

Magnetohydrodynamic Flow of a Dusty Viscous Fluid through Circular Tube

R. K. Khare and Neetu Singh

Department of Mathematics & Statistics
Allahabad Agricultural Institute Deemed-University
Allahabad 211007

Email: drrajeevkhare@gmail.com

(Received November 17, 2009)

Abstract: This study is concentrated on the effect of magnetic – field on the flow of dusty fluids through circular tube. After forming the equations for circular tube based on the required conditions with assumed parameters, the expression for mean velocity is found. It is observed that the increasing value of radius of channel(r) accelerates the resonance condition to occur for lower values of magnetic field (M) and the maximum mean velocity occurs for low values of r (near $M=2$). The dust particles, near the wall of the tube, seem to be more effected in this case.

1. Introduction

The flow of dusty fluids through circular channels in presence of magnetic field has got an important place in different sectors like industries, engineering and medical –science and therefore, it has attracted many mathematicians. Uchida¹ studied the pulsating flow in circular pipes; following him Micheal and Miller² examined the dusty fluid flow in a cylinder performing oscillatory motion. Verma and Mathur³ derived the expressions for the dusty viscous flow under a constant pressure gradient. Singh and Pathak⁴ investigated the flow through a tube under a time varying pressure Gradient. Later on, such flows influenced by magnetic field were studied by Guha⁵, Sastry and Seetharmaswamy⁶, Pathak and Upadhyay⁷ and Osiptsov⁸ taking different parameters and physical conditions, remarkable results were obtained. Recently, Conforto⁹ gave its theory based on axis-symmetric model. Attia¹⁰ gave important results about the Hall – effect in the flow dusty fluid in circular pipes. In this paper, motion of an incompressible dusty fluid through a circular pipe placed under a transverse magnetic field and its effect on the motion of dusty fluid have been

discussed with the help of graph and the results obtained have been verified with the physical nature of the problem.

2. Formulation of the Problem

Consider the flow of a dusty viscous incompressible fluid through a circular pipe of radius r_0 placed under a transversely applied magnetic field such that the flow occurs along the axis of the pipe. Then the governing equations of the flow based on Saffman- model of dusty viscous incompressible fluid are:

$$(2.1) \quad \frac{\partial u_1}{\partial t} = \frac{1}{\rho} \left(-\frac{\partial p}{\partial z} \right) + \nu \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} \right) + \frac{f}{\tau} (v_1 - u_1) - \frac{\sigma}{\rho} (B_0^2 u_1),$$

$$(2.2) \quad \tau \frac{\partial v_1}{\partial t} = u_1 - v_1,$$

where u_1 and v_1 are the velocity components respectively of fluid and dust particles along z - axis which is the axis of the pipe.

$f = (mN_0 / \rho)$ is the mass-concentration parameter of the dust –particles.

m is mass of the dust particles, N_0 is number density of the dust particles.

p is pressure applied in the motion, t is time, K is the Stokes resistance coefficient

τ is relaxation- parameter equal to (m/K) , ν is kinetic coefficient of viscosity

σ is electrical conductivity of the dust particles, ρ is density of the dust particles

μ is coefficient of viscosity of fluid particles, B_0 is magnetic field, and

λ is frequency parameter of the applied pressure.

The pressure gradient varying in magnitude only is assumed as

$$(2.3) \quad \frac{1}{\rho} \left(-\frac{\partial p}{\partial z} \right) = p_0 (1 + \varepsilon e^{i\lambda t}) \quad \text{such that} \quad 0 < \varepsilon < 1 \quad \text{and} \quad \lambda \text{ is the}$$

frequency parameter.

(2.4) The boundary conditions are

$u_1 = 0$ and $v_1 = 0$ when $r = r_0$ for $t > 0$, u_1, v_1 are finite when $r = 0$ for $t > 0$

To solve these equations, consider the following dimensionless quantities which on substitution in the equations (1) to (4); modify these equations and conditions to give them as (after dropping stars)

$$(2.5) \quad u_1^* = \frac{u}{u}, v_1^* = \frac{v_1}{u}, z = \frac{z}{L}, t^* = \frac{tu}{L}, \lambda^* = \frac{\lambda L}{u}, \tau^* = \frac{tu}{L}, \rho_0^* = \frac{\rho_0 L}{u^2},$$

$$r = \frac{r}{r_0}, L = r_0, M = B_0 L \sqrt{\frac{\sigma}{\rho g}}, R = \frac{Lu}{\nu},$$

$$(2.6) \quad \frac{\partial u_1}{\partial t} = \left(-\frac{\partial p}{\partial z} \right) + \frac{1}{R} \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} \right) + \frac{f}{\tau} (v_1 - u_1) - \frac{1}{R} (M^2 u_1),$$

$$(2.7) \quad \frac{\partial v_1}{\partial t} = u_1 - v_1,$$

$$(2.8) \quad -\frac{\partial p}{\partial z} = p_0 (1 + e^{i\lambda t}).$$

The boundary conditions are: $u_1 = 0$ and $v_1 = 0$ when $r = 1$ and for $t > 0$ and u_1, v_1 are finite when $r = 0$ for $t > 0$.

