

Numerical Study of Some Aspects of MHD Flow Past a Vertical Porous Plate

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Abstract: The problem of MHD free convection flow past a vertical porous plate in presence of constant suction/injection has been studied. The governing equations based on boundary layer approximations have been solved using method of finite differences as proposed by Crank Nicolson. The Numerical values for velocity and temperature under various physical situations are obtained and have been presented and analyzed graphically.

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1. Introduction

In recent years the problems of magnetohydrodynamics have attracted the attention of many researchers owing to its enormous applications in geophysical and astrophysical situations. The requirements of technology based modern developments have stimulated studies of such fluid flow problems, which involve an interaction of several phenomena. One such study is related to the effects of free convection flow through porous medium, which play an important role in agriculture, engineering and petroleum industries etc.

Stokes¹ initiated one of the earliest studies in this direction. He studied the flow of an incompressible viscous fluid past an impulsively started porous plate. Subsequently many researchers under different physical conditions have studied this problem.

Lighthill² considered the two-dimensional oscillatory flow of an incompressible viscous flow and he solved the problem by momentum method. Stuart³ extended this idea to study a two dimensional flow past an infinite porous plate when the free stream oscillates in time about a constant mean. Effects of convection process, forced or free, have been subject matter of the works done by many in varying physical situations. Young⁴ and Sparrow & Gregg⁵ have considered the laminar free convection on vertical plates and cylinders and obtained similarity solutions for problems. Sparrow & Gregg⁶ later considered the buoyancy effects in forced convection flow. Thermal stratification effects over a wedge

with suction or injection have been discussed by Anjali and Kandasamy⁷ quite recently. Combined free and forced convection flow about inclined surfaces in porous media has been studied by Ping Cheng⁸ in which he obtained the numerical solution and discussed the role of local Grashof and Reynolds numbers on the boundary layer flow. These problems have been studied for non-Newtonian fluids also, such as heat transfer in the flow of the viscoelastic fluid over a stretching sheet by Lawrence and Rao⁹. Gebhart¹⁰ studied viscous dissipative heat aspects in natural convection. Ede¹¹ considered the laminar natural convection boundary flows. Soundalgekar¹² considered the free convection effects due to heating or cooling of the plate on the Stokes problem for an infinite vertical porous plate. The problems of boundary layer have been extensively discussed by Schlichting¹³ & Herbert Oertel¹⁴. In recent years the problem of impulsively started permeable plate with suction/injection in viscous incompressible fluids has been discussed widely owing to its applications in technological fields.

In the present paper we are presenting a numerical study of some aspect of free convection MHD flow past a vertical porous plate. In most of the earlier works, the flow region has been taken to be porous where as, here, we have taken the plate to be porous for the allowance of the suction/injection and the flow region non porous. The governing equations in non-dimensional form were solved by an implicit finite difference method of CrankNicolson¹⁵ type. The Numerical procedure has been discussed by Smith¹⁶ and Carnahan et.al.¹⁷. The numerical values for velocity and temperature under various physical situations are obtained and have been analyzed graphically.

2. Symbols

B_0	=	Intensity of magnetic field
G	=	Grashof number $\left(\frac{\beta g v (T_p - T_\infty)}{V_0^3} \right)$
g	=	Acceleration due to gravity
M	=	Magnetic field parameter $\left(\frac{\sigma \beta^2 v}{\rho V_0^2} \right)$
P	=	Prandtl number $\left(\frac{v}{\alpha} \right)$
r	=	Suction/injection parameter $\frac{v_1}{V_0}$
T_1	=	Temperature of the fluid
T_p	=	Temperature of the plate
T_∞	=	Temperature of the fluid at infinity
t	=	Non –dimensional time $\left(\frac{t_1 V_0^2}{v} \right)$

