

Comparative Approach to Cost Computation of Arrival Control Queueing Model with Single Server Using SDM and GMIVM

Aabhas Kumar, Karma Jeet and S.S.Mishra*

Department of Mathematics & Statistics,

Dr Rammanohar Lohia Avadh University, Ayodhya, (U.P.), India

*Corresponding Author's Email: smssmishra5@gmail.com

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Abstract: In this article, we investigate the single server arrival control queueing system's cost optimization in an uncertain and fuzzy environment. This work aims to determine the optimal total-cost of the queueing model in the fuzzy environment, through its cost function. A single server arrival control queueing model was fuzzified using a triangular fuzzy number, and the Signed Distance Method (SDM) was employed to defuzzify the model. In order to achieve the paper's goal, we have additionally created parabolic and triangular, fuzzy numbers and used the Graded Mean Integration Value Method (GMIVM) for defuzzification of this model. In contrast to the standard crisp findings for the model, the computation of the fuzzy cost has been done in order to present a more realistic result to the model under discussion. Theoretical advancements for the systems that are crisp and fuzzy have been determined for each model independently, and estimated costs have made it easy to compare the two different approaches using computational tables. At the last, discussion and conclusion has also been given to gain better insight into the model.

Keywords: Fuzzy Optimization, Queueing Model, Cost Function, Signed Distance Method (SDM), Mean Integration Value Method (GMIVM).

AMS Subject Classification: 90B22, 60K25

1. Introduction

In certain conditions when ambiguity persists, fuzzy knowledge helps us make better judgments by providing more exact details. Fuzziness uses fuzzy logic and theory of fuzzy set, which have been studied by numerous authors. Bellman and Zadeh¹ have explored the challenges and remedies associated with decision-making in uncertain environments. Baby and Suvitha² have

described an examination of a standby server, single operating vacation server, and single server repairable queueing system with many service types. Bouchentouf *et al.*³ have created the arrival of the single server batch Bernoulli feedback queueing system with k-variant vacations, frustrated customers, and waiting servers. De and Rawat⁴ have given a triangle fuzzy numbers used in a fuzzy inventory model without shortages. Kaufmann⁵ investigated the fuzzy subset theory for many domains of applications. Zimmermann⁶ also discussed the theory of fuzzy set and its uses. Zadeh⁷ developed and used the possibility theory's underlying theory of uncertainty, and he proposed that the possibility theory is built upon fuzzy set theory. Markovian queues have significance for queueing theory research. Yao and Wu⁸ have investigated the decomposition concept and signed distance are used to rank fuzzy numbers. Khan and Paramasivam⁹ have analyzed decrease in a single server's delay time Markovian queueing promoted arrival model. Mishra and Yadav¹⁰ have developed and discussed by the computational method for queueing theory's, an examination of timed waiting circuits' expenses and earnings. Mishra and Shukla¹¹ described the theory of queues' computational approach to the machine interference model's cost analysis. Sanga *et al.*¹² have created the single server queue's fuzzy cost optimization under f-policy for admission control and balking. Shree *et al.*¹³ have analyzed cost analysis of a discrete-time modeling queue of priorities with Bernoulli feedback and cluster arrival. Seenivasan and Ramesh¹⁴ have investigated wingspans fuzzy ranking method is used to assess the costs of a phosphorescent bulk arrival queueing system. Vaithyanathan *et al.*¹⁵ gave the analysis of a single server system with heterogeneous arrival, heterogeneous service, system failure and maintenance. Sundari and Palaniammal¹⁶ have discussed the issue with employing an artificial neural network software to simulate an M/M/1 queueing system. The literature has demonstrated time and again that Markovian queues are crucial, and that fuzzy models for waiting are far more helpful than standard crisp theory since they offer more practical answers for a range of real-world scenarios. In many different application domains, fuzzy waiting models have more applications and are more realistic than crisp waiting models. Prado and Fuente¹⁷ have discussed the relevance of fuzzy set theory to the problems with queueing theory in Markovian decision systems. Li and Lee¹⁸ examined the fuzzy queues and talked about its uses in several fields. Buckley¹⁹ has investigated the fundamental theory of queueing based on possibility theory. Negi and Lee²⁰ work on these models for queueing and conducted an environment of uncertainty analysis. Dubois and Prade²¹ have analyzed fuzzy sets and systems. The control models play a crucial role in theory of queue.

