

Peristaltic Pumping of Maxwell Fluids in a Curved Channel: a Model for Intestinal Transport

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Abstract: Chyme is propelled through large and small intestines by peristalsis which is a wavelike propulsion of muscular contraction and relaxation. Chyme is a strongly rheological biofluid and requires non-Newtonian models for accurate simulation. Motivated by gastric transport applications, in this paper, a peristaltic transport model is developed to analyze the hydrodynamics of chyme movement through the large intestine. Chyme is modelled with the generalized (fractional) Maxwell viscoelastic model. The geometry for the large intestine is approximated as a two-dimensional curved channel. The analysis is carried out under the lubrication theory and low Reynolds number approximation. The approximate solutions of the governing equations under complex boundary conditions are obtained by deploying the Reduced Differential Transformation Method. Trapping phenomena are also analyzed graphically. The effects of fractional parameter, relaxation time, curvature and amplitude on the pressure difference and friction force are visualized graphically and interpreted in detail. The present simulations are applicable in gastric transport phenomena and the design of novel intestinal devices (colonoscopy) in biomedical engineering.

Keywords: Intestinal peristalsis; chyme; curved channel; viscoelastic fluids; reduced Differential Transformation Method; trapping; fractional derivative.

1. Introduction

Gastric disorders and gastrointestinal diseases constitute a growing concern in modern medicine¹. The physiological fluids featured in such processes are extremely complex as is their transport mechanism. To provide deeper understanding of the mechanics of gastric transport and, in order to, design more sustainable and less invasive technologies for the diagnosis and monitoring of diseases, theoretical and computational fluid dynamic models provide an excellent complimentary approach to clinical investigations. Sophisticated multi-physical models provide much needed insight into robust pathways for improving the design of biomedical devices which play a significant role in treatment of gastric illnesses. The intestine is a large part of the human digestive system. Chyme is a thick semi-fluid mass of partially digested food and digestive secretions which is synthesized naturally in the stomach and intestine during digestion. Healthy gastric motility requires a regulated delivery of gastric chyme to the duodenum at a rate which does not overload the digestive and absorptive functions of the small intestine, which has a curved and extremely sophisticated topology. The mechanism of propulsion of chyme is by peristalsis. Peristalsis is one of the most efficient methodologies deployed in nature for both internal propulsion (swallowing, digestion, blood flow) and external locomotion (snake movement, certain reptiles, worms etc). In gastric transport, it is characterized by the involuntary constriction and relaxation of the muscles of the intestine, creating wave-like movements which push the contents of the intestine forward. The contraction/relaxation waves generated are generally radially symmetric and propagate in an anterograde direction. To provide a realistic characterization of the fluid mechanical processes intrinsic to correct gastric dynamics of chyme, engineers and scientists have for some time developed multi-dimensional models for peristaltic propulsion of both the small and large intestine. One of the earliest analytical studies of peristaltic flow of chyme in the small intestine was reported by Lew et al.² for a Newtonian viscous liquid. They considered several scenarios including peristalsis without a net pressure gradient and peristaltic compression without net transport of the fluid. This model however neglected rheological properties of chyme. Srivastava and Srivastava³ used the Ostwald-DeWaele power-law non-Newtonian model to study chyme transport in both uniform and non-uniform tubes, observing

that pressure rise is reduced for pseudo-plastic fluids and, that for a given flow rate with greater wavelength the pressure rise is also lowered although the sensitivity of the transport to rheological effects is increased. Ravi Kiran et al.⁴ used the Eringen micropolar non-Newtonian model to study hydrodynamic reactive dispersion in peristaltic pumping of chyme in the intestine with wall deformability. A theoretical model was presented to discuss the effect of an inserted endoscope on chyme movement in the small intestine by Srivastava⁵ although only Newtonian fluids were considered. Heat transfer in chyme movement was reported by Akbar et al.⁶ for the case of non-periodic sinusoidal waves with different wavelengths on the inner and outer endoscope walls, although again the model was restricted to Newtonian fluids. Nadeem et al.⁷ deployed a Williamson viscoelastic fluid model to simulate chyme dynamics. Akbar et al.⁸ used a Jeffrey viscoelastic model for peristaltic pumping of chyme. Akbar and Nadeem⁹ investigated peristaltic flow of chyme with the Stokes couple stress fluid model. In these studies, however, the conduit was assumed to be straight i.e. curvature was ignored. Curvature is however a significant property of the intestinal tract. Ali et al.¹⁰ used a finite difference numerical method to compute the dynamics of Carreau fluids in peristaltic pumping in a curved channel. They observed that higher viscoelastic effect (Weissenberg number) creates a vortex near the innermost wall of the channel, which is enhanced with further increase in Weissenberg number and furthermore for shear-thickening bio-fluids, the flow is accelerated with greater Weissenberg number. Hoque et al.¹¹ employed a spectral algorithm to compute the magnetized blood flow in peristaltic pumping through a curved conduit, observing that curvature has a profound influence on vortex structure and bolus dynamics. Tripathi et al.¹² used integral methods and Maple numerical quadrature to study the peristaltic pumping of Nakamura-Sawada rheological fluids in a curved tube. Narla et al.¹³ have simulated the electro-osmotic transport of nanofluids propelled via peristaltic flow in a curved microchannel.