In view of the assumed conditions, we can write the solution in the form

$$(2.9) \quad u_1 = p_0 [\phi_0(r) + \varepsilon \phi_1(r) e^{i\lambda t}],$$

then the boundary conditions changes to

$$(2.10) \quad \phi_0(1) = 0 \text{ and } \phi_1(r) = 0 \text{ for all } t, \text{ and when } r = 0, \phi_0 \text{ and } \phi_1 \text{ are finite.}$$

Now to solve these equations, put the value of v_1 from the equation (2.7) into (2.6) and on simplifying, we get,

$$(2.11) \quad \frac{\partial^2 u_1}{\partial t^2} = \frac{\partial}{\partial t} \left(-\frac{\partial p}{\partial z} \right) + \frac{1}{R} \frac{\partial}{\partial t} \left(\frac{\partial^2 u_1}{\partial r^2} \right) + \frac{1}{Rr} \frac{\partial}{\partial t} \left(\frac{\partial u_1}{\partial r} \right) - \left(\frac{M^2}{R} + \frac{f}{\tau} + \frac{1}{\tau} \right) \left(\frac{\partial u_1}{\partial t} \right)$$

$$- \frac{1}{\tau} \left(\frac{\partial p}{\partial z} \right) + \frac{1}{R\tau} \left(\frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r} \frac{\partial u_1}{\partial r} \right) - \frac{M^2 u_1}{\tau R}.$$

Putting the values of different derivatives from the equations and comparing the coefficients of the terms independent of $e^{i\lambda t}$, we get

$$(2.12) \quad \phi_0''(r) + \frac{1}{r} \phi_0''(r) - M^2 \phi_0(r) = -R,$$

Now, we solve this equation as

$$r^2 \frac{d^2 \phi_0}{dr^2} + r \frac{d \phi_0}{dr} - r^2 M^2 \phi_0 = -R r^2,$$

$$(2.13) \quad r^2 \frac{d^2 F}{dr^2} + r \frac{d F}{dr} + i^2 r^2 M^2 F = 0,$$

where $F = \left(\phi_0 - \frac{R}{M^2} \right).$

Whose solution is given by (using mentioned boundary conditions)

$$(2.14) \quad \phi_0 = \frac{R}{M^2} \left(1 - \frac{J_0(i r M)}{J_0(i M)} \right).$$

Now comparing the coefficients of $e^{i\lambda t}$, we get

$$\phi_1''(r) + \frac{1}{r} \phi_1'(r) - \lambda R (b + ai) \phi_1(r) = -R r^2,$$

$$\text{where } a = \left(1 + \frac{f}{1 + \lambda^2 \tau^2} \right) \text{ and } b = \left(\frac{M^2}{\lambda R} + \frac{f \lambda \tau}{1 + \lambda^2 \tau^2} \right).$$

Whose solution is given by (using boundary conditions) where J_0 is the

$$\text{Bessel' equation and } \theta = \frac{1}{2} \tan^{-1} \frac{a}{b}$$

$$(2.15) \quad \phi_1 = \frac{(b - ia)}{\lambda (a^2 + b^2)} \left(1 - \frac{J_0 \left[r (\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]}{J_0 \left[(\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]} \right).$$

Hence on putting the values of ϕ_0 and ϕ_1 in the equations (9), we get

$$u_1(r, t) = p_0 \left[\frac{R}{M^2} \left(1 - \frac{J_0(i r M)}{J_0(i M)} \right) + \frac{\varepsilon e^{i\lambda t} (b - ia)}{\lambda (a^2 + b^2)} \left\{ 1 - \frac{J_0 \left[r (\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]}{J_0 \left[(\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]} \right\} \right],$$

$$v_1(r, t) = p_0 \left[\frac{R}{M^2} \left(1 - \frac{J_0(i r M)}{J_0(i M)} \right) + \frac{\varepsilon e^{i\lambda t} (b - ia)}{\lambda (1 + i \lambda \tau) (a^2 + b^2)} \left\{ 1 - \frac{J_0 \left[r (\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]}{J_0 \left[(\lambda R)^{1/2} (a^2 + b^2)^{1/4} e^{i\theta} \right]} \right\} \right].$$

The mean velocity is given by

$$(2.16) \quad u_m = \frac{1}{M^2} \left(1 - \frac{J_0(i r M)}{J_0(i M)} \right).$$

Now after taking approximations for the expansion of J_0 and giving suitable values to M and r , we have obtained the variations between mean velocity and magnetic field using simulation method. The data has been discussed graphically.

