

Comparative Transient Fuzzy Performance Measure of a Queue with Catastrophes by Utilizing Lenient Alpha-Cut Method and by L-R Method

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Abstract: The paper makes a comparative transient study of an infinite server queue with catastrophes and repair under fuzzy environment using two fuzzy methods— one is through lenient alpha-cuts mathematics and other one is through L-R mathematics. The study has been made by modelling a university campus with free Wi-Fi facility to the persons within the campus at any time. The model has numerical parameters and the model is based on the queueing system described by Bura¹. Average number of persons within the campus (at any time) has been computed and compared by the two methods mentioned above.

Keywords: Disaster, catastrophe, triangular fuzzy number, L-R triangular fuzzy number, lenient alpha-cuts method, L-R method.

1. Introduction

Catastrophes aka disasters are real-time phenomena which renders a system to a complete stoppage of its services. Sources of Catastrophes are various, e.g., natural (disruption of communication network due to bad weather), system overload (heavy power demand in peak hours), cyber-attacks in computer systems, human mistakes (road accidents due to rash driving), sudden failure of key machinery in manufacturing plants etc. Queueing researchers included this real-time happening phenomena into their research list in modelling various real-time systems that are capable of modelled as a queueing system. Other words that are used in place of disaster or catastrophe are— queue flushing, stochastic clearing etc. (see Yang *et al.*²). To get the knowledge of queues with disasters under various settings and with various applications one can study (and also the references there in), e.g.,

Kumar & Arivudainabi³, Kumar *et al.*⁴, Kalidas *et al.*⁵, Kumar *et al.*⁶, Dharmaraja & Kumar⁷ etc. While searching the literature on queues with disasters or catastrophe we found enough works on such queues from works of Towsley & Tripathi⁸, Chao⁹, Jain & Sigman¹⁰ etc. during last decade of previous century to the works of Sudhesh *et al.*¹¹, Bura¹, Laxmi & Bhavani¹², Remya *et al.*¹³ etc. upto on-going third decade of this century, to name a few.

Usually, we do not have sufficient, definite and precise information about the system parameters of a queueing system that models a real phenomenon e.g., we often hear “service rate is nearly 3 units per hour” or “arrival rate of clients is 2 clients per hour”. The reason lies in the complex nature of real systems and real phenomenon. To tackle the issue of impreciseness and bluntness of parameters of a real system fuzzy set theory, originated by Zadeh¹⁴, has emerged as a tool. Fuzzy queues are closer to real queues than crisp queues are. Fuzzy environment makes the system parameters possibilistic, whereas crisp parameters are probabilistic. In fuzzy system, fuzziness of a parameter is described by a membership function (also, in addition, a non-membership function too, in case of neutrosophic fuzzy system). As regards to fuzzy models of queues with disasters/catastrophes, we could find only two such works upto this time, viz., Shekhar *et al.*¹⁵ and Dhibar & Jain¹⁶. Both the investigations are steady state and used α -cuts and parametric nonlinear programming (PNLP).

During the course of searching the literature on fuzzy queues we came across two more methods besides alpha-cut-cum-PNLP method--one is “*Left-Right method (L-R method)*” introduced by Mukeba *et al.*¹⁷ and other one is “*flexible alpha-cuts method*” laid down by Mukeba Kanyinda¹⁸. Both the methods used interval arithmetic in conjugation with L-R fuzzy number (in former method) and alpha-cuts of a fuzzy number (in later method) for computations. This combination made almost all computations crisp. For interval analysis (or interval computations) and for their applications one can study books by, e.g., Moore¹⁹, Alefeld & Herzberger²⁰, Jaulin *et al.*²¹. Some other works on the above two methods, we may cite- Mukeba Kanyinda *et al.*²², Mueen *et al.*²³, Mukeba²⁴, Mukeba Kanyinda²⁵, Saini *et al.*²⁶ etc. It may be noted that all the above studies are steady-state fuzzy study. Lately w’Omatete *et al.*²⁷ and Mabela *et al.*²⁸ have, respectively, applied flexible alpha-cuts method and L-R method to a markovian queue in transient state to find transient state performance measures successfully. Above discussion-cum-literature review shows that no one has used the “*flexible alpha-cuts method*” and/or “*Left-Right method (L-R method)*” method in studying disaster fuzzy queue either in transient scene or in steady-state scene till date. In this paper, we humbly take this opportunity to fill the gap. In this paper we