In recent years, fractional calculus has been shown to be of considerable use in formulating more accurate mathematical models. This approach has been implemented in many diverse areas including viscoelasticity¹⁴, electromagnetics¹⁵, fractional imaging in neurosciences¹⁶ etc. In particular, fractional models have been found to be quite flexible in describing viscoelastic behavior of fluids and solids. The starting point of the fractional derivative model of non-Newtonian fluids is usually a classical differential equation which is modified by replacing the time derivative of an integer order by the so-called Riemann-Liouville fractional

operator. This generalization allows one to define precisely non-integer order derivatives. Tan et al.¹⁷ have investigated the unsteady channel flow of a viscoelastic fluid with the fractional Maxwell model. Qi and Jin¹⁸ have discussed unsteady flows between coaxial cylinders with fractional approaches. Qi and Xu¹⁹ have studied in the channel and solutions are obtained by using Laplace transform, Fourier transform and Weber transform. Hayat et al.²⁰ have constructed periodic unidirectional flows of a viscoelastic fluid with the fractional Maxwell model and solutions are solved by Fourier transform. Khan et al.²¹ have derived analytical solutions for the decay of a potential vortex for viscoelastic fluid with fractional Maxwell model with Hankel and discrete Laplace transforms. Several studies of fractional non-Newtonian peristaltic flows have also been communicated, in which the Caputo formulation has been used for fractional derivatives. Tripathi and Bég²² presented analytical solutions for peristaltic pumping of a fractional second order Reiner-Rivlin viscoelastic fluid through an inclined cylindrical conduit, noting that an increase in fractional parameter and Froude number decrease flow rates whereas an increase in viscoelastic material parameter, Reynolds number, inclination of angle and amplitude aid the peristaltic flow. Tripathi²³ studied the peristaltic transport of chyme in a tilted tube with the fractional Oldroyd-B elastic-viscous model. Other models for fractional viscoelastic peristalsis include the Maxwell model^{24,25}, the Darcy-Brinkman porous media fractional Oldroyd-B model²⁶ and the Burgers viscoelastic model²⁷.

The objective of this paper is to investigate the effect of fractional parameters of viscoelastic fluid on physiological transport through a curved channel. We employ the classical lubrication theory for physiological transport and the analysis is also restricted to low Reynolds number flow. Approximate analytical solutions of the fractional differential equation are obtained by the Reduced Differential Transformation Method (RDTM). Extensive graphical visualizations of the effects of the dominating physical parameters on peristaltic pumping characteristics are presented. The influence of viscoelasticity material constants and fractional parameters on pressure difference across one wavelength and friction force distributions are also examined. The study is relevant to gastric dynamics in the large intestine.

2. Mathematical Formulation

We consider the flow of a biological fluid (chyme) simulated with the Maxwell viscoelastic fluid model, for which the appropriate stress-strain relation is given by²⁷:

$$(2.1) \quad \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \right) \tau = \mu \dot{\gamma},$$

where λ_1 , α , t , τ , μ , $\dot{\gamma}$ are the relaxation time, fractional parameter, time, shear stress, viscosity, rate of shear strain.

The Cartesian co-ordinate system (X', Y', Z') is related to the toroidal co-ordinate system (x, r, z) by the relations:

$$(2.2) \quad X' = (R + r) \cos\left(\frac{x}{R}\right), Y' = (R + r) \sin\left(\frac{x}{R}\right).$$

The flow regime for a curved channel is shown in Figure 1. The curved channel has unperturbed width $2a$, filled with an incompressible viscoelastic fluid. The channel is coiled in a circle with center O and radius R . There is no component in the z -direction and the fluid motion within the channel is induced by two infinite trains of sinusoidal waves that are propagated along the flexible walls of the channel. The sinusoidal waves propagating along the channel walls are described as:

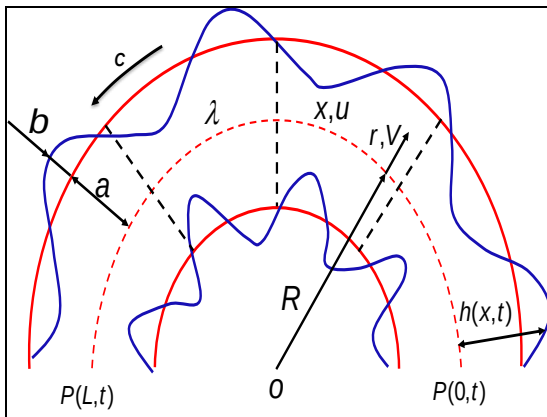


Figure 1. (a) Schematic of two-dimensional chyme pumping through the large intestine.

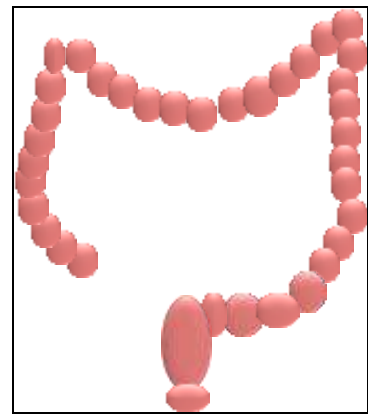


Figure 1. (b) Shape of human large intestine.

$$(2.3) \quad r = \pm H = \pm a \pm b \cos \left[2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \right].$$

