

Fluid Transport in a Tube of Variable Cross-Sectional Area by Peristaltic Waves of Dilating Amplitude: A Mathematical Model for Investigating Impact of Hiatus Hernia on Swallowing

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Abstract: In order to extract new inferences to help medical intervention, this paper aims to construct a mathematical model to suitably characterize swallowing in an oesophagus which suffers from sliding hiatus hernia. In such a state of dysfunction of oesophagus, the stomach moves upward through the hiatal orifice; due to which, there is a widening above the hiatus.

We make an attempt to mathematically formulate the circumstances created due to herniation. The formulation is in the dimensionless parameters using the long wavelength and low Reynolds number approximations modelling Newtonian flows in tubes, converging and diverging partially, or converging somewhere and diverging somewhere else.

The model validates the practical observations and gives some clues of the impact of partial convergence and divergence on pressure distribution. Less pressure is required for flow if the tube diverges but pressure has to be increased if the tube converges. It is further inferred from computer simulation that even if merely the lower part diverges, pressure is affected right from the beginning of flow. In case of sliding hiatal hernia, it is expected, pressure requirement for swallowing will be less due to oesophageal widening near the lower sphincter. It is even an experimental report that hiatus hernia reduces LES pressure. When the oesophagus converges, unlike this, pressure requirement for flow is more. This convergence may be an alarming situation. If hiatus hernia

goes unnoticed, it is suspected that the narrowed part is less than the widened part. This inference can also be useful in peristaltically driven engineering applications.

Keywords: Oesophagus; Peristalsis; Bolus transport; hiatus hernia; Lower oesophageal sphincter, dilating wave-amplitude, variable cross-sectional area.

Nomenclature

H'	wall displacement	λ	wavelength
t'	time	c	wave velocity
ϕ'_0	initial wave-amplitude	ρ	density
k'	amplitude dilation parameter	μ	coefficient of viscosity
z'	axial coordinate	θ	angular coordinate
r'	radial coordinate	a_0	tube radius at the inlet
u'	axial velocity	R_e	Reynolds number
v'	radial velocity	m	tube gradient
P'	pressure		

Parameters without asterisk are dimensionless counterparts

1. Introduction

The junction between oesophagus and stomach is a specific region comprising lower oesophageal sphincter (abbreviated as LES) and its neighbouring anatomic structures, gastric sling and diaphragm (Mittal and Balaban¹, see Figure 1a). Its length is 2-3cm in human beings (Homby et al.²). These organs act together to prevent reflux of gastric contents across the oesophago-gastric junction into the oesophagus while LES ensures the passage of consumed food into the stomach. However, the LES lets pass gastric contents into oesophagus in the reverse direction during vomiting. This complex task is accomplished by a high-pressure zone in the les that keeps the junction unceasingly closed but relaxes momentarily to allow passage (Boeckxstaens³). Sensory information is conveyed from the LES to the brain by spinal and vagal sensory afferents (Goyal et al.⁴).

Oesophageal hiatal orifice is an elliptically shaped opening through the diaphragm with its long axis in the sagittal plane through which the oesophagus and vagus nerves gain access to the abdomen (Kahrilas et al.⁵). Hiatus hernia is a state of dysfunction of the oesophagus in which the stomach moves upward through the hiatus due to reasons like, but not limited to, old age and obesity (Stene-Larsen et al.⁶, see Figure 1b). This dislocates

the lower oesophageal sphincter, physiologically known as cardiac sphincter.

The oesophagus enters the abdomen through diaphragmatic hiatus, anchored at the level of the oesophagogastric junction by the phreno-oesophageal membrane. The hernia is vulnerable to visceral herniation because it is directly subject to pressure stress between the abdomen and the chest. The diaphragmatic margin of the hiatus is formed by the right diaphragmatic crus. The right crus and the LES together form the oesophagogastric junction which acts as a barrier against the reflux of gastric content into the oesophagus (Roman and Kahrilas⁷).

Para-oesophageal hernia is less common (5-15%) than sliding hernia (85-95%). Diaphragmatic hiatus dilates to allow fundus to herniate upward. No symptom is ever specific for hiatus hernia. However, gastro-oesophageal reflux, heartburn, regurgitation or dysphagia may be suspected with the presence of hernia (Roman and Kahrilas⁷).