t_1	=	Time
(u_1, v_1)	=	Velocity of the fluid near the plate
U	=	Non dimensional velocity $\left(\frac{u_1}{V_0}\right)$
V_0	=	Velocity of the free stream
v_0	=	Suction velocity
X_1, Y_1	=	Co-ordinate axes parallel & normal to plate respectively
y	=	Non dimensional distance normal to the plate $\left(\frac{y_1 V_0}{v}\right)$
ν	=	Kinematics viscosity
σ	=	Electrical conductivity of the fluid
ρ	=	Density of the fluid
T	=	Non-dimensional temperature of the fluid $\left(\frac{T_1 - T_\infty}{T_p - T_\infty}\right)$
β	=	Coefficient of volume expansion
α	=	Thermal diffusivity

3. Mathematical Analysis

Here, an unsteady two-dimensional free convection flow of an electrically conducting viscous and incompressible fluid past an infinite, porous vertical plate with suction / injection is assumed. A magnetic field $\overline{B_0}$ is assumed to be applied perpendicular to the plate. A system of rectangular coordinate axes $OX_1Y_1Z_1$ is taken such that $Y_1 = 0$ is on the plate and X_1 is along its leading edge. All the fluid properties are considered constant except the influence of the density variation with temperature in the body force term. As the bounding surface is of infinite length, all the physical variables are functions of y_1 and t_1 only. Hence, by the usual boundary layer and Boussinesq approximations and following Sattar¹⁸, the problem is governed by the following system of equations:

$$(3.1) \quad \frac{\partial v_1}{\partial y_1} = 0$$

$$(3.2) \quad \frac{\partial u_1}{\partial t_1} + v_1 \frac{\partial u_1}{\partial y_1} = g\beta(T_1 - T_\infty) + \nu \frac{\partial^2 u_1}{\partial y_1^2} - \sigma \frac{\overline{B_0^2} u_1}{\rho}$$

$$(3.3) \quad \frac{\partial T_1}{\partial t_1} + v_1 \frac{\partial T_1}{\partial y_1} = \alpha \frac{\partial^2 T_1}{\partial y_1^2}$$

The initial and boundary conditions of the problems are

$$(3.4) \quad t_1 \leq 0: \quad u_1(y_1, t_1) = 0, \quad T_1(y_1, t_1) = T_\infty$$

$$(3.5) \quad t_1 > 0: \quad u_1 = V_0, \quad T_1 = T_p \text{ at } y_1 = 0$$

$$(3.6) \quad t_1 > 0: \quad u_1 = 0, \quad T_1 = T_\infty \text{ as } y_1 \rightarrow \infty$$

From equation (3.1) we observed that v_1 is independent of space co-ordinates and may be taken as constant $v_1 = v_0$ so that $v_0 > 0$ corresponds to suction, which can be interpreted as back flow into the plate. We define the following non-dimensional variables and parameters.

$$(3.7) \quad \begin{cases} t = \frac{t_1 V_0^2}{\nu}, & y = \frac{V_0 y_1}{\nu}, & U = \frac{u_1}{V_0}, & T = \frac{(T_1 - T_\infty)}{(T_p - T_\infty)} \\ r = \frac{v_1}{V_0}, & P = \frac{\nu}{\alpha}, & M = \frac{\sigma B_0^2}{\rho V_0^2} \nu, & G = \frac{\nu g \beta (T_p - T_\infty)}{V_0^3} \end{cases}$$

Now taking into account equations (3.7), equations (3.2) and (3.3) reduce to the following non-dimensional form

$$(3.8) \quad \frac{\partial u}{\partial t} - r \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + GT + MU$$

$$(3.9) \quad P \frac{\partial T}{\partial t} - Pr \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2}$$