Here, x is the axial distance, α the radius of the stationary curved channel, ϕ the wave amplitude, λ the wave length, T the wave period, and h the radial displacement of the wave from the centerline. The wavelength is large compared with the channel's width $\left(\frac{a}{\lambda} \ll 1\right)$. The governing equations for an ordinary Maxwell fluid through a curved channel with the velocity vector $\bar{V} = u \hat{e}_x + v \hat{e}_r$, are given by:

$$(2.4) \quad R \frac{\partial u}{\partial x} + \frac{\partial}{\partial r} \{(r+R)v\} = 0,$$

$$\begin{aligned} & \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial u}{\partial t} + (\bar{V} \cdot \nabla)u - \frac{uv}{r+R} \right) \\ &= - \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(\frac{R}{\rho(r+R)} \frac{\partial p}{\partial x} \right) + v \left[\nabla^2 u - \frac{u}{(r+R)^2} + \frac{2R}{(r+R)^2} \frac{\partial v}{\partial x} \right], \end{aligned}$$

$$\begin{aligned} (2.5) \quad & \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial v}{\partial t} + (\bar{V} \cdot \nabla)v - \frac{u^2}{r+R} \right) \\ &= - \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left(\frac{1}{\rho} \frac{\partial p}{\partial r} \right) + v \left[\nabla^2 v - \frac{v}{(r+R)^2} - \frac{2R}{(r+R)^2} \frac{\partial u}{\partial x} \right], \end{aligned}$$

where

$$(2.6) \quad (\bar{V} \cdot \nabla) = \frac{Ru}{(r+R)} \frac{\partial}{\partial x} + v \frac{\partial}{\partial r}, \text{ and } \nabla^2 = \left(\frac{R}{r+R} \right)^2 \frac{\partial^2}{\partial x^2} + \frac{1}{r+R} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}.$$

In the above equations p is pressure, ρ is fluid density, ν the kinematic viscosity, u and v are the velocity components in the radial r and axial x directions respectively.

The following dimensionless variables and parameters are introduced:

$$(2.7) \quad x' = \frac{x}{\lambda}, r' = \frac{r}{a}, u' = \frac{u}{c}, v' = \frac{v}{\delta c}, h' = \frac{h}{a}, \phi = \frac{b}{a}, \kappa = \frac{R}{a},$$

$$p' = \frac{a^2 p}{\mu c \lambda}, \text{Re} = \frac{ca\delta}{\nu}, \psi' = \frac{\psi}{ac}, Q' = \frac{Q}{2ac}, t' = \frac{t}{T'}$$

where, Re is Reynolds number, F is volume flow rate, $c = \lambda/T^*$ is the velocity of the wave, $\delta = a/\lambda$ defines wave number, ϕ is the amplitude ratio, κ is the curvature parameter. The stream function ψ can be defined by using the dimensionless variables and parameters in equation (2.7), which satisfies the continuity equation (2.4) as:

$$(2.8) \quad u = \frac{\partial \psi}{\partial r}, v = -\frac{\kappa}{r + \kappa} \frac{\partial \psi}{\partial x}.$$

The dimensionless equations are obtained (neglecting the primes for clarity) by applying long wavelength and negligible Reynolds number approximation using equation (2.1) in equations (2.5) and (2.6), as

$$(2.9) \quad \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right) \left(\frac{\kappa}{r + \kappa} \frac{\partial p}{\partial x}\right) - \nabla^2 \frac{\partial \psi}{\partial r} + \frac{1}{(r + \kappa)^2} \frac{\partial \psi}{\partial r} = 0,$$

$$(2.10) \quad \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha}\right) \frac{\partial p}{\partial r} = 0,$$

where the Laplacian operator is defined as:

$$\nabla^2 = \frac{1}{r + \kappa} \frac{\partial}{\partial r} \left\{ (r + \kappa) \frac{\partial}{\partial r} \right\}.$$

The dimensionless boundary conditions are given by:

$$(2.11) \quad \psi = \pm \frac{Q(x, t)}{2}, \frac{\partial \psi}{\partial r} = 0, \text{ at } r = \pm h(x, t),$$

where $Q(x, t)$ is volume flow rate and is defined as

$$Q = \int_{-h}^h u(x, r, t) dr.$$

The solution of equations (2.9) and (2.10) with the boundary conditions equation (2.11) are as follows

$$(2.12) \quad \psi = C_1 + C_2 \log(r + \kappa) + C_3(r + k)^2 + C_4(r + \kappa)^2 \log(r + \kappa),$$

Here:

$$C_1 = \frac{Q \left[2h\kappa(h^2 + \kappa^2) + \left\{ \left(\log \frac{\kappa + h}{\kappa - h} \right) + (\log(\kappa - h))^2 \right\} (h^2 - \kappa^2)^2 - (\log(\kappa + h))^2 \right]}{2 \left[-4h^2\kappa^2 + (h^2 - \kappa^2)^2 \left(\log \frac{\kappa - h}{\kappa + h} \right)^2 \right]},$$

$$C_2 = \frac{Q(h^2 - \kappa^2) \log \frac{\kappa - h}{\kappa + h}}{-4h^2\kappa^2 + (h^2 - \kappa^2)^2 \left(\log \frac{\kappa - h}{\kappa + h} \right)^2},$$

$$C_3 = \frac{Q \left[-2h\kappa + (h - \kappa)^2 \log(\kappa - h) - (h + \kappa)^2 \log(h + \kappa) \right]}{2 \left[-4h^2\kappa^2 + (h^2 - \kappa^2)^2 \left(\log \frac{\kappa - h}{\kappa + h} \right)^2 \right]},$$

$$C_4 = \frac{2Qh\kappa}{-4h^2\kappa^2 + (h^2 - \kappa^2)^2 \left(\log \frac{\kappa - h}{\kappa + h} \right)^2}.$$

It is clear from equation (2.12) that the stream function is independent of material constant λ_1 .