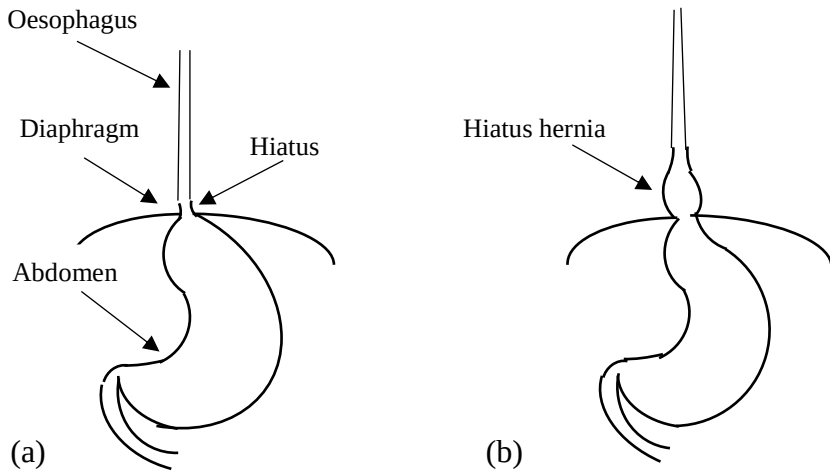


Figure 1. Diagrams for (a) Normal oesophagus and (b) Oesophagus suffering from sliding hiatus hernia

Together with the diaphragm, the LES functions as an anti-reflux barrier protecting the oesophagus from the caustic gastric content. It generates a tonic pressure of 15-30 mmHg above the intra-gastric pressure and accounts for approximately 90% of the basal pressure at the oesophagogastric junction. Hence, the rest is attributed to the crural diaphragm (Joel et al.⁸). Any dislocation makes the LES weak to withstand the pressure. Once the abdomen slips upward, the oesophagus widens, shrinks and gets curved in different parts.

Considering widening, shrinking and curving together initially might be complex for modelling. Thus, variation only in the cross-sectional area is considered while the curvilinear shape will be a matter of investigation in a separate communication.

The cross-sectional variation considered here is very mild. Whether it affects and if yes, how much the effects are felt and how it can be diagnosed, are issues of discussion in this paper.

The physical model

While investigating the presence of a high-pressure zone in the lower oesophagus pointed out by Kahrilas et al.⁹, Pandey et al.¹⁰, based on the anatomical measurements of Xia et al.¹¹ found that gradually dilating wave amplitude was a possible reason. We shall be considering this factor as a part of the model and also in the subsequent discussion. Considering progressively dilating peristaltic wave amplitude, swallowing of non-Newtonian fluids were also investigated (Pandey and Tiwari¹², Pandey and Chandra¹³). Pandey and Tiwari¹² concluded that reflux of Casson fluid takes place at high flow rates but with increasing wave amplitude Casson fluid is less prone to reflux. Pandey and Chandra¹³ concluded that pressure inside the tube enhances if coupling number and amplitude dilation parameter are increased, while micro-polar parameter reduces the pressure along the axis of the tube. Local wall shear stress too increases with amplitude dilation parameter. They also recommended that achalasia patients should avoid the consumption of micro-polar fluids. Another important result was that reflux action weakens with dilation of wave amplitude for micro-polar flows. A few more papers (Pandey and Singh¹⁴, Pandey and Tiwari¹⁵) dealt with the flow particles fluid motion. Pandey and Singh¹⁴ used a perturbation technique and presented results in the first order of the wave number (the ratio of the tube ratio to the wavelength). They observed that fluid moves faster everywhere except near the wall. They endorsed the doctors' advice to patients suffering from achalasia, oesophageal stricture and oesophageal tumors to take liquid or food items with lesser solid contents. Pandey and Tiwari¹⁵, who formulated axial pressure in terms of fixed time averaged flow rate, reported that although pressure rises with wave amplitude but diminishes with rising concentration of suspended particulate matter. However, none of these articles tried to test hiatus hernia.

Peristaltic flow through channels and tubes of variable cross-sectional areas by neglecting inertia terms was studied by Gupta and Seshadri¹⁶ who simulated their theoretical results against the experimental results obtained from a study conducted on rhesus monkeys by Guha et al.¹⁷ who had also

constructed a mathematical model for quantitative analysis. Some extensions were carried out by Srivastava and Srivastava¹⁸⁻²⁰ for physiological fluids. Misra and Pandey²¹ used a perturbation technique for small amplitudes duly considering inertia terms and found peristaltic contribution more ascribable than the previous findings for the spermatic flow reported by Guha et al.¹⁷. Pandey and Chaube²² extended the model for Maxwell fluids and the same authors²¹ further modified the model for asymmetric waves. Hariharan et al.²⁴ examined the roles of different waveforms when the tube diverges while propelling a fluid peristaltically. Several communications deal with different situations for flows through ducts of varying cross-sectional area (Manton²⁵; Safia et al.²⁶, Rao and Devanathan²⁷; Sisavath et al.²⁸) or asymmetric channel/tube.