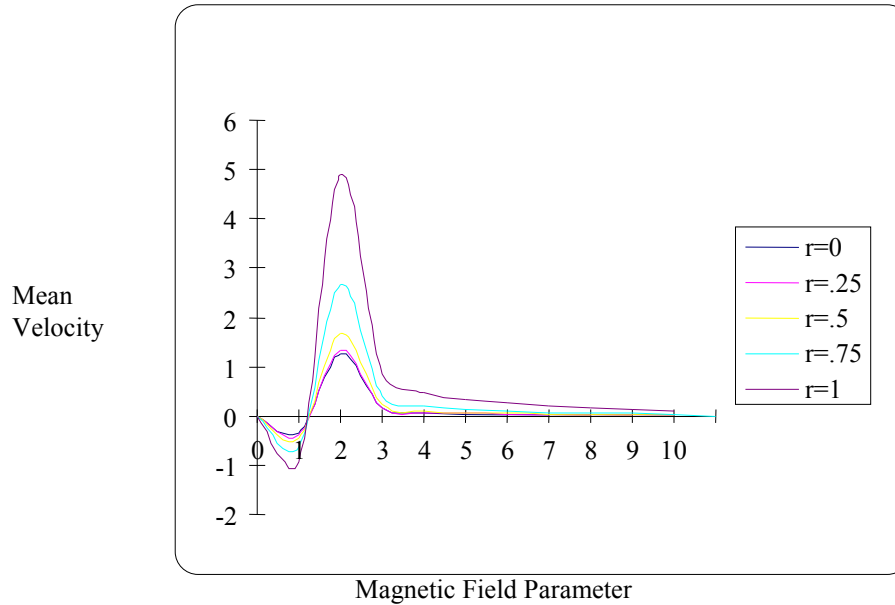


Figure no. 1

Mean Velocity distribution versus magnetic field parameter

3. Results and Discussion

It is observed from the figure1 that as the magnetic field increases for different value of the radius r , the mean velocity also increases first and taking a maximum first on negative side with little amplitudes, comes down to zero level and then this pattern is repeated on positive side with large amplitudes showing a wavy form with a resonance character which is obvious due to the fact that this velocity is dominated by the magnetic field. As r increases, the peak occurs at greater heights i.e the maximum mean velocity occurs for large values of r . Moreover the increasing value of r accelerates this condition to occur for low values of M and removes the negative part of mean velocity. It is observed that the maximum mean velocity occurs for low values of r (near $M=2$). The dust particles, near the walls, seem to be more effected in this case.

After proper analysis, it is observed that the motion of a dusty fluid having magnetic sensitive particles (two phase system) can be controlled effectively

by the application of magnetic field as expected. Hence, it gets an important role in the flow of dusty fluids through a circular channel e.g checking the metallic impurities in water used in irrigation through circular pipes, capillary rise of polluted water in plants, blood flow in human body.

References

1. S. Uchida, The viscous flow superposed on the steady laminar motion of incompressible fluid in a circular pipe., *Z. Angew. Maths. Phys.*, **7**(1956) 403- 422.
2. D. H. Micheal and D. A. Miller, Plane parallel flow of a dusty gas. *Mathematica*, **13** (1966) 97 – 109.
3. P. D. Verma and A. K. Mathur, Unsteady flow of a dusty viscous liquid in a circular tube, *I J P A M*, **4** (2) (1973) 133.
4. J .P.Singh and S.R.Pathak, Unsteady flow of a dusty viscous fluid through a uniform pipe with sector of a circle as a cross- section, *I J P A M*, **8** (6) (1977) 696 – 702.
5. D. K. Guha, Unsteady flow of a viscous conducting dusty gas between two coaxial cylinders, *Indian Journal of Technology*, **19** (1981)261-266.
6. D. V. S. Sastry and Seetharamaswamy, MHD Dusty Viscous flow through a circular pipe, *I J P A M*, **13**(1982) 7. 811-817.
7. R. S. Pathak and B. N. Upadhyay, Unsteady dusty fluid flow through a circular pipe under transverse magnetic field. *J. Ind. Acad. Math.*, **4** (1982) 39-47.
8. A. N. Osipov, Structure of the Laminar Boundary Layer of a Disperse Medium on a Flat plate, *Fluid Dynamics.*, **15** (1980) 512 - 517.
9. F. Conforto, Wave features and group analysis for axi - symmetric model of a dusty gas, *Int. J. Nonlinear. Mech.*, **5** (2000)925-930.
10. A. Attia, Hall Effect on the flow of a dusty Bingham fluid in a circular pipe. *Turkish J of Engg. Sci.*, **30**(2006) 14-21.
11. P. D. Verma and P. R. Sharma, Pulsatile flow of a viscous liquid loaded with particles in a thin –walled elastic tube. *Proc.Indian Navt. Sci. Acad., India.*, **49** (6) 622-631.
12. P. R. Sharma, P. D. Ariel and P. Chatmani, Pulastile flow of a viscous liquid premeated with particles in a thick –walled viscoelastic tube. “*Ganita Sandesh*”.*J.Raj Ganita Parishad, India*, **9** (1) (1995) 1-11.