The axial pressure gradient in terms of time-averaged flow can be obtained by substituting equation (2.12) in equation (2.9) leading to:

$$(2.13) \quad \left(1 + \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \right) \frac{\partial p}{\partial x} = \frac{8Qh}{-4\kappa^2 h^2 + (\kappa^2 - h^2)^2 \left(\log \frac{\kappa + h}{\kappa - h} \right)^2}.$$

The transformations between the wave frame and the laboratory (stationary) frame, in dimensionless form, are defined as:

$$(2.14) \quad X = x - ct, U = u - 1, V = v, \Psi = \psi - r,$$

where the parameters on the left side are in the wave frame and that on the right side are in the laboratory frame. The time-averaged flow rate over a single time-period emerges as:

$$(2.15) \quad Q_T = \int_0^1 Q(x, t) dt = \int_0^1 (q + 2h) dt = q + 2.$$

In view of equations (2.15), equation (2.13) is converted into a time-fractional nonhomogeneous linear differential equation:

$$(2.16) \quad \lambda_1^\alpha \frac{\partial^\alpha}{\partial t^\alpha} \left(\frac{\partial p}{\partial X} \right) + \frac{\partial p}{\partial X} + \frac{8(Q_T + 2(h-1))h}{4\kappa^2 h^2 - (\kappa^2 - h^2)^2 \left(\log \frac{\kappa + h}{\kappa - h} \right)^2} = 0,$$

3. Reduced Differential Transformation Method (RDTM) Solutions

The reduced differential transform technique is an iterative procedure for obtaining Taylor series solution of differential equations²⁸⁻³⁰. Equation (2.16) can be re-written as:

$$(3.1) \quad \frac{\partial^\alpha}{\partial t^\alpha} \tilde{u}(X, t) = -\frac{1}{\lambda_1^\alpha} \tilde{u}(X, t) + \frac{1}{\lambda_1^\alpha} \tilde{A}(X),$$

$$\tilde{u}(X, 0) = \tilde{f}(X) = 0, t > 0, \alpha > 0, X \in \mathbb{R},$$

Where

$$\tilde{u}(X, t) = \frac{\partial p}{\partial X}, \text{ and } \tilde{A}(X) = \frac{8(Q_T + 2(h-1))h}{-4\kappa^2 h^2 + (\kappa^2 - h^2)^2 \left(\log \frac{\kappa + h}{\kappa - h} \right)^2}.$$

Applying the Reduced Differential Transform to equation (3.1), we obtain the following iteration relation:

$$(3.2) \quad \frac{\Gamma(K\alpha + \alpha + 1)}{\Gamma(K\alpha + 1)} \frac{\partial^\alpha}{\partial t^\alpha} \tilde{U}_{K+1}(X) = -\frac{1}{\lambda_1^\alpha} \tilde{U}_K(X) + \frac{1}{\lambda_1^\alpha} \tilde{A}(X) \delta(K),$$

$$\tilde{U}_0(X) = \tilde{F}(X) = 0.$$

Using the iterative relation (3.2), we obtain the following values of $\tilde{U}_k(X)$ successively:

$$(3.3) \quad \begin{cases} \tilde{U}_0(X) = 0, \\ \tilde{U}_1(X) = \frac{\tilde{A}(X)}{\lambda_1^\alpha \Gamma(\alpha + 1)}, \\ \tilde{U}_2(X) = \frac{\tilde{A}(X)}{\lambda_1^{2\alpha} \Gamma(2\alpha + 1)}, \\ \tilde{U}_3(X) = \frac{\tilde{A}(X)}{\lambda_1^{3\alpha} \Gamma(3\alpha + 1)}, \\ \tilde{U}_4(X) = \frac{\tilde{A}(X)}{\lambda_1^{4\alpha} \Gamma(4\alpha + 1)}, \\ \vdots \\ \tilde{U}_{k+1}(X) = (-1)^k \frac{\tilde{A}(X)}{\lambda_1^{(k+1)\alpha} \Gamma((k+1)\alpha + 1)}, \end{cases}$$

The differential inverse transforms of $\tilde{U}_k(X)$ generate the following:

$$(3.4) \quad \tilde{u}(X, t) = \sum_{K=0}^{\infty} \tilde{U}_K(X) t^K = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\left[\left(\frac{t}{\lambda_1} \right)^\alpha \right]^n}{\Gamma(n\alpha + 1)} \tilde{A}(X)$$

$$= -\tilde{A}(X) \left[\sum_{n=0}^{\infty} \frac{\left[-\left(\frac{t}{\lambda_1} \right)^{\alpha} \right]^n}{\Gamma(n\alpha + 1)} - 1 \right] = -\tilde{A}(X) \left[E_{\alpha} \left[-\left(\frac{t}{\lambda_1} \right)^{\alpha} \right] - 1 \right],$$

where $E_{\alpha} \left[t^{\alpha} \right] = \sum_{r=0}^{\infty} \frac{t^{r\alpha}}{\Gamma(r\alpha + 1)}$ is the one-parameter Mittag-Leffler function.