and the initial and boundary conditions take the form

$$(3.10) \quad t \leq 0: \quad U(y, t) = 0, \quad T(y, t) = 0$$

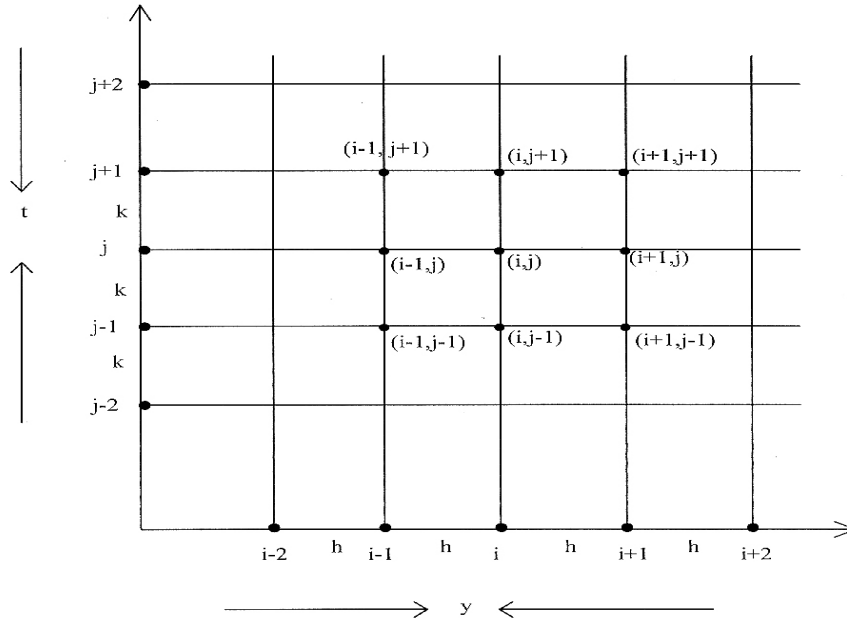
$$(3.11) \quad t > 0: \quad U(0, t) = 1, \quad T(0, t) = 1$$

$$(3.12) \quad t > 0: \quad U(\infty, t) = 0, \quad T(\infty, t) = 0$$

where the parameter r represents suction or injection depending on whether it is positive or negative. The Grashof number $G > 0$ represents external cooling of the plate and $G < 0$ denotes external heating of the plate.

4. Method of Solution

An implicit finite difference method has been used to solve the equations (3.8) and (3.9) subject to (3.10) to (3.12). To obtain the difference equations, the region of the flow is divided into a grid or mesh of lines parallel to y - and t - axes. Solutions of difference equations are obtained at the intersection of these mesh lines, called nodes, using the initial and boundary conditions.



In the above figure h and k are mesh sizes along y and t directions respectively. We find single values at next time level in terms of known values at an earlier time level. A forward difference approximation for the first order partial derivatives of T and U with respect to t and y and a central difference approximation for the second order partial derivative of U and T with respect to y are used.

$$\begin{aligned} \left(\frac{\partial T}{\partial y}\right)_{i,j} &= \frac{T_{i+1,j} - T_{i,j}}{h}; & \left(\frac{\partial U}{\partial y}\right)_{i,j} &= \frac{U_{i+1,j} - U_{i,j}}{h}; \\ \left(\frac{\partial T}{\partial t}\right)_{i,j} &= \frac{T_{i,j+1} - T_{i,j}}{k}; & \left(\frac{\partial U}{\partial t}\right)_{i,j} &= \frac{U_{i,j+1} - U_{i,j}}{k}; \\ \left(\frac{\partial^2 T}{\partial y^2}\right)_{i,j} &= \frac{T_{i+1,j} + T_{i-1,j} - 2T_{i,j}}{h^2}; & \left(\frac{\partial^2 U}{\partial y^2}\right)_{i,j} &= \frac{U_{i+1,j} + U_{i-1,j} - 2U_{i,j}}{h^2} \end{aligned}$$