propose to study the time-dependent mean customer number in a disaster (catastrophic) queueing model described by Bura¹ under fuzzy environment using both the “flexible alpha-cuts method” and “Left-Right method (L-R method)” taking triangular fuzzy numbers with a view to compare the results. The model will be studied by taking up a real situation of an university campus. In what follows, we use the term ‘lenient’ in place of ‘flexible’.

The course of paper follows following path: the model and the problem (to be studied) are described in section 2. Fuzzy preliminaries are detailed in section 3. Section 4. briefly describes α -cuts arithmetic with the help of interval arithmetic. Methodology of lenient alpha-cuts method and solution of the proposed problem are presented in section 5. L-R triangular fuzzy numbers and its arithmetic, L-R method as well as solution of the proposed problem through this method are given in section 6. A discussion on obtained results are provided in section 7. Section 8 concludes the paper with references.

2. The Crisp Queue with Disaster

We consider M/M/ ∞ queueing system that can meet a catastrophic breakdown with repairing of servers under transient regime. Mean rate of Poisson arrivals is λ — one by one. Service is exponential with rate η . Poisson catastrophic breakdowns (disasters) occur with rate θ . Instant repair starts for collapsed servers with exponential rate δ . Customers are not allowed to get into the system as long as repairing continues.

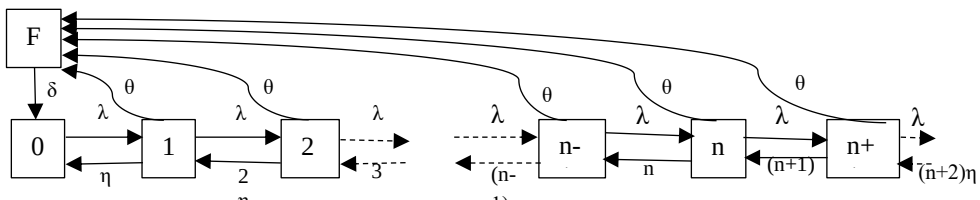


Fig.1: State-transitions

Problem: Let us consider a university campus providing cost-free Wi-Fi services to all persons within the campus. Entrants are entering into the campus with approximate Poisson rate 2. Exponential rate of Wi-Fi use by the persons within the campus is 12. The Wi-Fi connection is unreliable whose approximate Poisson rate of failure is 3. Whenever Wi-Fi failure occurs, it flushes out all the Wi-Fi users at the time. Immediate attempts are made to restore the Wi-Fi connections with some approximate exponential rate of repair. During the Wi-Fi failure, the persons within the campus are forbidden to enter into the campus till the repair is over. Find the approximate

average number of persons within the campus at any time 't'. (All the quantities are in appropriate units).

If we consider each Wi-Fi connection as a queue server then the above problem can be seen approximately as an infinite server Markovian queue (M/M/∞) having catastrophe and repair under non-deterministic environment with deterministic time. Thus, the queueing model of Bura¹, as described above, appropriately models the above problem and fuzzy mathematics is the appropriate mathematical tool to solve the above problem. In what follows we solve the above problem by using two types of fuzzy mathematics — one is alpha-cut mathematics and second one is L-R mathematics.