Finally, for $\alpha=1$, all the results can be reduced to the ordinary Maxwell model:

$$(3.5) \quad \tilde{u}(X, t) = \frac{t}{\lambda_1} - \frac{\left(\frac{t}{\lambda_1} \right)^2}{2!} + \frac{\left(\frac{t}{\lambda_1} \right)^3}{3!} - \frac{\left(\frac{t}{\lambda_1} \right)^4}{4!} + O \left[\frac{t}{\lambda_1} \right]^5 \tilde{A}(X),$$

$$(3.6) \quad \tilde{u}(X, t) = \left[1 - \exp \left(-\frac{t}{\lambda_1} \right) \right] \tilde{A}(X).$$

Eqns. (3.5), (3.6) constitute the exact solution. Furthermore, the case for a Newtonian fluid is retrieved with $\lambda_1 \rightarrow 0$. The pressure difference ΔP and friction force (F) across one wavelength are given by

$$(3.7) \quad \Delta P = \int_0^1 \frac{\partial p}{\partial x} dx,$$

$$(3.8) \quad F = \int_0^1 -h \frac{\partial p}{\partial x} dx.$$

4. Numerical Results and Discussion

Exact solutions have been derived for the stream function defining the flow of fractional Maxwell fluid under peristaltic waves in a curved channel. Approximate analytical solution has also been constructed for pressure gradient using the robust Reduced Differential Transformation

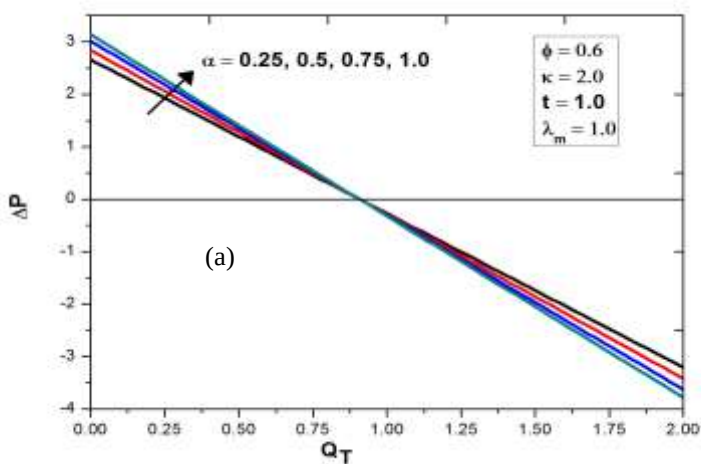
Method (RDTM), presented in terms of the one-parameter Mittag-Leffler function. These solutions can be adapted easily to obtain specific solutions for ordinary Maxwell and Newtonian fluids. Next, we conduct a parametric study of the characteristics of the flow regime. All solutions have been evaluated in Mathematica symbolic software and are presented in Figs. 2(a)-(d) to 5 (a)-(d).

The pressure difference across one wavelength is plotted against averaged flow rate in Figure 2 (a)-(d) at different fractional parameters α , with channel curvature κ , for relaxation time parameter λ_1 , and amplitude ratio ϕ , respectively. These figures reveals that the pressure-flow relationship is linear. The pressure difference ΔP_λ increases with increasing values of fractional parameter α and amplitude ratio ϕ whereas it decreases with increasing values of relaxation time λ_1 and curvature parameter κ when the flow rate is below a critical value, $Q_T < Q_c$. If the flow rate is above this critical value Q_c , the characteristic of ΔP_λ demonstrates the opposite behaviour at these parameter values. An increase in the flow rate reduces the pressure difference; thus maximum ΔP_λ is obtained at zero flow rate i.e. $Q_T = 0$ and is denoted by $P_{\max}$. When the pressure difference $\Delta P_\lambda = 0$, the corresponding time- averaged flux is denoted by $Q_{\max}$, and this zone is termed the free pumping zone. The pressure assists the flow when $\Delta P_\lambda < 0$ and this is known as co-pumping. It is observed from the Figures 2(a) and 2(d) that $P_{\max}$ increases with fractional parameter and amplitude ratio parameter. This means that peristalsis has to work against greater pressure rise when α and ϕ increases. When fractional parameter $\alpha = 1$, the model reduces to classical Maxwell flow. This means that pressure increases as one moves from the fractional model to classical model. Figure 2(b) indicates that peristalsis in a curved channel works against greater pressure rise as compared to flow in straight channel. It is seen from Figure 2(c) that the maximum pressure rise against which peristalsis pumping works decreases with an increase in the relaxation time parameter.

Figures 3 (a)-(d) shows the variations of friction force F with the averaged flow rate Q_T under the influence of all parameters $\alpha, \kappa, \lambda_1$ and ϕ . It is observed that friction forces exhibit the opposite behavior in comparison with pressure. The results for the case of a Newtonian fluid flow in a straight channel can be recovered by choosing $\lambda_1 \rightarrow \infty$ and $\kappa \rightarrow \infty$.

Figures 4a,b show the behaviour of curvature and time-averaged flow rate on velocity. It is observed that the velocity distribution is not symmetric in the channel for small curvature parameter, whereas the velocity profile is symmetric for large curvature parameter, i.e. in straight channel (Fig. 4a). It is also evident that the maximum velocity in the curved channel is achieved towards the lower wall of the channel and this is corroborated by a number of other studies in the literature for different non-Newtonian fluids – see e.g. Ali et al.¹⁰. Fig. 4(b) analyzes the behaviour of time-averaged flow rate Q_T in the curved channel and it is apparent that the flow is accelerated i.e. the chyme velocity is elevated with flow rate in the channel.

Trapping is an important phenomenon which is described as the translocation of a bolus of fluid as a whole with the wave³⁰. Trapping may be observed in curved channels by plotting the streamlines in a wave frame³¹. The streamline patterns in the wave frame for Newtonian fluid with $Q_T = 1.75$ and $\phi = 0.6$ for different values of curvature parameter κ are shown in Figure 5. It is observed that for small values of κ only one stagnation zone (bolus) is formed that is near to upper wall. As κ increases two stagnation zones with different magnitudes are formed. The bolus near the upper wall increases. However the bolus near the lower wall disappears eventually as the channel curvature increases. Overall, nevertheless the result agree closely to those computed for a straight channel ($\kappa \rightarrow \infty$) as the bolus splits with equal magnitudes and these are symmetrical about $r = 0$ ^{31, 32}.