In terms of these finite differences, the equations (3.8) and (3.9) reduce to

$$\begin{aligned} (4.1) \quad & 2 \left[Ph(T_{i,j+1} - T_{i,j}) - Prk(T_{i+1,j} - T_{i,j}) \right] \\ & = h \left[T_{i+1,j} + T_{i-1,j} - 2T_{i,j} + T_{i+1,j+1} + T_{i-1,j+1} - 2T_{i,j+1} \right] \end{aligned}$$

$$\begin{aligned} (4.2) \quad & 2 \left[h(U_{i,j+1} - U_{i,j}) - rk(U_{i+1,j} - U_{i,j}) \right] = h[U_{i+1,j} + U_{i-1,j} - 2U_{i,j} + U_{i+1,j+1} \\ & + U_{i-1,j+1} - 2U_{i,j+1} + GT_{i,j} + MU_{i,j}] \end{aligned}$$

Computations of U and T at a time level $(k+1)$ using the values at previous time level (k) are carried out. The finite difference equations (4.1) at every internal nodal point on a particular i th level constitute tridiagonal system of equations. This system of equations could be solved either by Gauss elimination method or matrix method. In this paper such system of equations from equation (4.1) were solved by using matrix method, thus the values of T are known at every nodal point at a particular i at $(k+1)$ th time level. By using the value of T obtained from equation (4.2). We get the values of U in similar manner. After computing values corresponding to each i at a time level, the values at the next time level are determine

5. Results and discussion

Fig. 1 and Fig. 2 show the variations in temperature of the fluid with suction and injection respectively. From Fig.1 it is clear that the temperature of the fluid decreases with the increase in suction velocity where as from Fig.2 we observe that temperature of the fluid increases with an increase in the injection velocity.