Average number of persons within the campus at any time 't', $\mathcal{M}(t)$, is given by

$$\mathcal{M}(t) = \frac{\lambda}{\eta + \theta} [1 - e^{-(\eta + \theta)t}]$$

3. Fuzzy Sets, Fuzzy Numbers, Fuzzy Function and Related Concepts

In this section we present some essential preliminaries regarding fuzzy sets, fuzzy numbers, extension principle etc. and for details, we refer the interested readers to the relevant literature mentioned:

Fuzzy Set: [Syropoulos and Grammenos²⁹] Let U be a classical set (crisp set) of discourse or a universe. A fuzzy set (or a fuzzy subset) $\bar{\mathcal{A}}$ of U is characterized by a function, called membership function, $\mu_{\bar{\mathcal{A}}}: U \rightarrow [0, 1]$. The function $\mu_{\bar{\mathcal{A}}}(x)$ is called grade of x or membership degree of x to the fuzzy set $\bar{\mathcal{A}}$ for $\forall x \in U$. A fuzzy set $\bar{\mathcal{A}}$ is said to be convex if and only if $\mu_{\bar{\mathcal{A}}}[\lambda x + (1 - \lambda)y] \geq \min[\mu_{\bar{\mathcal{A}}}(x), \mu_{\bar{\mathcal{A}}}(y)]$, $\forall x, y \in U, \lambda \in [0, 1]$.

For a fuzzy set $\bar{\mathcal{A}}$ in U we define following terms through crisp sets [Hanss³⁰]:

Core of $\bar{\mathcal{A}}$: $core(\bar{\mathcal{A}}) \stackrel{\text{def}}{=} \{x \in U: \mu_{\bar{\mathcal{A}}}(x) = 1\}$

Height of $\bar{\mathcal{A}}$: $hgt(\bar{\mathcal{A}}) \stackrel{\text{def}}{=} \sup\{\mu_{\bar{\mathcal{A}}}(x): x \in U\}$

Support of $\bar{\mathcal{A}}$: $supp(\bar{\mathcal{A}}) \stackrel{\text{def}}{=} \{x \in U: \mu_{\bar{\mathcal{A}}}(x) > 0\}$

Alpha-cut (or α -cut) of $\bar{\mathcal{A}}$: $\bar{\mathcal{A}}_{\alpha} \stackrel{\text{def}}{=} \{x \in U: \mu_{\bar{\mathcal{A}}}(x) \geq \alpha\}, \alpha \in [0, 1]$

A fuzzy set $\bar{\mathcal{A}}$ is normal if $hgt(\bar{\mathcal{A}}) = 1$. Alpha-cuts are also called α -level sets in the literature³¹. The $\mu_{\bar{\mathcal{A}}}(x)$, the membership function of the fuzzy set $\bar{\mathcal{A}}$, can also be expressed through the alpha-cuts of the fuzzy set $\bar{\mathcal{A}}$ using characteristic function of the alpha-cuts: $\mu_{\bar{\mathcal{A}}}(x) = \sup_{\alpha \in [0, 1]} \inf(\alpha, \chi_{\bar{\mathcal{A}}_{\alpha}}(x))$,

where $\chi_{\bar{\mathcal{A}}_{\alpha}}$ is characteristic (or, indicator) function of $\bar{\mathcal{A}}_{\alpha}$:

$$\chi_{\bar{\mathcal{A}}_{\alpha}}(x) = \begin{cases} 1 & \text{if } x \in \bar{\mathcal{A}}_{\alpha} \\ 0 & \text{if } x \notin \bar{\mathcal{A}}_{\alpha} \end{cases}$$

Fuzzy Number: [Buckley and Eslami³²] A fuzzy subset $\bar{\mathcal{A}}$ of the set of real number $\mathcal{R}$ defines a fuzzy number if

- (i) the core of $\bar{\mathcal{A}}$ is non-void;
- (ii) α -cuts $\bar{\mathcal{A}}_\alpha$ are all closed and bounded intervals of $\mathcal{R}$; and
- (iii) the support of $\bar{\mathcal{A}}$ is bounded.