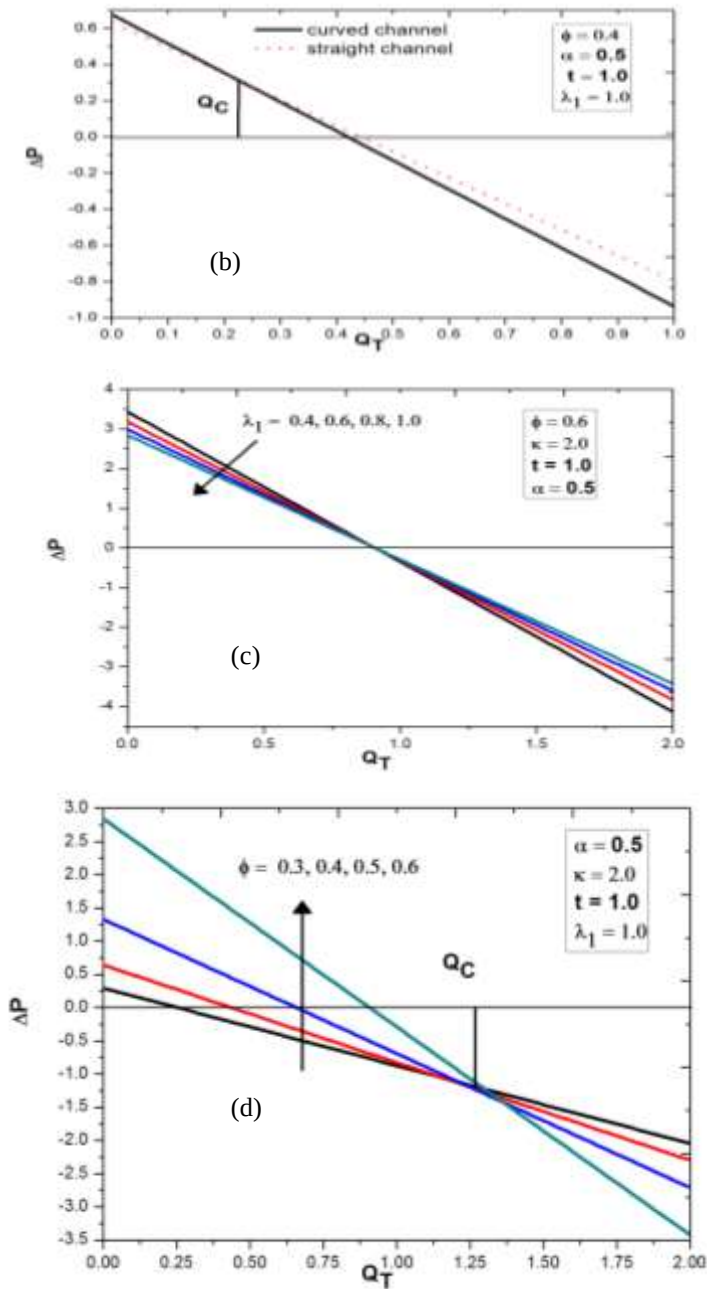
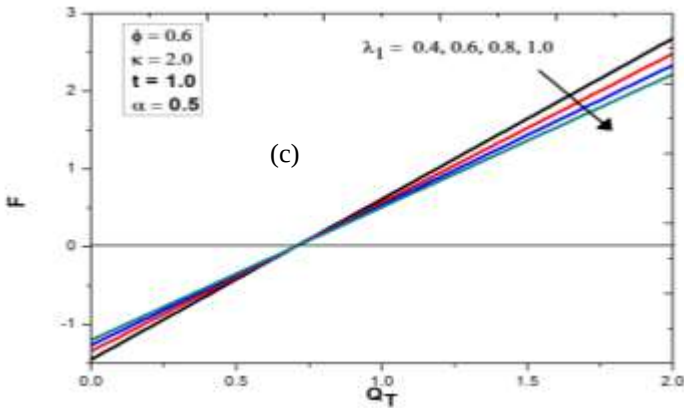
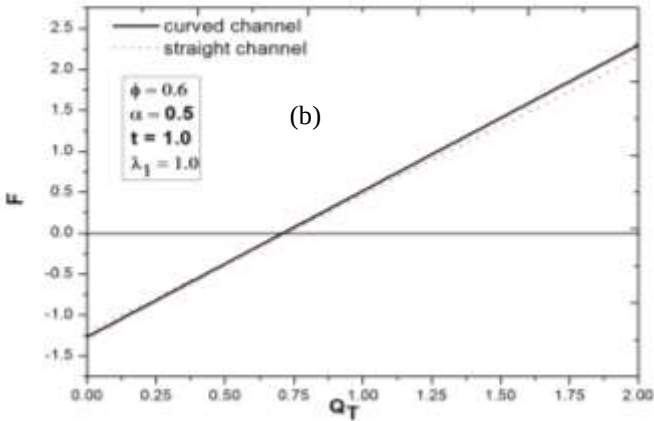
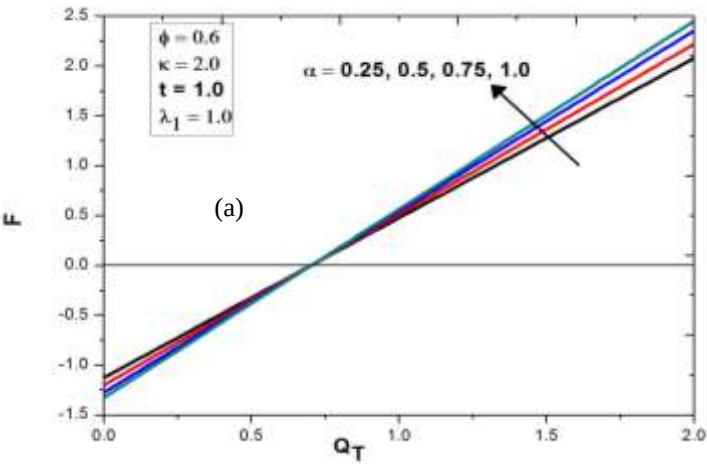


Figure 2. Pressure difference across one wavelength against time-averaged flow rate for (a) different values of fractional parameter (b) comparison between curved channel and straight channel (c) different values of relaxation parameter (d) different values of amplitude.