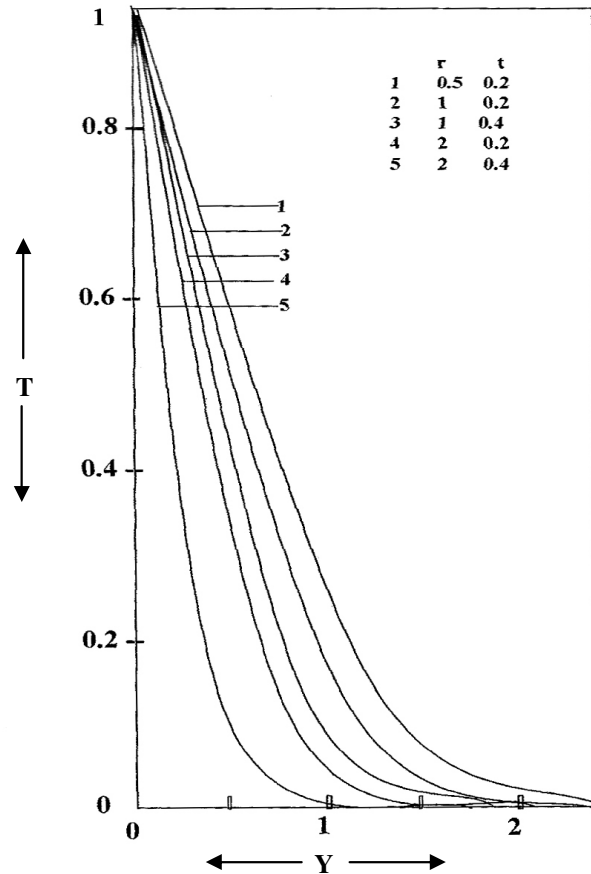


Fig. 1

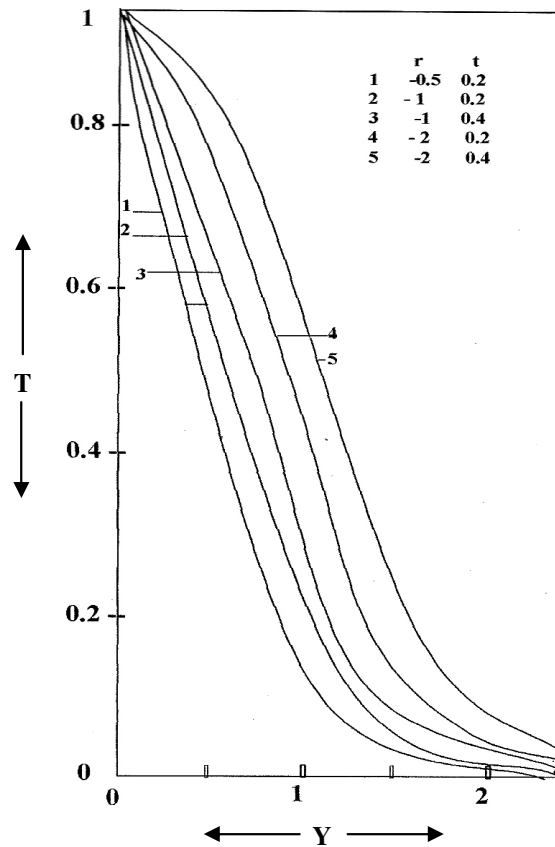


Fig. 2

Fig. 3 and Fig. 4 show the velocity profile. From Fig. 3 and Fig. 4 it is clear that an increase in the Grashof number G has a positive effect on the velocity. In other words an increase in the plate temperature follows a corresponding increase in the velocity of the flow. It is again observed that an increase in suction velocity causes to decrease the fluid velocity while the increase in the injection velocity results in increasing the fluid velocity. From both the figures it is also clear that the magnetic field parameter has an adverse effect on the velocity profile of the fluid in both the cases. The presence of the free convection currents, which results in greater heat addition by viscous dissipation, causes the velocity to rise.

6. Conclusion

From the above discussions it is clear that suction has adverse effect both on the velocity and as well as the temperature of the flow region under consideration and the Grashof number that is the plate temperature has positive effect on the velocity profile.

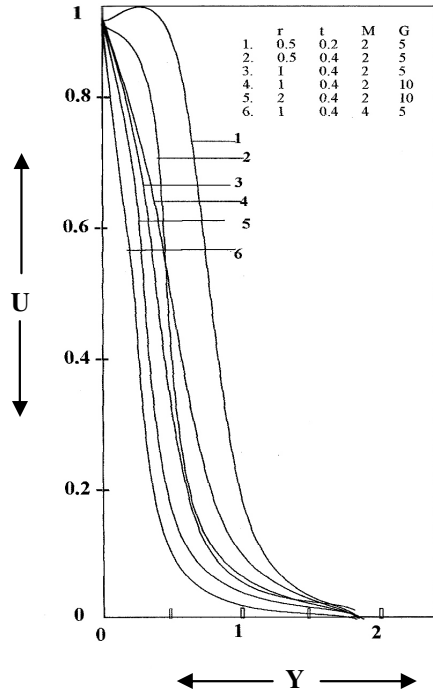


Fig. 3

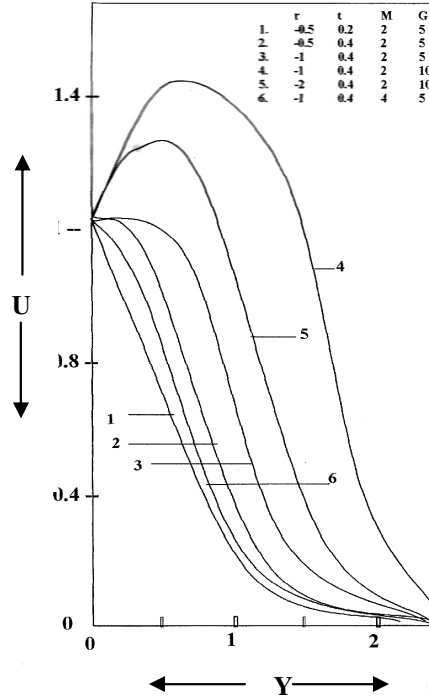


Fig. 4

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