Among these frameworks the control of the service is dependent on a number of different metrics, including service rate, server count, queue management, or a combination of these. Arrival management, which is accomplished by allocating arriving clients to certain servers or dispersing them among the customers, is another crucial aspect of queueing theory. The control of arrivals can be achieved either by toll devices or by practical limits, such as defining the parameters for physical space and working shift, among other things. Such parameters have been developed in queueing theory by Buckley *et al.*²², who also offered a fuzzy expert system solution. Chen²³ has described the fuzzy set theory approach to solve the problem of bulk arrival queueing model with different batch sizes and fuzzy parameters. Panta *et al.*²⁴ have investigated in a fuzzy environment, optimize the M/M/S/N waiting model with reneging. Ke and Lin²⁵ have explored the fuzzy analysis using nonlinear programming of queueing systems with an unreliable server. Chirkova and Mazalov²⁶ have analyzed optimal arrivals to preemptive queueing system. Laxmi and Hasan²⁷ analysis of the general service queue for batch arrivals including feedback, balking, and subsequent alternative service.

Later, a new trend in queueing models has swung to develop fuzzy parameters and coefficients in the models of waiting which are optimized by the utilization of fuzzy expert system optimization techniques. Fazlollahtabar and Gholizadeh²⁸ discussed the price model of the M/M/1/N waiting system economic analysis in an ambiguous environment. Prameela and Kumar²⁹ explored the Erlang service in the FM/ FE_k /1 queueing Approach under different fuzzy number types. Palpandi and Geetharamani³⁰ worked on the robust ranking approach is utilized to evaluate bulk arrival queue performance metrics with fuzzy parameters. Sanga and Jain³¹ worked on using a parametric nonlinear programming technique, the FM/FM/1 double orbit retrial queue with customers' joining strategies. Barak and Fallahnezhad³² discussed an evaluation of costs of fuzzy waiting techniques. Fathi-Vajargah *et al.*³³ worked on the analysis of a random fuzzy waiting system with several servers using simulation. Enrique and Enrique³⁴ have created fuzzy queueing systems simulation with changing arrival, service, and number of servers. Kannadasan and Sathiyamoorth³⁵ have developed the M/M/1 queue analysis in a fuzzy environment with working vacation. Chen *et al.*³⁶ worked on the examination of customers' strategic behavior in ambiguous waiting structures. Keith and Darryl³⁷ carried out a survey of decision making and optimization under uncertainty. Singh *et al.*³⁸ worked on the Markovian arrival and service queueing model's optimization under fuzziness. Zheng *et al.*³⁹ have explored an M/M/1 queue with server failures and adaptive service rate management.

Mishra⁴⁰ discussed the signed distance approach and ANN were used to analyze the costs of the M/M/1 and MF/MF/1 queuing models. Subhapiya and Thiagarajan⁴¹ have created M/M/1 interdependent waiting system that includes controlled arrival rates and vacation periods.

Recently cost optimization of arrival service control queueing model has been attempted by Mishra *et al.*⁴² under fuzzy environment by using trapezoidal fuzzy numbers and Signed Distance Method for defuzzification of the model. In this paper, an endeavor has been made to present the comparative study of two different methods of defuzzification on previous model for fuzzification, triangular fuzzy numbers in both of methods namely Signed Distance Method (SMD) and Graded Mean Integration Value Method (GMIVM) have been used. Optimum overall cost of the model has been completed by applying different methods of defuzzification SDM, GMIVM and has been subjected to comparison in order to determine more efficient method of defuzzification applied on this queueing model. Apart from applying fuzzification by triangular number, we have also used parabolic fuzzy number for fuzzification to further measure the impact on optimal total cost of the model.

2. Preliminaries

A fuzzy set A is defined by a membership function $\mu_A(x)$ which maps each and every element of X to [0, 1], i.e. $\mu_A(x) \rightarrow [0,1]$, the underlying ground set is denoted by X. To put it simply, a fuzzy set is one whose boundaries are unclear. However, as previously said, a fuzzy set is one whose elements are defined by a membership function.

