

Analytical Approach to Fractional Order Newell-Whitehead-Segel Equations by Sumudu transform Iterative Method

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(Received April 9, 2023, Accepted August 27, 2023)

Abstract: The present paper aims to solve the linear and nonlinear time-fractional Newell-Whitehead-Segel equations using the Sumudu transform iterative method. The time-fractional derivatives are considered in the Caputo sense. In addition, the approximate analytical solutions derived in series form are graphically represented in this investigation, and the solution graphs show that the approximate solution is closely related to the exact solution.

Keywords: Sumudu Transform, Iterative Method, Caputo fractional Derivative, Newell-Whitehead-Segel Equations, Fractional Differential Equations.

2010 Mathematics Subject Classification: 33E12, 34A08, 35A20, 35C10

1. Introduction

In the last decade, fractional differential equations have gained a considerable amount of attention due to their applicability in physics, biology, engineering, and other disciplines. Numerous numerical and analytical techniques have been developed to investigate the solutions of fractional partial differential equations, such as the Adomian decomposition method (ADM)¹, the Laplace Adomian decomposition method (LADM)², the iterative Laplace transform method (ILTM)³, the homotopy perturbation

method (HPM)⁴, the Laguerre wavelet method (LWM)⁵, the homotopy analysis Sumudu transform method (HASTM)⁶, the fractional variational iteration method (FVIM)⁷, etc. In 1993, Watugala introduced the Sumudu transform method (STM)⁸ for solving engineering problems. This approach was used by Weerakoon⁹ to solve partial differential equations. Weerakoon¹⁰ later derived the formula for the inverse of this transform. Demiray *et al.*¹¹ used the Sumudu transform method to discover exact solutions to fractional differential equations. Recently, Wang and Liu proposed the Sumudu transform iterative method (STIM)¹², which combines the Sumudu transform with an iterative technique, in order to discover approximate analytical solutions to time-fractional Cauchy reactions-diffusion equations. The Sumudu transform iterative approach was successfully utilized to solve fractional Fokker-Planck equations¹³, generalized fractional biological population models¹⁴, fractional diffusion equations¹⁵, fractional Schrodinger equations¹⁶, third-order fractional dispersive equations¹⁷ etc.

In this paper, we consider the following fractional order Newell-Whitehead-Segel equation in operator form as¹⁸

$$(1.1) \quad \frac{\partial^\alpha u}{\partial t^\alpha} = \lambda \frac{\partial^2 u}{\partial x^2} + au - bu^q,$$

with initial condition

$$(1.2) \quad u(x, 0) = f(x),$$

where a, b and λ are real numbers with $\lambda > 0$, q is a positive integer and α is a parameter describing the order of the fractional derivative. The fractional derivative is used in the Caputo sense. In particular for $\alpha = 1$, the fractional order Newell-Whitehead-Segel equation reduces to the classical Newell-Whitehead-Segel equation.

In this paper, we apply the Sumudu transform iterative technique (STIM) to solve the Newell-Whitehead-Segel equation of fractional order. This approach is an excellent mixture of the iterative method and the Sumudu transform method. It provides the solution in terms of convergent series with components that are readily computable in a straightforward manner that does not need linearization, perturbation, or restricted assumptions.

2. Basic Definitions and Notations

In this section, we provide some of the fundamental definitions, notations, and properties of fractional calculus and Sumudu transform theory that will be utilized in subsequent sections of this paper.

Definition 2.1. The fractional derivative of $u(x, t)$ in the Caputo sense is defined as¹⁹⁻²⁰

$$(2.1) \quad D_t^\alpha u(x, t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\eta)^{m-\alpha-1} u^{(m)}(x, \eta) d\eta, \quad m-1 < \alpha \leq m, m \in \mathbb{N}.$$

Definition 2.2. The Sumudu transform is defined over the set of functions

$$\left\{ f(t) \mid \exists M, \rho_1 > 0, \rho_2 > 0, |f(t)| < Me^{|t|/\rho_j} \text{ if } t \in (-1)^j \times [0, \infty), j=1, 2 \right\}$$

by the following formula^{8, 21}

$$(2.2) \quad S[f(t)] = F(\omega) = \int_0^\infty e^{-t} f(\omega t) dt, \quad \omega \in (-\rho_1, \rho_2).$$

Definition 2.3. The Sumudu transform of the Caputo fractional derivative is defined as¹¹⁻¹²

$$(2.3) \quad S[D_t^\alpha u(x, t)] = \omega^{-\alpha} S[u(x, t)] - \sum_{k=0}^{m-1} \omega^{-\alpha+k} u^{(k)}(x, 0), \quad m-1 < \alpha \leq m, m \in \mathbb{N},$$

where $u^{(k)}(x, 0)$ is the k -order derivative of $u(x, t)$ with respect to t at $t = 0$.

