

On HGF- Structure Manifolds

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Abstract: In this paper, recurrence and symmetry of different kinds in HGF-manifold have been defined. Some theorems establishing relationship between different kinds of recurrent HGF-manifolds have been stated and proved. Furthermore, theorems on different kinds of recurrent and symmetric HGF-manifolds involving equivalent conditions with respect to curvature tensors have also been discussed.

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1. Introduction

We consider a differentiable manifold M_n of differentiability class C^∞ . Let there be in M_n a vector-valued linear function F of class C^∞ satisfying the equation

$$(1.1) \quad \bar{\bar{X}} = -a^2 X$$

for arbitrary vector field X , where $\bar{X} = FX$ and a is a complex number. Then $\{F\}$ is said to give to M_n a hyperbolic differentiable structure, briefly known as HGF-structure, defined by the equation (1.1) and then M_n is called HGF-manifold¹.

The equation (1.1) gives different structures for different values of a . If $a \neq 0$, it is hyperbolic π -structure. If $a = \pm 1$, it is an almost complex or an almost hyperbolic product structure. If $a = \pm i$, it is an almost product or an almost hyperbolic complex structure and if $a = 0$, it is an almost tangent or hyperbolic almost tangent structure.

Let the HGF-structure be endowed with a metric tensor g , such that

$$(1.2) \quad g(\bar{X}, \bar{Y}) - a^2 g(X, Y) = 0.$$

Then $\{F, g\}$ is said to give to M_n a Hyperbolic metric or hyperbolic Hermite structure.

Agreement 1.1: In what follows and above, the equations containing $X, Y, Z, \dots$ etc. hold for these arbitrary vectors in M_n .

The curvature tensor K with respect to the connection D is given by²

$$(1.3a) \quad K(X, Y, Z) = D_X D_Y Z - D_Y D_X Z - D_{[X, Y]} Z,$$

where

$$(1.3b) \quad [X, Y] = D_X Y - D_Y X - S(X, Y).$$

The Ricci tensor in M_n is given by

$$(1.4) \quad Ric(Y, Z) = (C_1^1 K)(Y, Z).$$

By $(C_1^1 K)(Y, Z)$, we mean the contraction of $K(X, Y, Z)$ with respect to first slot.

For Ricci tensor, we also have

$$(1.5a) \quad Ric(Y, Z) = Ric(Z, Y),$$

$$(1.5b) \quad Ric(Y, Z) = g(r(Y), Z) = g(Y, r(Z)),$$

$$(1.5c) \quad (C_1^1 r) = R$$

Let W, C, L and V be the Projective, Conformal, Conharmonic and Concircular curvature tensors respectively given by²

$$(1.6) \quad W(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-1)} [Ric(Y, Z)X - Ric(X, Z)Y],$$

$$(1.7) \quad C(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-2)} [Ric(Y, Z)X - Ric(X, Z)Y - g(X, Z)r(Y) + g(Y, Z)r(X)] + \frac{R}{(n-1)(n-2)} [g(Y, Z)X - g(X, Z)Y],$$

$$(1.8) \quad L(X, Y, Z) = K(X, Y, Z) - \frac{1}{(n-2)} [Ric(Y, Z)X - Ric(X, Z)Y - g(X, Z)r(Y) + g(Y, Z)r(X)].$$

$$(1.9) \quad V(X, Y, Z) = K(X, Y, Z) - \frac{R}{n(n-1)} [g(Y, Z)X - g(X, Z)Y].$$

A manifold is said to be recurrent² if

$$(1.10) \quad (\nabla K)(X, Y, Z, T) = A(T) K(X, Y, Z),$$

where $A(T)$ is a non-vanishing C^∞ - function.

The recurrent manifold is said to be symmetric² if

$$(1.11) \quad A(T) = 0$$

in the equation (1.10).

The manifold is said to be Ricci-recurrent if

$$(1.12) \quad (\nabla Ric)(Y, Z, T) = A(T) Ric(Y, Z).$$

The Ricci-recurrent manifold is said to be symmetric if

$$(1.13) \quad A(T) = 0,$$

in the equation (1.12).

2. Recurrence and Symmetry of Different Kinds

Let the vector-valued trilinear function Q be any one of the curvature tensors K , W , C , L or V . Then we will define recurrence of different kinds as follows:

Definition 2.1: An HGF- manifold is said to be (1)- recurrent in Q if

$$(2.1) \quad a^2(\nabla Q)(X, Y, Z, T) - Q((\nabla F)(\bar{X}, T), Y, Z) = a^2 A(T) Q(X, Y, Z),$$

where $A(T)$ is a non-vanishing C^∞ function.