However, here we intend to present a model of peristaltic transport concerned with hiatal hernia (Fig.1), which is different from those discussed above for the following reasons:

- The peristaltic waves have dilating amplitude while propagating along oesophageal wall (Pandey et al.¹⁰, Xia et al.¹¹).
- The cross-section of the oesophagus does not stay uniform throughout its length due to hernia but gets swelled or widened at least near the distal end.

In light of new investigations, further analyses and modified mathematical models are required to deal with the circumstances arising out of the presence of hiatus hernia. Unlike previous investigations of considering cross-sectional variation along the entire length, we shall also consider cases where cross-sectional variations are limited somewhere in the tubular length. This is an addition to the consideration that the wave amplitude gradually increases.

2. The Mathematical Model

2.1. Mathematical model for wall motion:

We model the oesophagus suffering from hiatus hernia as a circular cylindrical tube whose cross-sectional area varies as the bolus moves from the neck to the stomach. The peristaltic waves travelling along the walls of the oesophagus are created by the contraction of circular muscles and stretching of longitudinal muscles of the oesophagus. This is a natural occurrence. The peristaltic waves are further assumed to increase their amplitude to make sure that the delivery of the food bolus to the stomach is done by creating some additional pressure in the lower oesophagus. All aspects are modelled in the following equation proposed by Pandey et al.¹⁰

with an additional consideration that the tube radius is dependent on the axial coordinate, i.e. $a(z')$:

$$(2.1) \quad H'(z', t') = a(z') - \phi_0' e^{k'z'} \cos^2 \frac{\pi}{\lambda} (z' - ct')$$

where $H', z', t', \phi_0', k', \lambda$ and c respectively stand for the wall displacement from the centre line of the tube, the axial coordinate, time, the initial wave-amplitude, the amplitude dilation parameter, the wavelength, the axial and the wave-velocity. The tube radius $a(z')$ will be defined in sequel.

2.2 Mathematical model for fluid motion

We consider an axisymmetric unsteady flow of a viscous Newtonian incompressible fluid in a circular cylindrical tube of length L , with slightly varying cross-sectional area (see Figures 2 & 4). The equations governing the motion for unsteady flow are given by

$$(2.2) \quad \frac{1}{r'} \frac{\partial}{\partial r'} (r' v') + \frac{\partial u'}{\partial z'} = 0,$$

$$(2.3) \quad \rho \left(\frac{Dv'}{Dt'} \right) = - \frac{\partial P'}{\partial r'} + \mu \left(\frac{\partial^2}{\partial r'^2} + \frac{\partial^2}{\partial z'^2} + \frac{1}{r'} \frac{\partial}{\partial r'} - \frac{1}{r'^2} \right) v',$$

$$(2.4) \quad \rho \left(\frac{Du'}{Dt'} \right) = - \frac{\partial P'}{\partial z'} + \mu \left(\frac{\partial^2}{\partial z'^2} + \frac{\partial^2}{\partial r'^2} + \frac{1}{r'} \frac{\partial}{\partial r'} \right) u',$$

where the parameters $\rho, r', u', v', P', \mu, \theta$ represent the density, the radial coordinate, the axial and the radial velocity components, the pressure, the coefficient of viscosity, and the angular coordinate respectively. These equations of motion are subject to the following boundary conditions:

$$(2.5) \quad \begin{cases} u = 0 & \text{at } r = H(z, t) \quad (\text{i.e. the no-slip condition at the wall}), \\ \frac{\partial u}{\partial r} = 0 & \text{at } r = 0 \quad (\text{i.e. the regularity condition}), \\ v = 0 & \text{at } r = 0 \quad (\text{i.e. the radial velocity vanishes at the centre of the tube}), \\ v = \frac{\partial H(z, t)}{\partial t} & \text{at } r = H(z, t) \quad (\text{i.e. the wall undergoes a rhythmic motion with the waves}) \end{cases}$$