2.1 Triangular Fuzzy Numbers (TFN) : Let $u, v, w \in \mathcal{R}$ s.t $u < v < w$, then a TFN (Triangular Fuzzy Number) is denoted by $T=(u, v, w)$ and is defined through the membership function. De et al⁴.

$$\mu_T(x) = \begin{cases} 0, & u \leq x, \\ \frac{x-u}{v-u}, & u \leq x \leq v, \\ \frac{w-x}{w-v}, & v \leq x \leq w, \\ 0, & x \geq w, \end{cases}$$

2.2 Arithmetic of TFNs: Triangular Fuzzy Numbers can be subjected to the following four operations:

Let $L=(a'_1, a'_2, a'_3)$ and $M=(b'_1, b'_2, b'_3)$ subsequently

(i) **Addition:** $L + M = (a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3).$

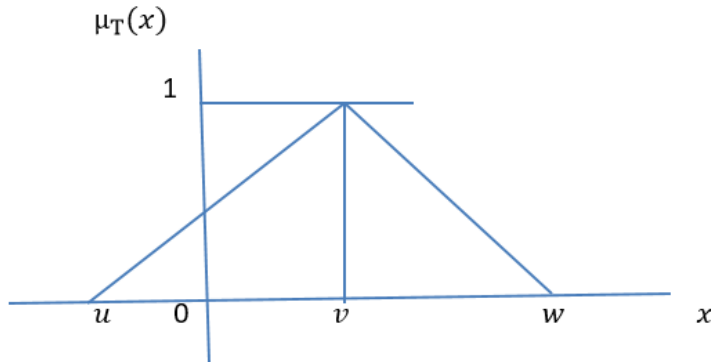
(ii) **Subtraction:** $L - M = (a'_1 - b'_3, a'_2 - b'_2, a'_3 + b'_1)$

(iii) Multiplication:

$$L \times M = [\min(a'_1 b'_1, a'_1 b'_3, a'_3 b'_1), a'_2 b'_2, \max(a'_1 b'_1, a'_1 b'_3, a'_3 b'_1)].$$

(iv) Division:

$$L \div M = \left[\min\left(\frac{a'_1}{b'_1}, \frac{a'_1}{b'_3}, \frac{a'_3}{b'_1}\right), \frac{a'_2}{b'_2}, \max\left(\frac{a'_1}{b'_1}, \frac{a'_1}{b'_3}, \frac{a'_3}{b'_1}\right) \right].$$

Fig 1: Variation of membership degree w.r.to x

2.3 Alpha- cuts of a TFN: The Alpha-cuts A_α of a TFN $T=(u, v, w)$, $\alpha \in [0, 1]$ are given by $A_\alpha = [AL_\alpha, AR_\alpha] = [u + \alpha(v - u), w - \alpha(w - v)]$.

2.4 Definitions

2.4.1. Let $d^*(\alpha, o) = ; \alpha, o \in \mathbb{R}$. Geometrically, $\alpha > 0$ means that α falls to the right of the origin o and the distance then is given by $\alpha, \alpha = d^*(\alpha, o)$. Likewise, $\alpha < 0$ say that α falls to the left of the origin o and the corresponding distance between then is given by $-\alpha = d^*(\alpha, o)$. Hence $d^*(\alpha, o)$ represents the signed distance of α from o .

2.4.2 Signed-distance TFN for $A = (r'_1, r'_2, r'_3)$ measured from $\widetilde{O}_1$ is

$$d(A, \widetilde{O}_1) = \frac{1}{4}(r'_1 + 2r'_2 + r'_3).$$

Property 1. Let $A = (p, q, r)$ and $B = (u, v, w) \in F_N$. Then, direct sum is given by and signed-distance between $A \oplus B$ and $\widetilde{O}_1$ is defined by

$$d(A \oplus B, \widetilde{O}_1) = d(A, \widetilde{O}_1) + d(B, \widetilde{O}_1).$$

3. Formulation of Arrival Control Model

We consider a model of arrival control, both in environments that are crisp and fuzzy. We first create an arrival control model in a crisp environment, and then we use fuzzy parameters to examine the identical problem in a fuzzy environment. Particularly we have paid our attention to ordering and service rate as uncertain quantities. They are demonstrated by TFN (triangular fuzzy

numbers), and the signed distance approach is used to determine the overall cost for defuzzification. Eventually, the outcome was compared to the precise values that were obtained for both the arrival control model and the overall cost.

3.1 Symbols Used in the Model.

The arrival model of control related to this work utilizes the following symbols.

- TC = Optimum overall cost
- k = cost of service per unit
- c = cost of waiting per unit (WC)
- λ =rate of consumer arrival
- μ = service rate
- $\tilde{k}$ = fuzzified cost of service per unit
- $\tilde{c}$ = fuzzified cost of waiting per unit (FWC)
- $\tilde{\lambda}$ = fuzzified arrival rate of customer
- E(n) = The system has an average number of customers.