A fuzzy number $\bar{\mathcal{A}}$ is said to be positive, if $\mu_{\bar{\mathcal{A}}}(x) = 0, \forall x < 0$ and is said to be negative, if $\mu_{\bar{\mathcal{A}}}(x) = 0, \forall x > 0$ [Syropoulos and Grammenos²⁹]. For two fuzzy numbers $\bar{\mathcal{A}}$ and $\bar{\mathcal{B}}$, we define $\bar{\mathcal{A}} \leq \bar{\mathcal{B}} \Leftrightarrow u \leq v, \forall u \in \text{supp}(\bar{\mathcal{A}}), \forall v \in \text{supp}(\bar{\mathcal{B}})$ [Mukeba *et al.*¹⁷].

Extension Principle: Extension principle is a fundamental idea of theory of fuzzy sets which allows us to transform nonfuzzy (or, crisp) mathematics to fuzzy mathematics. The extension principle may be defined as given below [Hanss³⁰]:

Let $X = X_1 \times X_2 \times X_3 \dots \times X_n$ be cartesian product of universes (crisp sets) and F be a function from X to a universe Y defined by $y = F(x_1, x_2, x_3, \dots, x_n)$ and suppose $\bar{\mathcal{A}}_i \subseteq X_i, i = 1, 2, 3, \dots, n$, are fuzzy sets with membership functions $\mu_{\bar{\mathcal{A}}_i}(x_i), x_i \in X_i, i = 1, 2, 3, \dots, n$, respectively. Then the grade of $y \in Y, \mu_{\bar{\mathcal{B}}}(y)$ of the fuzzy set $\bar{\mathcal{B}} \subseteq Y$ such that $\bar{\mathcal{B}} = F(\bar{\mathcal{A}}_1, \bar{\mathcal{A}}_2, \bar{\mathcal{A}}_3, \dots, \bar{\mathcal{A}}_n)$ is given by

$$\mu_{\bar{\mathcal{B}}}(y) = \begin{cases} \sup_{y=F(x_1, x_2, x_3, \dots, x_n)} \min\{\mu_{\bar{\mathcal{A}}_i}(x_i) : i = 1, 2, 3, \dots, n\}, & \text{if } \exists y = F(x_1, x_2, x_3, \dots, x_n) \\ 0, & \text{otherwise} \end{cases}$$

Triangular Fuzzy Number: A fuzzy number $\bar{\mathcal{A}}$ is said to be a triangular fuzzy number if its membership function $\mu_{\bar{\mathcal{A}}}(x)$ fulfils following requirements:

$$\mu_{\bar{\mathcal{A}}}(x) = \begin{cases} \frac{x-p}{q-p} & \text{for } p \leq x \leq q \\ \frac{r-x}{r-q} & \text{for } q \leq x \leq r \\ 0 & \text{otherwise} \end{cases}$$

This triangular fuzzy number is often denoted by the triad $\bar{\mathcal{A}} = (p, q, r)$. The α -cuts of such a triangular fuzzy number are given by $\bar{\mathcal{A}}_\alpha = [p + (q - p)\alpha, r - (r - q)\alpha], \alpha \in [0, 1]$.

Fuzzy Function: [w'Omatete *et al.*²⁷ and the references therein] Suppose $\mathfrak{F}(\mathcal{R})$ be the fuzzy numbers set. A function $\bar{\mathcal{G}}: \mathcal{R} \rightarrow \mathfrak{F}(\mathcal{R})$ is called a fuzzy-valued function (or fuzzy function). Alpha-cuts representation of $\bar{\mathcal{G}}$ is defined by $\bar{\mathcal{G}}_\alpha(x) = [\bar{\mathcal{G}}_\alpha^l(x), \bar{\mathcal{G}}_\alpha^u(x)] = [\bar{\mathcal{G}}^l(x, \alpha), \bar{\mathcal{G}}^u(x, \alpha)], \alpha \in [0, 1]$. The support of $\bar{\mathcal{G}}$ is given by $\text{supp}(\bar{\mathcal{G}}(x)) =]\bar{\mathcal{G}}^l(x, 0), \bar{\mathcal{G}}^u(x, 0)[$ and the kernel (or core)