Definition 2.4. The Mittag-Leffler function, a generalization of the exponential function, is defined as²²

$$(2.4) \quad E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha + 1)}, \quad \alpha \in \mathbb{C}, \operatorname{Re}(\alpha) > 0,$$

where $\Gamma(\cdot)$ is the well-known Gamma function.

3. Basic Idea of Sumudu Transform Iterative Method

In order to illustrate the key concept of this method¹², we consider the general fractional partial differential equation with initial conditions of the type

$$(3.1) \quad D_t^\alpha u(x, t) + Ru(x, t) + Nu(x, t) = g(x, t), \quad m-1 < \alpha \leq m, m \in \mathbb{N},$$

$$(3.2) \quad u^{(k)}(x, 0) = h_k(x), \quad k = 0, 1, 2, \dots, m-1,$$

where $D_t^\alpha u(x, t)$ is the Caputo fractional derivative of order α , $m-1 < \alpha \leq m$, $m \in \mathbb{N}$, defined by equation (2.1), R is a linear operator and may include other fractional derivatives of order less than α , N is a non-linear operator which may include other fractional derivatives of order less than α and $g(x, t)$ is a known function.

Applying the Sumudu transform on both sides of equation (3.1), we have

$$(3.3) \quad S[D_t^\alpha u(x, t)] + S[Ru(x, t) + Nu(x, t)] = S[g(x, t)].$$

By using the equation (2.3), we get

$$(3.4) \quad S[u(x, t)] = \sum_{k=0}^{m-1} \omega^k u^{(k)}(x, 0) + \omega^\alpha S[g(x, t)] - \omega^\alpha S[Ru(x, t) + Nu(x, t)].$$

On taking inverse Sumudu transform on equation (3.4), we have

$$(3.5) \quad u(x, t) = S^{-1} \left[\omega^\alpha \left(\sum_{k=0}^{m-1} \omega^{-\alpha+k} u^{(k)}(x, 0) + S[g(x, t)] \right) \right] \\ - S^{-1} \left[\omega^\alpha S[Ru(x, t) + Nu(x, t)] \right].$$

Furthermore, we apply the iterative method proposed by Daftardar-Gejji and Jafari²³, which represents a solution in an infinite series of components as

$$(3.6) \quad u(x, t) = \sum_{i=0}^{\infty} u_i(x, t).$$

As R is a linear operator, so we have

$$(3.7) \quad R \left(\sum_{i=0}^{\infty} u_i(x, t) \right) = \sum_{i=0}^{\infty} R[u_i(x, t)],$$

and the non-linear operator N is decomposed as follows

$$(3.8) \quad N \left(\sum_{i=0}^{\infty} u_i(x, t) \right) = N[u_0(x, t)] + \sum_{i=1}^{\infty} \left\{ N \left(\sum_{j=0}^i u_j(x, t) \right) - N \left(\sum_{j=0}^{i-1} u_j(x, t) \right) \right\}.$$

Substituting the results given by equations from (3.6) to (3.8) in the equation (3.5), we get

$$(3.9) \quad \sum_{i=0}^{\infty} u_i(x, t) = S^{-1} \left[\omega^\alpha \left(\sum_{k=0}^{m-1} \omega^{-\alpha+k} u^{(k)}(x, 0) + S[g(x, t)] \right) \right] \\ - S^{-1} \left[\omega^\alpha S \left[\sum_{i=0}^{\infty} R[u_i(x, t)] + N[u_0(x, t)] + \sum_{i=1}^{\infty} \left\{ N \left(\sum_{j=0}^i u_j(x, t) \right) - N \left(\sum_{j=0}^{i-1} u_j(x, t) \right) \right\} \right] \right].$$