Definition 2.2: An HGF- manifold is said to be (12)- recurrent in Q if

$$(2.2a) \quad a^2(\nabla Q)(X, \bar{Y}, Z, T) - Q((\nabla F)(\bar{X}, T), \bar{Y}, Z) + a^2 Q(X, (\nabla F)(Y, T), Z) \\ = a^2 A(T) Q(X, \bar{Y}, Z),$$

or equivalently

$$(2.2b) \quad a^2(\nabla Q)(\bar{X}, Y, Z, T) + a^2 Q((\nabla F)(X, T), Y, Z) - Q(\bar{X}, (\nabla F)(\bar{Y}, T), Z) \\ = a^2 A(T) Q(\bar{X}, Y, Z).$$

Definition 2.3: An HGF- manifold is said to be (123)- recurrent in Q if

$$(2.3a) \quad a^2(\nabla Q)(\bar{X}, Y, \bar{Z}, T) - Q((\nabla F)(\bar{X}, T), \bar{Y}, \bar{Z}) + a^2 Q(X, (\nabla F)(Y, T), \bar{Z}) \\ + a^2 Q(X, \bar{Y}, (\nabla F)(Z, T)) = a^2 A(T) Q(\bar{X}, Y, \bar{Z}),$$

or equivalently

$$(2.3b) \quad a^2(\nabla Q)(\bar{X}, Y, \bar{Z}, T) + a^2 Q((\nabla F)(X, T), Y, \bar{Z}) - Q(\bar{X}, (\nabla F)(\bar{Y}, T), \bar{Z}) \\ + a^2 Q(\bar{X}, Y, (\nabla F)(Z, T)) = a^2 A(T) Q(\bar{X}, Y, \bar{Z}),$$

or equivalently

$$(2.3c) \quad a^2(\nabla Q)(\bar{X}, \bar{Y}, Z, T) + a^2 ((\nabla F)(X, T), \bar{Y}, Z) + a^2 Q(\bar{X}, (\nabla F)(Y, T), Z) \\ - Q(\bar{X}, \bar{Y}, (\nabla F)(\bar{Z}, T)) = a^2 A(T) Q(\bar{X}, \bar{Y}, Z).$$

Note 2.1: Similarly (2), (3), (23) and (13) recurrence in Q can also be defined.

Definition 2.4: A recurrent HGF- manifold is said to be symmetric if

$$(2.4) \quad A(T) = 0.$$

Theorem 2.1: A Q -(123)-recurrent HGF-manifold is Q -(12)- recurrent for the same recurrence parameter provided

$$(2.5) \quad a^2 Q(X, \bar{Y}, (\nabla F)(Z, T)) = 0.$$

Proof: Barring Z in equation (2.2a), we get

$$(2.6) \quad \begin{aligned} a^2(\nabla Q)(X, \bar{Y}, \bar{Z}, T) - Q((\nabla F)(\bar{X}, T), \bar{Y}, \bar{Z}) + a^2Q(X, (\nabla F)(Y, T), \bar{Z}) \\ = a^2A(T)Q(X, \bar{Y}, \bar{Z}). \end{aligned}$$

Using equation (2.5) in equation (2.3a), we get the equation (2.6). This shows that the manifold is Q -(12) recurrent.

Note 2.2: Similar theorems can be stated and proved by taking Q -(13), Q -(23), Q -(1), Q -(2) and Q -(3) recurrent HGF-manifold in place of Q -(12) recurrent HGF-manifold.

Theorem 2.2: In a Q -(1) recurrent HGF-manifold, if any two of the following conditions hold for the same recurrence parameter, then the third also holds:

- (i) it is conformal (1)-recurrent,
- (ii) it is conharmonic (1)-recurrent
- (iii) it is concircular (1)-recurrent

Proof: From the equations (1.7), (1.8) and (1.9), we have

$$(2.7) \quad C(X, Y, Z) = L(X, Y, Z) + \frac{n}{(n-2)}[K(X, Y, Z) - V(X, Y, Z)]$$

Barring X twice in equation (2.7), using equation (1.1) and multiplying the resulting equation by $A(T)$, we get

$$(2.8) \quad a^2A(T)C(X, Y, Z) = a^2A(T)L(X, Y, Z) + a^2A(T)\frac{n}{n-2}[K(X, Y, Z) - V(X, Y, Z)].$$

Barring X twice in equation (2.7) and differentiating the resulting equation with respect to T , we get

$$(2.9) \quad \begin{aligned} a^2(\nabla C)(X, Y, Z, T) - C((\nabla F)(\bar{X}, T), Y, Z) &= a^2(\nabla L)(X, Y, Z, T) \\ &- L((\nabla F)(\bar{X}, T), Y, Z) + \frac{n}{(n-2)}[a^2(\nabla K)(X, Y, Z, T) - K((\nabla F)(\bar{X}, T), Y, Z) \\ &- a^2(\nabla V)(X, Y, Z, T) + V((\nabla F)(\bar{X}, T), Y, Z)]. \end{aligned}$$

Subtracting equation (2.8) from equation (2.9) and then using the fact that the (1)-recurrent HGF-manifold is conformal (1)-recurrent and conharmonic (1)-recurrent for the same recurrence parameter, we have

$$a^2(\nabla V)(X, Y, Z, T) - V((\nabla F)(\bar{X}, T), Y, Z) = a^2A(T)V(X, Y, Z),$$

that is the manifold is concircular (1)-recurrent.

The proof of the remaining two cases follows similarly.

Theorem 2.3: In a Q -(1) symmetric HGF-manifold, if any two of the following conditions hold, then the third also hold:

- (i) it is conformal (1)-symmetric,
- (ii) it is conharmonic (1)-symmetric,
- (iii) it is concircular (1)-symmetric.

Proof: The statement follows from the theorem (2.2) and equation (1.11).

Note 2.3: Theorems similar to the theorems(2.2) and (2.3) can also be stated and proved for (2), (3), (12), (13), (23), (123) recurrent and symmetric HGF-manifolds.

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