of $\bar{g}$ is given by $core(\bar{g}(x)) = [\bar{g}^l(x, 1), \bar{g}^u(x, 1)]$. The membership function $\mu_{\bar{g}(t)}$ of the fuzzy function $\bar{g}(t)$ is given by

$$\mu_{\bar{g}(t)}(t, x_t) = \begin{cases} \frac{x_t - \bar{g}^l(x, 0)}{\bar{g}^l(x, 1) - \bar{g}^l(x, 0)}, & \text{if } \bar{g}^l(x, 0) \leq x_t \leq \bar{g}^l(x, 1) \\ \frac{\bar{g}^u(x, 0) - x_t}{\bar{g}^u(x, 0) - \bar{g}^u(x, 1)}, & \text{if } \bar{g}^u(x, 1) \leq x_t \leq \bar{g}^u(x, 0) \\ 0, & \text{otherwise} \end{cases}$$

where x_t is real function of time 't'.

4. Interval Arithmetic and α -Cuts

Interval arithmetic is concerned with fundamental arithmetic operations (addition, subtraction, multiplication and division) on closed intervals. These arithmetic operations on closed intervals are subject matter of *interval analysis*- an accepted field of classical mathematics.

Let * denote any one of the four fundamental arithmetic operations- addition (+), multiplication (.), subtraction (-), division (/). Then [Hanss³⁰],

$$[a, b] * [e, f] = \{g * h | a \leq g \leq b, e \leq h \leq f\} = [\min(G), \max(G)],$$

where G is given by

$$G = \{a * e, a * g, b * e, b * f\} \text{ (for division } 0 \notin [e, f] \text{)}.$$

Arithmetic on α -cuts is performed with the help of above interval arithmetic on closed intervals as α -cuts are themselves closed intervals.

5. Methodology of Lenient α -Cuts Method and the Solution of the Problem

For methodology we refer the reader to w'Omatete *et al.*²⁷ together with references therein. We solve the problem using triangular fuzzy number and their alpha-cuts. The relevant parameters of the problem given in section 2 in can be given by following triangular fuzzy numbers:

$$\bar{\lambda} = (1, 2, 3), \bar{\eta} = (11, 12, 13), \bar{\theta} = (2, 3, 4)$$

The α -cuts of above fuzzy numbers are

$$\bar{\lambda}_\alpha = [1 + \alpha, 3 - \alpha], \bar{\eta}_\alpha = [11 + \alpha, 13 - \alpha], \bar{\theta}_\alpha = [2 + \alpha, 4 - \alpha]$$

respectively.

Fuzzy expression for average number of persons within the campus at any time t is given by (bar over a letter denotes fuzziness)

$$\bar{\mathcal{M}}(t) = \frac{\bar{\lambda}}{\bar{\eta} + \bar{\theta}} [1 - e^{-(\bar{\eta} + \bar{\theta})t}]$$

Putting the values in above expression, we get α -cuts of $\bar{\mathcal{M}}(t)$

$$[\bar{\mathcal{M}}(t)]_{\alpha} = \frac{[1 + \alpha, 3 - \alpha]}{[11 + \alpha, 13 - \alpha] + [2 + \alpha, 4 - \alpha]} [1 - e^{-([11 + \alpha, 13 - \alpha] + [2 + \alpha, 4 - \alpha])t}]$$

Simplifying above expression using α -cuts arithmetic (interval arithmetic), we finally get

$$[\bar{\mathcal{M}}(t)]_{\alpha} = \left[\frac{1 + \alpha - (3 - \alpha)e^{x_t}}{17 - 2\alpha}, \frac{3 - \alpha - (1 + \alpha)e^{x_t}}{13 + 2\alpha} \right]$$

where $x_t = -([13 + 2\alpha, 17 - 2\alpha])t, t \geq 0, \alpha \in [0, 1]$.