We have formulated the recurrence relations as

$$(3.10) \quad u_0(x, t) = S^{-1} \left[\omega^\alpha \left(\sum_{k=0}^{m-1} \omega^{-\alpha+k} u^{(k)}(x, 0) + S(g(x, t)) \right) \right],$$

$$(3.11) \quad u_1(x, t) = -S^{-1} \left[\omega^\alpha S \left[R(u_0(x, t)) + N(u_0(x, t)) \right] \right],$$

$$(3.12) \quad u_{m+1}(x, t) = -S^{-1} \left[\omega^\alpha S \left[R(u_m(x, t)) - \left\{ N \left(\sum_{j=0}^m u_j(x, t) \right) - N \left(\sum_{j=0}^{m-1} u_j(x, t) \right) \right\} \right] \right],$$

$$m \geq 1.$$

Thus, the approximate analytical solution for equations (3.1) and (3.2) in truncated series form is given by

$$(3.13) \quad u(x, t) \cong \lim_{N \rightarrow \infty} \sum_{m=0}^N u_m(x, t) = u_0(x, t) + u_1(x, t) + u_2(x, t) + u_2(x, t) + \dots$$

In general, solutions in the preceding series converge rapidly. The classical approach to the convergence of this type of series was provided by Daftardar-Gejji and Jafari²³ and Bhalekar and Daftardar-Gejji²⁴.

4. Solutions of the fractional order Newell-Whitehead-Segel equations

In this part, the linear and non-linear time-fractional Newell-Whitehead-Segel equations with initial conditions are solved using the STIM technique.

Example 1. We consider the following linear time-fractional Newell-Whitehead-Segel equation^{6,18}

$$(4.1) \quad D_t^\alpha u = u_{xx} - 2u, \quad 0 < \alpha \leq 1,$$

with the initial condition

$$(4.2) \quad u(x, 0) = e^x.$$

Taking the Sumudu transform of the above equation (4.1), we have

$$(4.3) \quad S[D_t^\alpha u] = S[u_{xx}] - 2S[u].$$

By using equation (2.3), we have

$$(4.4) \quad S[u(x, t)] = u(x, 0) + \omega^\alpha S[u_{xx}] - 2\omega^\alpha S[u].$$

Applying inverse Sumudu transform to the equation (4.4), we obtain

$$(4.5) \quad u(x, t) = S^{-1}[u(x, 0)] + S^{-1}[\omega^\alpha S[u_{xx}] - 2\omega^\alpha S[u]].$$

Substituting the results from equations (3.6) to (3.8) in the equation (4.5) and applying the equations (3.10) to (3.12), we determine the components of the STIM solution as follows

$$(4.6) \quad u_0(x, t) = S^{-1}[u(x, 0)] = e^x,$$

$$(4.7) \quad u_1(x, t) = S^{-1}[\omega^\alpha S[(u_0)_{xx}] - 2\omega^\alpha S[u_0]] = -e^x \frac{t^\alpha}{\Gamma(\alpha + 1)},$$

$$(4.8) \quad u_2(x, t) = S^{-1}[\omega^\alpha S[(u_1 + u_0)_{xx}] - 2\omega^\alpha S[(u_1 + u_0)]] \\ - S^{-1}[\omega^\alpha S[(u_0)_{xx}] - 2\omega^\alpha S[u_0]] = -e^x \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)},$$

$$(4.9) \quad u_3(x, t) = S^{-1}[\omega^\alpha S[(u_2 + u_1 + u_0)_{xx}] - 2\omega^\alpha S[(u_1 + u_0)]] \\ - S^{-1}[\omega^\alpha S[(u_1 + u_0)_{xx}] - 2\omega^\alpha S[u_1 + u_0]] = -e^x \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)},$$

and so on. The other components can be determined in the same manner. Therefore, the approximate analytical solution in series form can be derived as

$$(4.10) \quad u(x,t) \cong \lim_{N \rightarrow \infty} \sum_{m=0}^N u_m(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \dots$$

$$= e^x \left[1 - \frac{t^\alpha}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \dots \right] = e^x \sum_{n=0}^{\infty} \frac{(-t^\alpha)^n}{\Gamma(n\alpha+1)}.$$

Hence, the exact solution can be expressed as

$$(4.11) \quad u(x,t) = e^x E_\alpha[-t^\alpha].$$

The same result was achieved by Bairwa and Tyagi¹⁸ using the ILTM technique.

Remark 1. For $\alpha = 1$, the result in equation (4.11) reduces to the following simple form

$$(4.12) \quad u(x,t) = e^{x-t}.$$

This result was previously reported by Nourazar *et al.*²⁵ using the HPM approach.