The set of equations together with the boundary conditions and wall equations are transformed by the following dimensionless parameters:

$$(2.6) \quad z = \frac{z'}{\lambda}, \quad t = \frac{ct'}{\lambda}, \quad \phi_0 = \frac{\phi_0'}{a_0}, \quad k = k'\lambda, \quad r = \frac{r'}{a_0}, \quad a(z) = \frac{a(z')}{a_0},$$

$$u = \frac{u'}{c}, \quad v = \frac{v'\lambda}{ca_0}, \quad p = \frac{p'a_0^2}{\mu c \lambda}, \quad H = \frac{H'}{a_0}, \quad Re = \frac{\rho c a_0^2}{\mu \lambda}.$$

where a_0 is the radius of the tube at the inlet. The non-dimensional parameter R_e is the Reynolds number. Since the flow is creeping, Equations (2.2 - 2.4), under the application of the long wavelength and low Reynolds number approximations, reduce to

$$(2.7) \quad \frac{1}{r} \frac{\partial(rv)}{\partial r} + \frac{\partial u}{\partial z} = 0,$$

$$(2.8) \quad \frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right),$$

$$(2.9) \quad \frac{\partial P}{\partial r} = 0.$$

The corresponding dimensionless form of the wall equation (2.1) is as follows:

$$(2.10) \quad H(z, t) = a(z) - \phi_0 e^{kz} \cos^2 \pi(z - t),$$

while the boundary conditions for the present model reduce to the following forms:

$$(2.11) \quad u = 0 \text{ at } r = H(z, t), \quad \frac{\partial u}{\partial r} = 0 \text{ at } r = 0, \quad v = 0 \text{ at } r = 0, \quad v = \frac{\partial H(z, t)}{\partial t} \text{ at } r = H(z, t),$$

It is worth noticing that the long wavelength approximation (i.e., $a \ll \lambda$) eliminates curvature effects. Under the first and second conditions of equation (2.11), the axial velocity is derived from equation (2.8) together with equation (2.9), as

$$(2.12) \quad u = \frac{\partial P}{\partial z} \frac{1}{4} \left\{ r^2 - a^2(z) + 2\phi_0 a(z) e^{kz} \cos^2 \pi(z - t) - \phi_0^2 e^{2kz} \cos^4 \pi(z - t) \right\}.$$

Equation (2.7), by substituting the axial velocity from equation (2.12) under the third conditions of equation (2.11), gives radial velocity as

$$(2.13) \quad v = \frac{r}{4} \left[\frac{\partial P}{\partial z} \left(a(z) - \phi_0 e^{kz} \cos^2 \pi(z - t) \right) \left\{ \frac{\partial a(z)}{\partial z} + \phi_0 e^{kz} (\pi \sin 2\pi(z - t) + k \cos^2 \pi(z - t)) \right\} + \frac{1}{2} \frac{\partial^2 P}{\partial z^2} \left\{ a^2(z) - \frac{r^2}{2} - 2a(z)\phi_0 e^{kz} \cos^2 \pi(z - t) + \phi_0^2 e^{2kz} \cos^4 \pi(z - t) \right\} \right]$$

The general axial pressure is defined as

$$(2.14) \quad P(z, t) = P(0, t) + \int_0^z \frac{\partial P}{\partial s}(s, t) ds,$$

which requires pressure gradient for determination. The pressure gradient may be obtained from equation (2.13), subject to the fourth condition of equation (2.11), as

$$(2.15) \quad \frac{\partial P}{\partial z} = \frac{\Omega_0(t) + 16\pi\phi_0 \int_0^z e^{kz} \sin 2\pi(z-t) \{a(z) - \phi_0 e^{kz} \cos^2 \pi(z-t)\} dz}{\{a(z) - \phi_0 e^{kz} \cos^2 \pi(z-t)\}^4},$$

where $\Omega_0(t)$ can be determined from equation (2.14), by evaluating the pressure difference $\Delta P(t) = P_{z=0} - P_{z=L}$ at the two ends of the tube, as

$$(2.16) \quad \Omega_0(t) = \frac{\Delta P(t)}{\int_0^L \{a(z) - \phi_0 e^{kz} \cos^2 \pi(z-t)\}^{-4} dz} - \frac{16\pi \int_0^L \{a(z) - \phi_0 e^{kz} \cos^2 \pi(z-t)\}^{-4} \int_0^z \{\phi_0 e^{ks} \sin 2\pi(s-t)(a(z) - \phi_0 e^{ks} \cos^2 \pi(s-t)) ds\} dz}{\int_0^L \{a(z) - \phi_0 e^{kz} \cos^2 \pi(z-t)\}^{-4} dz}$$