3.2 Model of Arrival Control: Crisp Formulation.

The model's TCF (total cost function) is described as

$$TC = k\mu + \lambda E(n)$$

This gives

$$TC = F_{\mu}(k, c, \lambda) = k\mu + \frac{\lambda c}{\mu}$$

For minimum cost relative to service, we have, for stationary values; this ultimately yields along with having satisfied for minimum cost.

$$\frac{dT_C}{d\mu} = \frac{d}{d\mu} \left[k\mu + \frac{\lambda c}{\mu} \right], \quad \frac{dT_C}{d\mu} = k - \frac{\lambda c}{\mu^2},$$

For stationary values, $\frac{dT_C}{d\mu} = 0$, this ultimately yields $\mu = \left[\frac{c\lambda}{K} \right]^{\frac{1}{2}}$ along with having satisfied, $\frac{d^2TC}{d\mu^2} = \frac{2c\lambda}{\mu^3} \geq 0$ for minimum cost.

3.3 Model of Arrival Control: Crisp Numerical Analysis

Relevant computations are presented in following table 1.

Table-1 (Computation table for k, TC)

k	c	λ	μ	TC
5	75	4	7.74	77.71
7	75	4	6.54	91.65
9	75	4	5.77	103.92
11	75	4	5.22	114.89

4. Fuzzy Mathematical Model

Now, we want to use a triangular fuzzy number to fuzzify the coefficients of cast and leading rate k, c, λ & $\tilde{k}, \tilde{c}$ and $\tilde{\lambda}$ respectively in LR form. We have further as

$$\tilde{k} = (k, k_1, k_2), \tilde{c} = (c, c_1, c_2), \tilde{\lambda} = (\lambda, \lambda_1, \lambda_2)$$

Consequently, total cost fuzzified is

$$\widetilde{TC} = \tilde{k}\mu \oplus \frac{\tilde{\lambda} \tilde{c}}{\mu} \text{ which given } \widetilde{TC} = [\tilde{k}\mu] \oplus \left[\frac{\tilde{\lambda} \tilde{c}}{\mu} \right]$$

$$\widetilde{TC} = [(k - k_1, k, k + k_2) \otimes \mu] \oplus \left[\frac{[(\lambda - \lambda_1, \lambda, \lambda + \lambda_2) \otimes (c - c_1, c, c + c_2)]}{\mu} \right]$$

$$\widetilde{TC} = [(k\mu - k_1\mu), k\mu, (k\mu + k_2\mu)] + \left[\frac{[(\lambda - \lambda_1)(c - c_1), \lambda c, (\lambda + \lambda_2)(c + c_2)]}{\mu} \right]$$

$$\widetilde{TC} = \left[(k\mu - k_1\mu) + \frac{(\lambda - \lambda_1)(c - c_1)}{\mu}, \right.$$

$$\left. k\mu + \frac{\lambda c}{\mu}, (k\mu + k_2\mu) + \frac{(\lambda + \lambda_2)(c + c_2)}{\mu} \right]$$

$$\widetilde{TC} = \left[\left(k\mu + \frac{\lambda c}{\mu} \right) - \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right), \left(k\mu + \frac{\lambda c}{\mu} \right), \right. \\ \left. \left(k\mu + \frac{\lambda c}{\mu} \right) + \left(k_2\mu + \frac{((c\lambda_2 + \lambda c_2 + c_2\lambda_2))}{\mu} \right) \right]$$

$$\widetilde{TC} = \left[F_\mu(k, c, \lambda) - \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right), F_\mu(k, c, \lambda), F_\mu(k, c, \lambda) \right. \\ \left. + \left(k_2\mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2)}{\mu} \right) \right]$$

$$\widetilde{TC} = [\alpha, \beta, \gamma], \text{ where } \alpha = F_\mu(k, c, \lambda) - \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right),$$

$$\beta = F_\mu(k, c, \lambda), \gamma = F_\mu(k, c, \lambda) + \left(k_2\mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2)}{\mu} \right)$$

Applying defuzzification using signed distance method,

$$d[F_\mu(\tilde{k}, \tilde{c}, \tilde{\lambda}), 0] = \frac{1}{4} [\alpha + 2\beta + \gamma]$$

$$d[F_\mu(\tilde{k}, \tilde{c}, \tilde{\lambda}), 0]$$

$$= \frac{1}{4} \left[F_\mu(k, c, \lambda) - \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right) \right. \\ \left. + 2F_\mu(k, c, \lambda) + F_\mu(k, c, \lambda) + \left(k_2\mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2)}{\mu} \right) \right]$$

$$\begin{aligned} d[F_{\mu}(\tilde{k}, \tilde{c}, \tilde{\lambda}), 0] &= F_{\mu}(k, c, \lambda) + \frac{1}{4} [(k_2 - k_1)\mu \\ &\quad + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu}] \\ &= F_d(\mu) \text{ Say} \end{aligned}$$