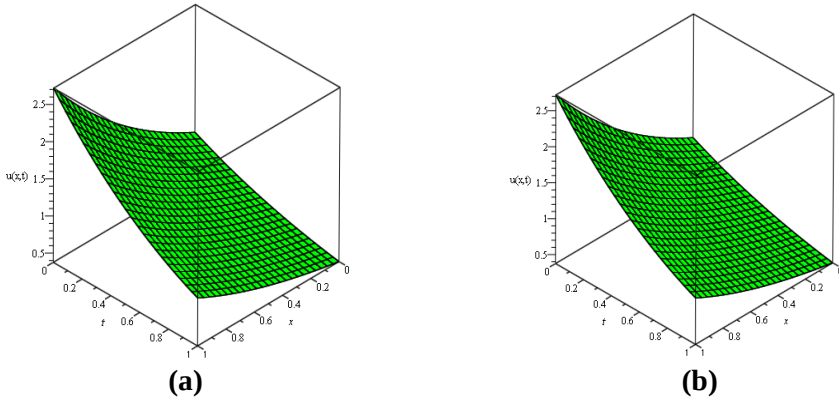


Figure 1: Solution graph for Example 1: **(a)** The Exact solution **(b)** The Approximate solution at $\alpha = 1$.

Example 2. We consider the following non-linear time-fractional Newell-Whitehead-Segel equation^{6,18}

$$(4.13) \quad D_t^\alpha u = u_{xx} + 2u - 3u^2.$$

with the initial condition

$$(4.14) \quad u(x,0) = \lambda,$$

where λ is a constant.

Taking the Sumudu transform of the above equation (4.13), we have

$$(4.15) \quad S[D_t^\alpha u] = S[u_{xx} + 2u - 3u^2].$$

Using equation (2.3), we get

$$(4.16) \quad S[u(x,t)] = u(x,0) + \omega^\alpha S[u_{xx} + 2u - 3u^2].$$

Applying the inverse Sumudu transform to the equation (4.16), we obtain

$$(4.17) \quad u(x, t) = S^{-1}[u(x, 0)] + S^{-1}[\omega^\alpha S[u_{xx} + 2u - 3u^2]].$$

Substituting the results from equations (3.6) to (3.8) in the equation (4.17) and applying the equations (3.10) to (3.12), we determine the components of the STM solution as follows

$$(4.18) \quad u_0(x, t) = S^{-1}[u(x, 0)] = \lambda,$$

$$(4.19) \quad u_1(x, t) = S^{-1}[\omega^\alpha S[(u_0)_{xx} + 2(u_0) - 3(u_0)^2]] = (2\lambda - 3\lambda^2) \frac{t^\alpha}{\Gamma(\alpha + 1)},$$

$$(4.20) \quad u_2(x, t) = S^{-1}[\omega^\alpha S[(u_1 + u_0)_{xx} + 2(u_1 + u_0) - 3(u_1 + u_0)^2]] \\ - S^{-1}[\omega^\alpha S[(u_0)_{xx} + 2(u_0) - 3(u_0)^2]] \\ = 2(1 - 3\lambda)(2\lambda - 3\lambda^2) \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)} - 3(2\lambda - 3\lambda^2)^2 \frac{\Gamma(2\alpha + 1)}{\{\Gamma(\alpha + 1)\}^2} \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)},$$