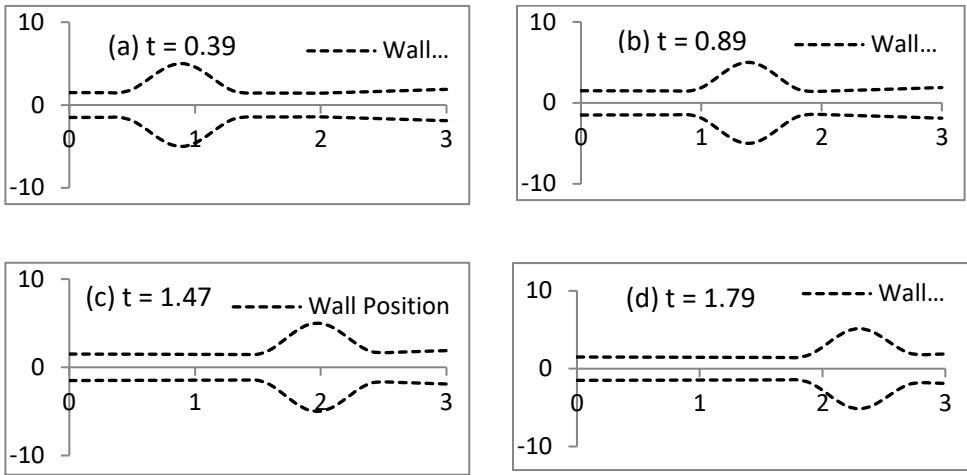
This completes the required model.

3. Results and Discussion

The main purpose of this investigation is confined to determine/ model/ examine the impact on swallowing in the varying cross-sectional area of a oesophagus suffering from hiatus hernia. Let the oesophagus have a slightly widened cross-section near the hiatus through which the stomach slightly slips upward above the hiatus. The swelled portion is symbolically modelled as $a(z) = 1 + m \times z$; where m is the tube-gradient.

Suppose that the oesophageal length is thrice the bolus, i.e., at a time the oesophagus can accommodate at most three boluses in its length. However, only one is considered to be present at a time for the entire discussion. This is because, in reality, mostly only one bolus moves/ remains in the oesophagus while eating. A train of swallowing boluses may be observed only when the fluid is Newtonian such as water.

Further consideration is that the pressures at the two ends of the oesophagus are zero. Then the pressure generated to swallow the food bolus is due only to peristalsis which is caused by progressive waves of dilating amplitude.



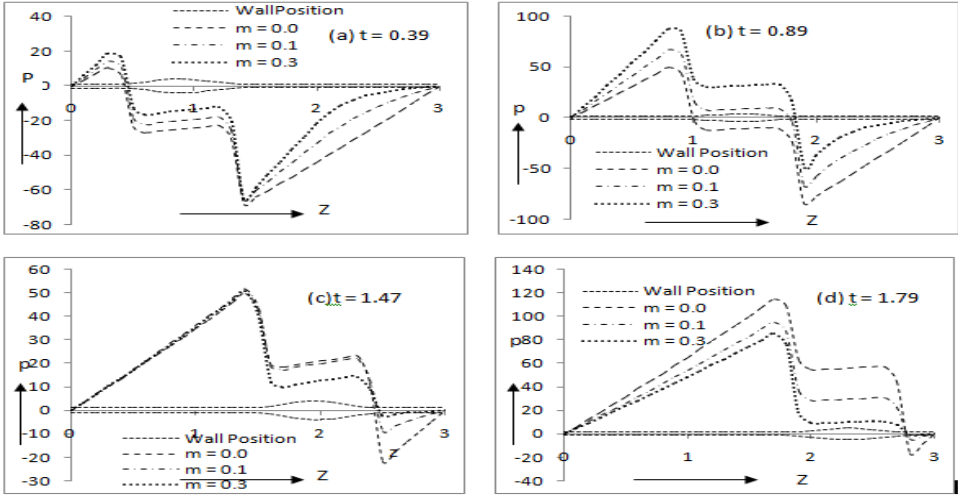
Figures 2(a-d). The diagrams exhibit the proper shapes of the tube carrying moving boluses and diverging towards the end at various instants given in the diagrams.

Pressure in the oesophagus is computed numerically along its axis when a single bolus moves in the oesophagus and is shown in Figures (3&5). There are two curves depicting pressures corresponding to two values of the wave amplitude dilating parameter k . The increment in k is being considered as small as 0.01.