At $\alpha = 0$ the interval $x_t = -[13, 17]t$ goes to center-valued $-15t$ so writing $x_t = -15t$, we get

$$[\bar{\mathcal{M}}(t)]_{\alpha=0} = \text{supp}(\bar{\mathcal{M}}(t)) = \left[\frac{1 - 3e^{-15t}}{17}, \frac{3 - e^{-15t}}{13} \right], t \geq 0$$

and in similar way, we get

$$[\bar{\mathcal{M}}(t)]_{\alpha=1} = \text{core}(\bar{\mathcal{M}}(t)) = \frac{2 - 2e^{-15t}}{15}, t \geq 0$$

Therefore, the average number of persons within the campus is a fuzzy function of time t whose membership function is given by

$$\mu_{\bar{\mathcal{M}}(t)}(t, x_t) = \begin{cases} \frac{x_t - \left(\frac{1-3e^{-15t}}{17}\right)}{\left(\frac{2-2e^{-15t}}{15}\right) - \left(\frac{1-3e^{-15t}}{17}\right)}, & \text{if } \frac{1-3e^{-15t}}{17} \leq x_t \leq \frac{2-2e^{-15t}}{15} \\ \frac{\left(\frac{3-e^{-15t}}{13}\right) - x_t}{\left(\frac{3-e^{-15t}}{13}\right) - \left(\frac{2-2e^{-15t}}{15}\right)}, & \text{if } \frac{2-2e^{-15t}}{15} \leq x_t \leq \frac{3-e^{-15t}}{13} \\ 0, & \text{otherwise} \end{cases}$$

6. L-R Method and the Solution of the Problem

A fuzzy number $\bar{\mathcal{A}}$ is said to be a L-R fuzzy number iff its membership function $\mu_{\bar{\mathcal{A}}}(x)$ can be written in terms of two parametrized reference functions L and R in the format [Hanss³⁰, Zimmerman³³] -

$$\mu_{\bar{\mathcal{A}}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & \text{if } x \leq m, \alpha > 0, \\ R\left(\frac{x-m}{\beta}\right) & \text{if } x \geq m, \beta > 0, \end{cases}$$

where m is called modal or mean value, α is called left spread and β is called right spread. Symbolically, $\bar{\mathcal{A}}$ is written as $(m, \alpha, \beta)_{LR}$. An ordinary fuzzy number $\bar{\mathcal{A}} = (p, q, r)$ can always be written in the form of L-R fuzzy number

as follows if we take $m = p, \alpha = q - r, \beta = r - q$. Support of $\bar{\mathcal{A}} = (m, \alpha, \beta)_{LR}$ is given by $\text{supp}(\bar{\mathcal{A}}) =]m - \alpha, m + \beta[$.

Arithmetic of L-R Fuzzy Numbers: Let $\bar{\mathcal{A}} = (m, \alpha, \beta)_{LR}$ and $\bar{\mathcal{B}} = (n, \gamma, \delta)_{LR}$ are two L-R fuzzy numbers then their addition, subtraction, multiplication and division, respectively, are given as follows:

$$\bar{\mathcal{A}} \oplus \bar{\mathcal{B}} = (m + n, \alpha + \beta, \gamma + \delta)_{LR}$$

$$\bar{\mathcal{A}} \ominus \bar{\mathcal{B}} = (m - n, \alpha + \beta, \gamma + \delta)_{LR}$$

$$\bar{\mathcal{A}} \odot \bar{\mathcal{B}} \approx (mn, m\gamma + n\alpha - \alpha\gamma, m\delta + n\beta + \beta\delta)_{LR}$$

$$\frac{\bar{\mathcal{A}}}{\bar{\mathcal{B}}} \approx \left(\frac{m}{n}, \frac{m\delta}{n(n + \delta)} + \frac{\alpha}{n} - \frac{\alpha\delta}{n(n + \delta)}, \frac{m\gamma}{n(n - \gamma)} + \frac{\beta}{n} + \frac{\beta\gamma}{n(n - \gamma)} \right)_{LR}$$