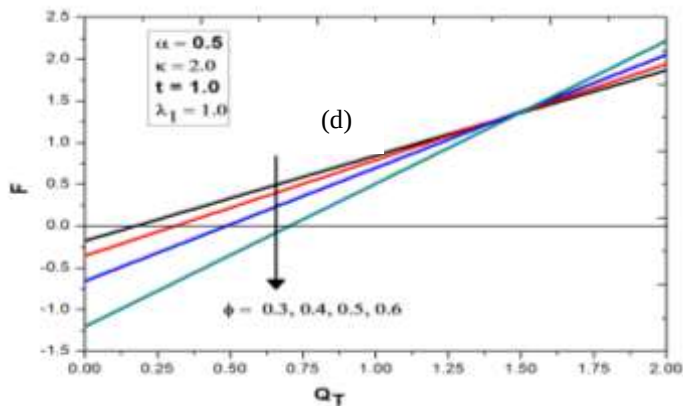


Figure 3. Friction force across one wavelength against time-averaged flow rate for (a) different values of fractional parameter (b) comparison between curved channel and straight channel (c) different values of relaxation parameter (d) different values of amplitude.

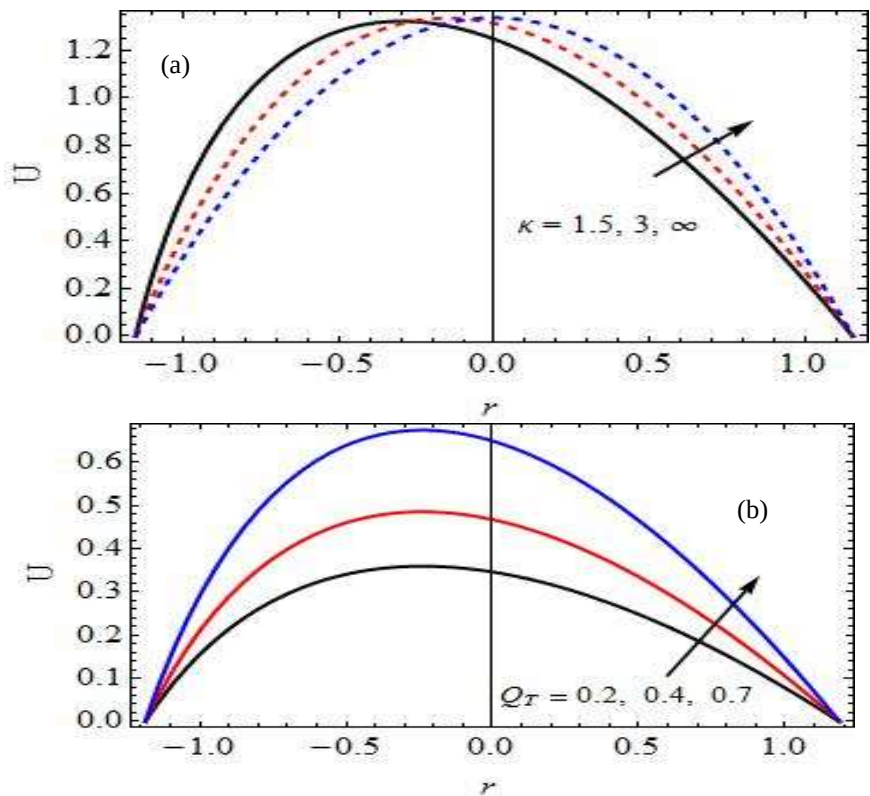
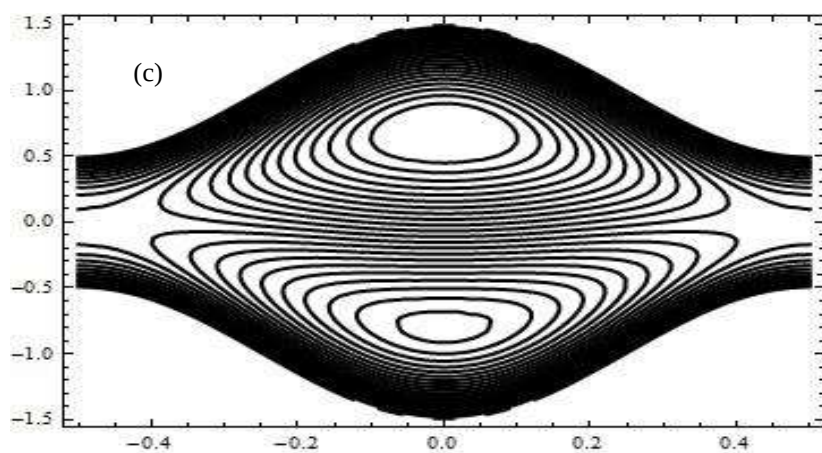
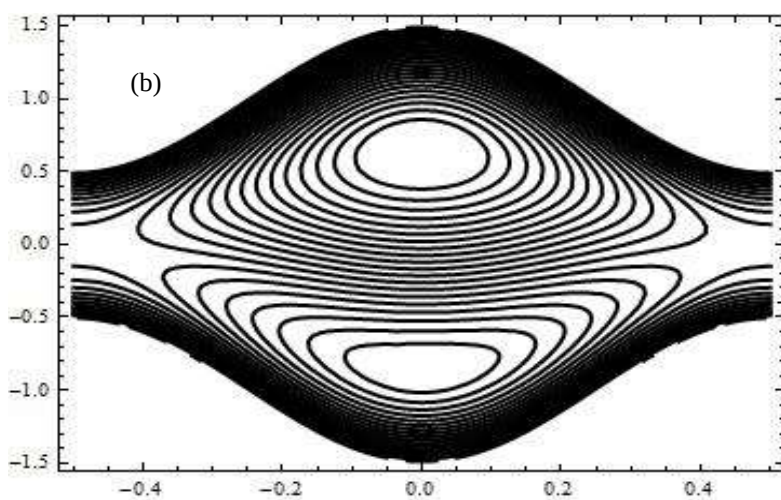
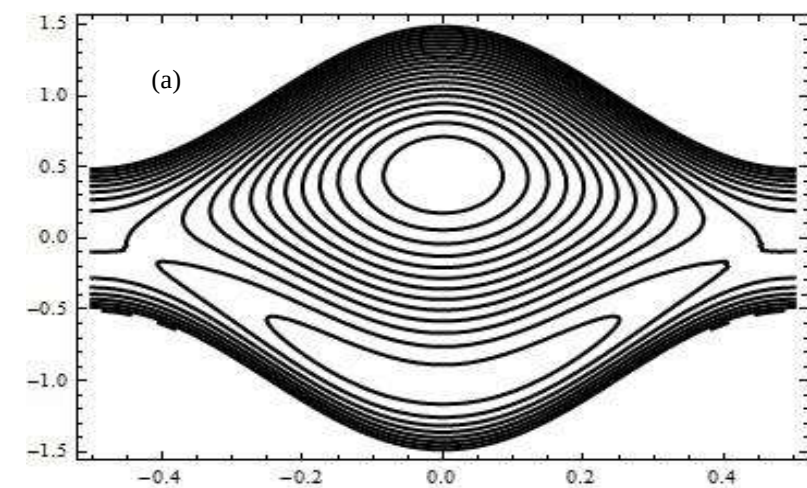