Then are drawn diagrams for axial pressure to estimate temporal variations of pressure along the axis. The following cases are examined one by one:

- (a) A single bolus moves in the tube whose cross-sectional divergence is located towards the end which is caused owing to the presence of hiatus hernia (see Figures 2 &3). For the sake of analysis, we consider that the last third of the length of the oesophagus diverges. Pressure is estimated at different instants.
- (b) A single bolus moves in the tube whose cross-sectional convergence is located towards the end. Partly this may be a case due to the presence of hiatus hernia (see Figures 4 &5). Hence, unlike the previous case, we consider that the last third of the length of the oesophagus converges. Pressure is estimated at different instants.
- (c) A single bolus moves in the tube which diverges right from the beginning. Pressure is estimated at different instants (see Figure 6). This is being considered as a benchmark to compare with the previous cases.

The instants of time are randomly chosen. We consider four time instants $t = 0.39, 0.89, 1.47, 1.79$.



Figures 3(a-d). The diagrams exhibit the variation of pressure along the axis of the tube at different time instants $t = 0.39, 0.89, 1.47, 1.79$ when the oesophagus has diverged only close to its end but at different rates $m = 0.0, 0.1, 0.2$ while the wave amplitude dilates continuously with the wave amplitude dilating parameter $k = 0.01$.

Case (a): The diagrams concerning this case are given in Figures 2(a-d) for geometric display of the tube and in Figs.3(a-d) for pressure distribution. The wall positions are not clearly visible in the location where the tube diverges. Hence, for clarity, the tube shapes are separately drawn and given in Figures 2(a-d). We consider three arbitrary values of the gradient parameter $m = 0, 0.1$ & 0.3 . A uniform tube corresponds to $m = 0$ while $m = 0.1$ & 0.3 represent diverging tubes, of course, towards the end only. The wave amplitude dilation parameter is fixed as $k = 0.01$.

For $t = 0.39$ (see Figure 3(a)), a keen observation of the pressure curves reveals that behind the bolus, the pressure in a diverging tube (i.e., $m = 0.1$ & 0.3) rises more than that in the uniform tube (i.e., $m = 0$), but falls less compared to that in the uniform tube, although the divergence is located apart. In terms of magnitude, the pressure required for flow in the diverging tube is less than that in the uniform tube. As usual, the pressure rises to zero as prescribed in all the cases. But the pressure rises at a faster rate in the case of diverging tube making a concave shape while, in the uniform tube, it rises linearly.

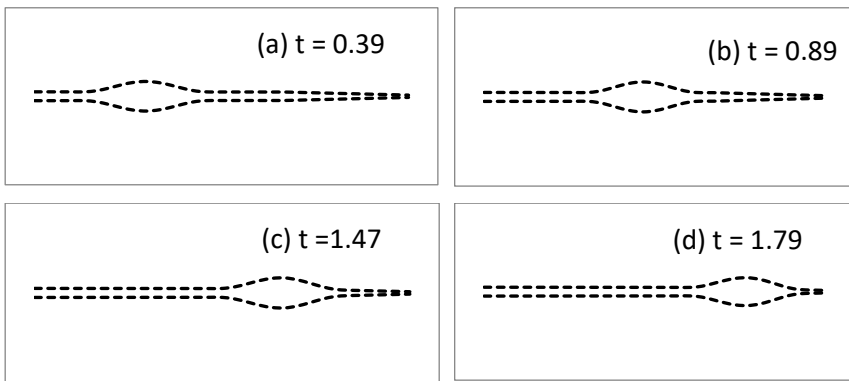
For $t = 0.89$ (see Figure 3(b)), when the bolus is farther from the initial end but a little behind the middle of the tube, such a tendency of pressure curves is more clearly visible while the magnitude of the pressure, which is the difference between the maximum and the minimum, is much larger than

that in the previous case. However, the difference of pressure for the two situations is not significant.

But once the bolus crosses the middle of the tube, pressure diminishes and it is distinct in the two different situations, pressure being less in the divergent case. This can be observed in Figure 2(c) when $t = 1.47$.

At $t = 1.79$, the bolus is near the outlet and is situated where the tube has diverged. Pressure rises here but it is less for the divergent tube in comparison with the uniform tube.