Calculating μ so that $F_d(\mu)$ is at its lowest

$F_d(\mu)$ is minimum at $F'_d(\mu) = 0$, where $F''_d(\mu)$ is positive. This implies that

$$\begin{aligned} F'_d(\mu) &= \frac{d}{d\mu} F_d(\mu) = k - \frac{c\lambda}{\mu^2} \\ &\quad + \frac{1}{4} \left[(k_2 - k_1) - \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu^2} \right] \\ F'_d(\mu) &= \frac{d}{d\mu} F_d(\mu) \\ &= \{4k\mu^2 - 4c\lambda + (k_2 - k_1)\mu^2 - [(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)]\} \end{aligned}$$

Now $F'_d(\mu) = 0$ will imply

$$\begin{aligned} \{4k\mu^2 - 4c\lambda + (k_2 - k_1)\mu^2 \\ - [(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)]\} = 0 \end{aligned}$$

i.e.

$$\begin{aligned} [4k + (k_2 - k_1)]\mu^2 \\ = 4c\lambda + [(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)] \end{aligned}$$

which implies

$$\mu = \sqrt{\frac{4c\lambda + c\lambda_2 + \lambda c_2 + c_2\lambda_2 - c\lambda_1 - \lambda c_1 - c_1\lambda_1}{4k + k_2 - k_1}}$$

4.1 Arrival Control Model: Fuzzy Computation

The computation of the cost function in a fuzzy environment is represents in table 2 below.

Table-2 (Computation table for $\tilde{k}, \tilde{Tc}$)

$\tilde{k}$	$\tilde{c}$	$\tilde{\lambda}$	$\tilde{\mu}$	$\tilde{Tc}$
(k, k_1, k_2)	(c, c_1, c_2)	$(\lambda, \lambda_1, \lambda_2)$		
(4, 5, 6)	(74, 75, 76)	(3, 4, 5)	7.81	66.63
(6, 7, 8)	(74, 75, 76)	(3, 4, 5)	7.05	80.56
(8, 9, 10)	(74, 75, 76)	(3, 4, 5)	5.62	97.84
(10, 11, 12)	(74, 75, 76)	(3, 4, 5)	5.04	103.48

4.2 Use of Graded Mean Integration Method: The same arrival control queuing model, whose costs are distinct fuzzy numbers, may be studied by applying another significant defuzzification method of GMIV. By this method of defuzzification, we have as,

4.2.1 For triangular fuzzy number (TFN) $\tilde{A} = (\alpha, \beta, \gamma)$

$$P_{dG}(\tilde{A}) = \frac{1}{6}(\alpha + 4\beta + \gamma)$$

$$P_{dG}(\tilde{A}) = \frac{1}{6} \left[F_{\mu}(k, c, \lambda) - \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right) + 4F_{\mu}(k, c, \lambda) \right. \\ \left. + F_{\mu}(k, c, \lambda) + \left(k_2\mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2)}{\mu} \right) \right]$$

$$P_{dG}(\tilde{A}) = F_{\mu}(k, c, \lambda) + \frac{1}{6}[(k_2 - k_1)\mu \\ + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu}] \\ = F_{d1}(\mu) \text{ Say}$$

$F_{d1}(\mu)$ is minimum at $F'_{d1}(\mu) = 0$ where $F''_{d1}(\mu)$ is positive.

$$6k\mu^2 - 6c\lambda + (k_2 - k_1)\mu^2 \\ - [(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)] = 0$$

$$\text{and } \mu^2 = \frac{6c\lambda + c\lambda_2 + \lambda c_2 + c_2\lambda_2 - c\lambda_1 - \lambda c_1 - c_1\lambda_1}{6k + k_2 - k_1}$$

finally

$$\mu = \sqrt{\frac{6c\lambda + c\lambda_2 + \lambda c_2 + c_2\lambda_2 - c\lambda_1 - \lambda c_1 - c_1\lambda_1}{6k + k_2 - k_1}}$$

The cost function for Defuzzifying a TFN (triangular fuzzy number) under the GMIVM (Graded Mean Integration Value Method) is computed in Table 3 below.