$$(4.21) \quad u_3(x, t) = S^{-1}[\omega^\alpha S[(u_2 + u_1 + u_0)_{xx} + 2(u_2 + u_1 + u_0) - 3(u_2 + u_1 + u_0)^2]] \\ - S^{-1}[\omega^\alpha S[(u_1 + u_0)_{xx} + 2(u_1 + u_0) - 3(u_1 + u_0)^2]] \\ = 4(1 - 3\lambda)^2 (2\lambda - 3\lambda^2) \frac{t^{3\alpha}}{\Gamma(3\alpha + 1)} - 6(2\lambda - 3\lambda^2) \times \\ \left[(1 - 6\lambda^2 + 9\lambda^3) \frac{\Gamma(2\alpha + 1)}{\{\Gamma(\alpha + 1)\}^2} + 2(1 - 3\lambda)(2\lambda - 3\lambda^2) \frac{\Gamma(3\alpha + 1)}{\Gamma(\alpha + 1)\Gamma(2\alpha + 1)} \right] \frac{t^{4\alpha}}{\Gamma(4\alpha + 1)} \\ - \left[12(1 - 3\lambda)^2 (2\lambda - 3\lambda^2)^2 \frac{\Gamma(4\alpha + 1)}{\{\Gamma(2\alpha + 1)\}^2} - 18(2\lambda - 3\lambda^2)^2 \frac{\Gamma(2\alpha + 1)\Gamma(4\alpha + 1)}{\{\Gamma(\alpha + 1)\}^3 \Gamma(3\alpha + 1)} \right] \frac{t^{5\alpha}}{\Gamma(5\alpha + 1)} \\ + 36(1 - 3\lambda)(2\lambda - 3\lambda^2)^2 \frac{\Gamma(5\alpha + 1)}{\{\Gamma(\alpha + 1)\}^2 \Gamma(3\alpha + 1)\Gamma(6\alpha + 1)} t^{6\alpha} \\ - 27(2\lambda - 3\lambda^2)^2 \frac{\Gamma\{(2\alpha + 1)\}^2 \Gamma(6\alpha + 1)}{\{\Gamma(\alpha + 1)\}^4 \{\Gamma(3\alpha + 1)\}^2} \frac{t^{7\alpha}}{\Gamma(7\alpha + 1)}.$$

and so on. The other components can be determined in the same manner. Therefore, the approximate analytical solution in series form can be derived as

$$\begin{aligned}
 (4.22) \quad u(x,t) &\cong \lim_{N \rightarrow \infty} \sum_{m=0}^N u_m(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \dots \\
 &= \lambda + (2\lambda - 3\lambda^2) \frac{t^\alpha}{\Gamma(\alpha+1)} + 2(1-3\lambda)(2\lambda - 3\lambda^2) \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\
 &\quad + (2\lambda - 3\lambda^2) \left[4(1-3\lambda)^2 - 3(2\lambda - 3\lambda^2) \frac{\Gamma(2\alpha+1)}{\{\Gamma(\alpha+1)\}^2} \right] \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \dots
 \end{aligned}$$

The same result was achieved by Bairwa and Tyagi¹⁸ using the ILTM technique.

Remark 2. For $\alpha = 1$, the result in equation (4.22) reduces to the following simple form

$$\begin{aligned}
 (4.23) \quad u(x,t) &= \lambda + 2(1-3\lambda)(2\lambda - 3\lambda^2) \frac{t^2}{2!} + 2\lambda(2-3\lambda)(27\lambda^2 - 18\lambda + 2) \frac{t^3}{3!} + \dots \\
 &= \frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3} + \lambda - \lambda e^{2t}}.
 \end{aligned}$$

This result was previously reported by Nourazar *et al.*²⁵ using the HPM approach.

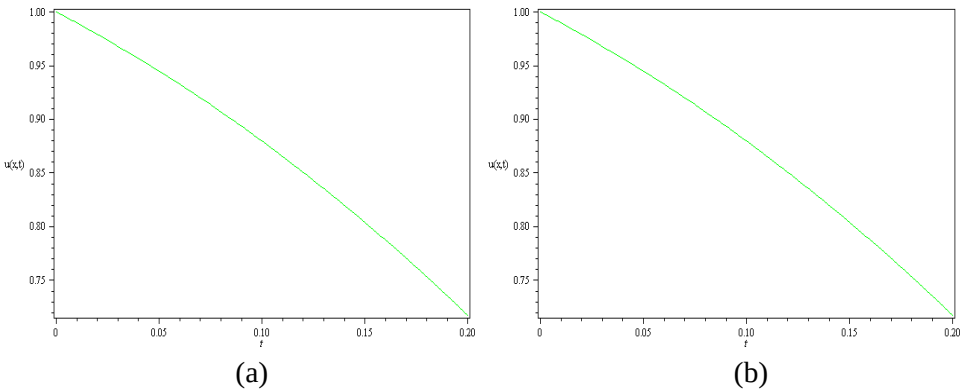


Figure 2 : Solution graph for Example 2, when $\lambda = 1$: **(a)** The Exact solution **(b)** The Approximate solution at $\alpha = 1$.

6. Conclusion

In this work, we have derived analytical solutions to the fractional model of the Newell-Whitehead Segel equations using the Sumudu transform iterative technique. The time fractional derivatives are considered in the sense of

Caputo. In addition, the outcomes of this investigation are shown graphically, and the solution graphs reveal that the approximate solutions are closely related to the exact solutions.

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