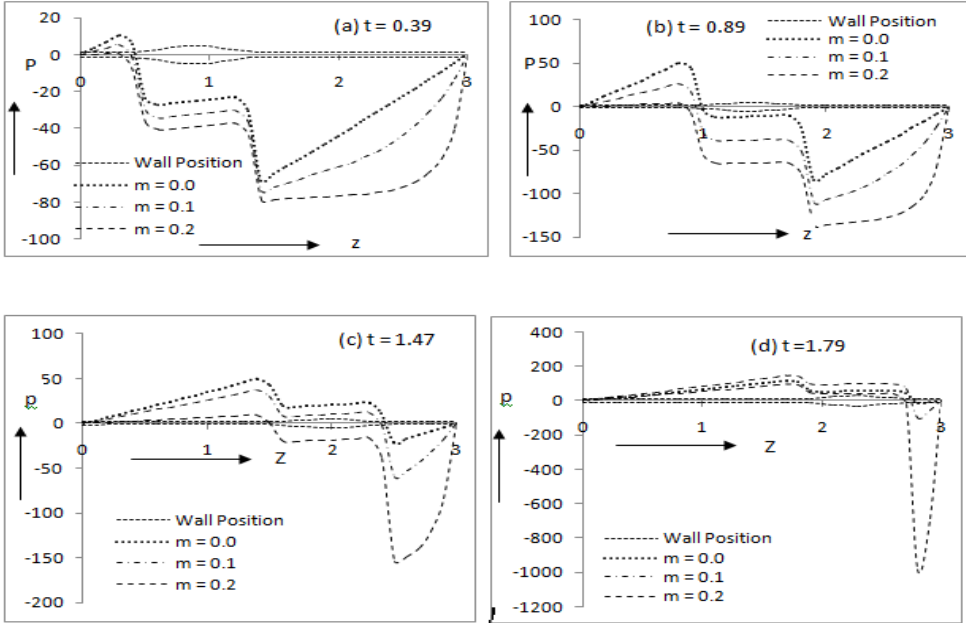
It is inferred from the analysis that although the tube diverges near the end only, its impact is felt on the pressure distribution right from the beginning. Pressure keeps increasing due to wave amplitude dilation. This divergence results in a slight increase in pressure but it is less exactly where the tube diverges. The results are interesting. Thus, a patient suffering from sliding hiatus hernia can swallow easily. The inference supports the observation that in many of the cases, sliding hernia goes unnoticed. Symptoms are rare and not very much convincing about the occurrence of herniation.



Figures 4(a-d). The diagrams exhibit the proper shapes of the tube carrying moving boluses and diverging towards the end at various instants given in the diagrams.

Case (b): We observed in the previous case that although the tube diverges towards the other end, pattern of pressure distribution differs from that of the uniform tube. This made us curious to examine the case when the tube converges only near the lower end (see Figures 4(a-d), In order to investigate this, we drew separate graphs under the following considerations:

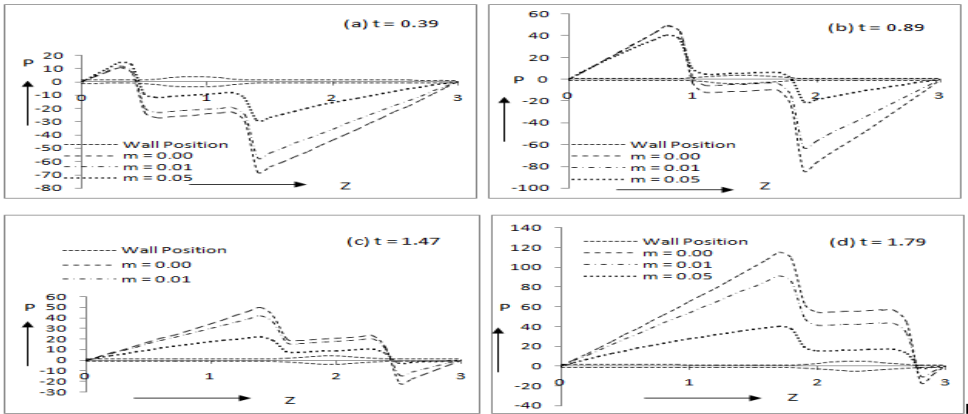
Identical time instants are taken into consideration. The wave amplitude dilating parameter is also considered the same as in the previous case, i.e., $k = 0.01$ and the tube is allowed to converge at similar rates (i.e., $m = 0.1$ & 0.3).



Figures 5(a-d). The diagrams exhibit the variation of pressure along the axis at different time instants $t = 0.39, 0.89, 1.47, 1.79$ when the oesophagus has converged close to its end but at different rates $m = 0.0, 0.1, 0.2$ while the wave amplitude dilates continuously with the wave amplitude dilating parameter $k = 0.01$.