Table-3 (Computation table for $\tilde{k}, \tilde{c}$)

$\tilde{k}$	$\tilde{c}$	$\tilde{\lambda}$	$\tilde{\mu}$	$\tilde{TC}$
(k, k_1, k_2)	(c, c_1, c_2)	$(\lambda, \lambda_1, \lambda_2)$		
(4, 5, 8)	(74, 75, 78)	(3, 4, 7)	8.18	68.52
(6, 7, 10)	(74, 75, 78)	(3, 4, 7)	6.84	97.49
(8, 9, 12)	(74, 75, 78)	(3, 4, 7)	5.95	111.13
(10, 11, 14)	(74, 75, 78)	(3, 4, 7)	5.63	122.82

4.2.2 Parabolic Fuzzy Number (PFN) $\tilde{A} = (\alpha, \beta, \gamma)$

$$P_{dG}(\tilde{A}) = \frac{1}{15}(4\alpha + 7\beta + 4\gamma)$$

$$P_{dG}(\tilde{A}) = \frac{1}{15} \left[4F_{\mu}(k, c, \lambda) - 4 \left(k_1\mu + \frac{(c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \right) + 7F_{\mu}(k, c, \lambda) \right. \\ \left. + 4F_{\mu}(k, c, \lambda) + 4 \left(k_2\mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2)}{\mu} \right) \right]$$

$$P_{dG}(\tilde{A}) = \left[F_{\mu}(k, c, \lambda) + \frac{4}{15} \{ (k_2 - k_1) \mu + \frac{(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)}{\mu} \} \right] = F_{d2}(\mu)$$

$F_{d2}(\mu)$ is minimum at $F'_{d2}(\mu) = 0$ where $F''_{d2}(\mu)$ is positive.
 $15k\mu^2 - 15c\lambda + 4(k_2 - k_1)\mu^2 - [(c\lambda_2 + \lambda c_2 + c_2\lambda_2) - (c\lambda_1 + \lambda c_1 + c_1\lambda_1)] = 0$
and

$$\mu^2 = \frac{15c\lambda + c\lambda_2 + \lambda c_2 + c_2\lambda_2 - c\lambda_1 - \lambda c_1 - c_1\lambda_1}{15k + 4k_2 - 4k_1}$$

finally

$$\mu = \sqrt{\frac{15c\lambda + c\lambda_2 + \lambda c_2 + c_2\lambda_2 - c\lambda_1 - \lambda c_1 - c_1\lambda_1}{15k + 4k_2 - 4k_1}}$$

Following table 4 represents computation of cost function under by using GMIVM (Graded Mean Integration Value Method) for defuzzification of PFN (Parabolic Fuzzy Number).

Table-4 (Computation table for $\tilde{k}, \tilde{TC}$)

$\tilde{k}$	$\tilde{c}$	$\tilde{\lambda}$	$\tilde{\mu}$	$\tilde{TC}$
(k, k_1, k_2)	(c, c_1, c_2)	$(\lambda, \lambda_1, \lambda_2)$		
(4, 5, 8)	(74, 75, 78)	(3, 4, 7)	7.27	82.96
(6, 7, 10)	(74, 75, 78)	(3, 4, 7)	6.08	95.35
(8, 9, 12)	(74, 75, 78)	(3, 4, 7)	5.37	106.09
(10, 11, 14)	(74, 75, 78)	(3, 4, 7)	5.02	115.93

5. Discussion and Conclusion

In this paper, we have been able to establish cost function of the queueing model. To find ideal overall expense of the queueing system in the fuzzy queueing model's fuzzy environment is among its most crucial features. When one or more parameters are uncertain or fuzzy in nature, we fuzzify the queueing models by using various fuzzy sets or number and defuzzify it using different methods to provide a scalar quantity for begin use for appropriate decision making.

Here, in the work, we have fuzzified the queueing model by using triangular and parabolic fuzzy numbers and two different methods of defuzzification- Signed Distance Method and Graded Mean Integration Value Methods have been applied and ultimately optimal total cost of the queueing model under consideration has been obtained in computational tables 2, 3 and 4. We find that though these tables total optimal cost obtained

by applying SDM is lesser that optimal total cost obtained by applying GMIVM which leads to the conclusion that for such category of queueing models, SMD is more suitable and efficient than GMIVM.

For future research, we will study such queueing models under intuitionistic fuzzy environment and prepare two comparable settings of fuzzy and intuitionistic fuzzy queueing models.

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