Figure 4. Variations of the longitudinal velocity against radial displacement for (a) different values of curvature parameter (b) different flow rates



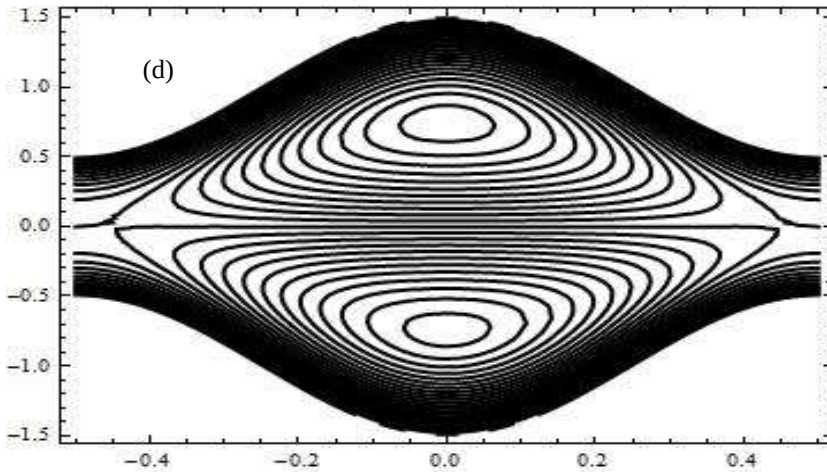


Figure 5. Streamline patterns at different channel curvatures (a) $k=2$, (b) $k=5$, (c) $k=10$, (d) $k=500$.

5. Conclusions

The effects of fractional parameters, channel curvature and viscoelastic material properties on peristaltic pumping of chyme in a two-dimensional channel model of the large intestine have been described. The chyme rheology has been studied with a fractional viscoelastic Maxwell. Analytical solutions have been derived for stream function at low inertial effect when the wavelength is moderately large compared to the channel width. An approximate analytical solution has also been obtained by using the Reduced Differential Transformation Method (RDTM) in terms of a one-parameter Mittag-Leffler function. The features of the flow characteristics are analyzed by plotting graphs and discussed in detail. The main conclusions can be summarized as follows:

1. The relation between pressure and flow is observed to be linear.
2. The pressure-flow function increases with increasing values of fractional parameter α and amplitude ratio ϕ , whereas it is reduced with increasing values of curvature parameter κ and relaxation time λ_1 .
3. The pressure profiles for fractional Maxwell model ($0 < \alpha < 1$) and classical Maxwell model ($\alpha=1$) have been compared and show substantial differences.
4. The variations of friction force against flow rate show opposite behavior to that of pressure.

5. The results for the case of Newtonian fluid flow in a straight channel can be recovered by choosing $\lambda_1 \rightarrow 0$ and $\kappa \rightarrow \infty$.

This model is applicable for the designing the smart pumping devices for the gastrointestinal dialysis however this study is limited to one of the fractional model of the viscoelastic fluids. This model could be further modified for the other viscoelastic fluids and thermal effects³³⁻³⁸ with nanofluids models can also be explored in future studies.

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