In Figures 5(a-d) we observe that even in this case pressure is affected right from the beginning of the tube; but unlike the diverging tube case, more pressure is required to transport fluid inside the tube when it converges. The impact of convergence is more than divergence in quantitative terms, even when the degrees of divergence and convergence are maintained the same as the uniform cross-sectional area case. It is further observed that in contrast with the concave rise of pressure in the diverging tube case from the lowest point indicating the fall of pressure, the pressure rise traverses a convex curvature and it is huge in quantitative terms.

This indicates that swallowing is difficult where oesophagus narrows but in the rest of the oesophagus, swallowing is smooth. The reports say that in case of hernia the lower sphincter is weak by 10%. This will facilitate gastroesophageal reflux. At that junction oesophagus widens; hence the reverse flow is easy. For some time, peristalsis may be difficult in the reverse direction but later there will be no difficulty. Thus, occurrence of gastroesophageal reflux, including heartburn, regurgitation, or dysphagia cannot be ruled out.



Figures 6(a-d): The diagrams exhibit variation of pressure along the axis at different time instants $t = 0.39, 0.89, 1.47, 1.79$ when the oesophagus has diverges from the beginning to the end with at different rates $m = 0.0, 0.01, 0.05$ while the wave amplitude dilates continuously with the wave amplitude dilating parameter $k = 0.01$.

Case (c): In this case, the tube is let diverge from the beginning to the end (Figures 6(a-d)). The trends are similar with the exception that pressure grows with time as the bolus moves towards the end (cf. Figure 6(d)). The pressure corresponding to diverging tube is always less. Thus, the larger the divergence of the tube, the easier is swallowing. This favours the conclusion that sliding hiatus hernia may not give any significant symptom.

Physical interpretation

Since the situation considered in Case (a), *i.e.*, divergence adjacent to the end of the tube resembles the geometry of oesophagus suffering from hiatus hernia, the results obtained give some interesting clues. In case the hiatus hernia is mild, the oesophagus may diverge a bit near the hiatus since the stomach protrudes a little bit upward. In such a situation, pressure required for carrying a bolus is, in fact, less giving no trouble to the patient. Hence it may go unnoticed.

Further, Kahrilas et al.⁵ compared endoscopic, fluoroscopic and manometric landmarks of the gastro-oesophageal junction of patients with hiatus hernia and volunteer subjects to examine the influence of hiatus hernia on the intra-luminal pressure profile. They concluded on the basis of the examination that hiatus hernia reduces *LES* pressure. Therefore, since the *LES* pressure is less when affected by hiatus hernia, pressure in the distal part of the oesophagus is enough for swallowing into the stomach. Thus the inference drawn from the mathematical model endorses the experimental reports of Kahrilas et al.⁵ However, it is suspected that the situation may be grave when the degree of herniation is large. This is because not only belching will be

easier, gastric contents will find it easy to move back to the oesophagus causing severe heart burn.

Contrary to divergence, if the oesophagus converges, the pressure required for flow will be more. This may be an alarming situation. If hiatus hernia goes unnoticed, it is expected that the converging area is quite less in comparison with the divergent one. Kahrilas et al.⁵ validate this inference. Of course, when the problem is aggravated, a large part of the stomach moves upward and the oesophagus becomes loose, resulting in a more troublesome situation.

The finding that a tube, diverging either throughout or at some locations, requires lesser pressure for transportation, gives a clue from engineering point of view; however, it is somewhat similar to losing control on transportation.

4. Conclusions

The analysis endorses experimental reports on hiatus hernia that hiatus hernia reduces *LES* pressure. Hiatus hernia widens the distal oesophagus near *LES*. The useful result is that the requirement of pressure for transportation becomes less if the tube diverges. The more interesting result is that pressure requirement is less even if the divergence is located to a small portion of the tube. Moreover, more pressure is required for swallowing if the tube converges either throughout or only towards the end.

Thus, divergence of oesophagus is the probable reason that sometimes patients suffering from mild hiatus hernia do not recognize its presence; it goes unnoticed and is not even troublesome. In fact, by dilating the oesophagus at some locations, swallowing may be eased. But the patient will feel the pain if his oesophagus has a narrowed portion. This knowledge is useful in artificial intervention for medical treatment. This may also be useful for artificially devised peristaltic pumps.

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