Kernel (core) and the support of $\frac{\bar{\mathcal{A}}}{\bar{\mathcal{B}}}$ is given, respectively, by

$$\text{ker} \left(\frac{\bar{\mathcal{A}}}{\bar{\mathcal{B}}} \right) = \frac{m}{n}$$

and

$$\text{supp} \left(\frac{\bar{\mathcal{A}}}{\bar{\mathcal{B}}} \right) =]\frac{m - \alpha}{n + \delta}, \frac{m + \beta}{n - \gamma}[$$

For details of above arithmetic and L-R method as well as procedure we refer the reader to Mabela *et al.*²⁸ and the references therein. The relevant parameters of the problem given in section 2 in can be given by following L-R fuzzy numbers:

$$\bar{\lambda} = (2, 1, 1)_{LR}, \bar{\eta} = (12, 1, 1)_{LR}, \bar{\theta} = (3, 1, 1)_{LR}$$

L-R fuzzy expression for average number of persons within the campus at any time t is given by (bar over a letter denotes fuzziness)

$$\bar{\mathcal{M}}_{LR}(t) = \frac{\bar{\lambda}_{LR}}{\bar{\eta}_{LR} + \bar{\theta}_{LR}} [1 - e^{-(\bar{\eta}_{LR} + \bar{\theta}_{LR})t}]$$

Putting the values in above expression and simplifying, we get

$$\bar{\mathcal{M}}_{LR}(t) \approx \left(\frac{2 - 2e^{-15t}}{15}, \frac{19 + 11e^{-15t}}{255}, \frac{19 + 11e^{-15t}}{195} \right)_{LR}$$

$$\text{ker}(\bar{\mathcal{M}}_{LR}(t)) = \frac{2 - 2e^{-15t}}{15}$$

$$\text{supp}(\bar{\mathcal{M}}_{LR}(t)) =]\frac{15 - 45e^{-15t}}{255}, \frac{45 - 15e^{-15t}}{195}[=]\frac{1 - 3e^{-15t}}{17}, \frac{3 - e^{-15t}}{13}[$$

Therefore, in this case also, the average number of persons within the campus is a fuzzy function of time 't' with identical core and support to the α -cuts case. Hence the membership function will also be identical in both the cases.

7. Results and Discussion

In the α -cuts method support of time-dependent average number of customers $\mathcal{M}(t)$ in the campus shows that it cannot go below the time-boundary defined by the curve $\frac{1-3e^{-15t}}{17}$ and cannot go above the time-boundary defined by the curve $\frac{3-e^{-15t}}{13}$. Also, most probably this average lies on the time variable curve $\frac{2-2e^{-15t}}{15}$. In the long run, i.e., when time goes to infinite, this average lies in the open interval $\left[\frac{1}{17}, \frac{3}{13}\right]$ with most probable value $\frac{2}{15}$. In the L-R case, these same boundaries are defined, respectively, by the curves $\frac{15-45e^{-15t}}{225} = \frac{1-3e^{-15t}}{17}$, $\frac{45-15e^{-15t}}{195} = \frac{3-e^{-15t}}{13}$ and $\frac{2-2e^{-15t}}{15}$ which are identical to the α -cuts case. Therefore, both the methods give identical fuzzy results with respect time variation and in the long too.

8. Conclusion

In this paper we have discussed a queueing model in transient fuzzy regime. The model is discussed through a real-time university campus situation providing free Wi-Fi to its entrants. Our tools have been two types of fuzzy numbers following two types of fuzzy arithmetic. Both methods yield same results. L-R method is easier to apply in comparison to α -cuts method.

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