere in a nonuniform flow, *The Physics of Fluids*, 26-4 (1983), 883-889.
10. Dharmendra Tripathi, D. S. Bhandari and O. Anwar Bég; Thermal effects on SARS-CoV-2 transmission in peristaltic blood flow: Mathematical modeling, *Physics of Fluids*, 34-6 (2022), 061904.
11. M. S. Islam, P. Larpuenrudee, A. R. Paul, G. Paul, T. Gemci, Y. Gu and S. C. Saha; SARS CoV-2 aerosol: How far it can travel to the lower airways, *Physics of Fluids*, 33-6 (2021), 061903.
12. S. H. Smith, G. A. Somsen, C. Van Rijn, S. Kooij, L. Van Der Hoek, R. A. Bem and D. Bonn; Aerosol persistence in relation to possible transmission of SARS-CoV-2, *Physics of Fluids*, 32-10 (2020), 107108.
13. V. K. Narla, Dharmendra Tripathi and D. S. Bhandari; Thermal analysis of micropolar fluid flow driven by electroosmosis and peristalsis in microchannels, *International Journal of Ambient Energy*, (2022), 1-13.
14. D. Tripathi, J. Prakash, O. A. Bég and R. Kumar; by 2 Peristaltic Wave Propagation with Electroosmotic and Magnetohydrodynamic Effects: Applications in Nanotechnological Energy Systems, *Energy Systems and Nanotechnology*, (2021), 223.
15. D. Tripathi, J. Prakash, M. G. Reddy and J. C. Misra; Numerical simulation of double diffusive convection and electroosmosis during peristaltic transport of a micropolar nanofluid on an asymmetric microchannel; *Journal of Thermal Analysis and Calorimetry*, 143-3 (2021), 2499-2514.
16. D. Tripathi, J. Prakash, O. Anwar Bég and R. Kumar; Thermal Analysis of $\gamma\text{Al}_2\text{O}_3/\text{H}_2\text{O}$ and $\gamma\text{Al}_2\text{O}_3/\text{C}_2\text{H}_6\text{O}_2$ Elastico-Viscous Nanofluid Flow Driven by Peristaltic Wave Propagation with Electroosmotic and Magnetohydrodynamic Effects: Applications in Nanotechnological Energy Systems, *Energy Systems and Nanotechnology*, (2021), 223-259.
17. J. Prakash and Dharmendra Tripathi, Study of EDL phenomenon in Peristaltic pumping of a Phan-Thien-Tanner Fluid through asymmetric channel, *Korea-Australia Rheology Journal*, 32-4 (2020), 271-285.
18. I.J. Cook, W.J. Dodds, R.O. Dantas, B. Massey, M.K. Kern, I.M. Lang, J.G. Brasseur and W.J. Hogan; Opening mechanisms of the human upper esophageal sphincter, *American Journal of Physiology-Gastrointestinal and Liver Physiology*, 257-5 (1989), G748-G759.

19. Jiménez-Lozano, Joel, Mihir Sen and Patrick F. Dunn; Particle motion in unsteady two-dimensional peristaltic flow with application to the ureter, *Physical Review E*, 79-4 (2009), 041901.
20. Sethuraman Govindaraj, M. Jonathan Daniel, Srinivasan Subramanian Vasudevan; Changes in salivary flow rate, pH, and viscosity among working men and women, *Dentistry and Medical Research*, 7-2 (2019), 56. O(37).