

Propagation of Spherical Shock Waves in a Dusty Gas under Influence of Gravitation

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(Received July 07, 2008)

Abstract: The problem of propagation of explosion waves in a gas containing small solid particles has been discussed by using the method developed by Whitham¹, including the effects due to gravitational force. The analytical solutions of the shock strength, shock velocity and flow variables for weak and strong shocks have been obtained and particular cases are discussed.

1. Introduction

Recently the study of the flow field behind a propagating blast wave in a mixture of gas and solid particles has attracted a large number of investigators because of its numerous applications in underground explosions, nozzle flow, lunar ash flow, bomb blast and many other science and engineering problems. By using the gas and solid particles mixture model described by Pai², several workers have studied the effects of the presence of solid particles on the flow field behind the propagating blast wave. Narasimhulu et al³ obtained an approximate analytical solution for a self similar flow behind a strong shock wave propagating outward in a particle laden gas with variable total energy release by employing integral method of Lambach and Probst⁴. Suzuki et al⁵, Pai et al⁶ found similarity solutions for plane or line source explosions in a dusty gas. Higashino and Suzuki⁷, extended the analysis by expanding the self-similar flow variables to a series of inverse squares of shock Mach numbers to a first order of approximation. Jan and Sharma⁸ have obtained necessary conditions for the existence of similarity solutions for shocks of arbitrary strength as well as for strong shocks by using the Lie group method described in Bluman and Cole⁹, Bluman and Kumei¹⁰ Logan and Perez¹¹ and Radha and Sharma¹². A similarity solution has been obtained by Gretlar and Regenfelder¹³⁻¹⁴ for a laser-driven strong shock wave propagating in a mixture of gas and small solid particles. Miura and Glass¹⁵, Vishwakarma¹⁶ and Steiner and Hirschler¹⁷ have obtained analytic solutions for the one dimensional self-similar flow field between the strong shock wave and moving piston behind it by considering different physical conditions in a gas containing solid particles.

Ojha & Srivastava¹⁸ have studied the point explosion problem by using the method of energy hypothesis of Thomas¹⁹ in a gas containing solid particles and obtained the expressions for the velocity, pressure and radius of the blast wave. In present problem our aim is to discuss the explosion problem in a mixture of gas and small solid particles under the influence of gravitational force by using the method developed by Whitham¹, we have

assumed that the viscous stresses and heat conduction of the mixture of the gas and solid particles are negligible and that the equilibrium flow condition is maintained in the flow field. The analytical solution of the shock strength has been obtained and particular cases are discussed.

2. Basic Equations

The fundamental equations for the one dimensional spherical unsteady flow in a mixture of gas and small solid particles under the influence of gravitational force can be written as (Gretler & Regenfelder^{13,14}).

$$(2.1) \quad \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0,$$

$$(2.2) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{g_o r_o}{r^2},$$

$$(2.3) \quad \frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0,$$

where u is the velocity of the mixture, p the pressure of the mixture, ρ the density of the mixture and e the internal energy of the mixture per unit mass at distance r and time t . Further r_o and g_o be the reference distance and acceleration due to gravity at r_o .

The equation of state of the mixture under equilibrium condition is:

$$(2.4) \quad p = \frac{(1-k_p)}{(1-z)} \rho R T,$$

where k_p is the mass concentration of the solid particles in the mixture taken as a constant in the whole flow field, z the volume fraction of the solid particles, R the gas constant and T the temperature. The relation between k_p and z is given by Pai et al⁶,

$$(2.5) \quad k_p = \frac{z \rho_{sp}}{\rho},$$

where $z = \frac{\rho}{\rho_o} z_o$, while ρ_{sp} is the species density of the solid particles and subscript o refers to the initial values of z and ρ . z_o is completely determined by the dust parameters k_p and G , as follows:

$$z_o = \frac{k_p}{(1-k_p)G + k_p},$$

where G is the dust volumetric extension.

The internal energy of the mixture is related to the internal energy of the two species and may be written as:

$$(2.6) \quad e = C_{vm} T = \{k_p C_{sp} + (1 - k_p) C_v\} T,$$

where C_{sp} is the specific heat of the solid particles, C_v the specific heat of the gas at constant volume and C_{vm} is the specific heat of the mixture at constant volume. For the equilibrium condition, the specific heat of the mixture at constant pressure is:

$$(2.7) \quad C_{pm} = [k_p C_{sp} + (1 - k_p) C_p],$$

where C_p is the specific heat of the gas at constant pressure. The ratio of the specific heat of the mixture is:

$$(2.8) \quad \Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma + \delta\beta}{1 + \delta\beta},$$

$$\text{where } \gamma = \frac{C_p}{C_v}, \beta = \frac{C_{sp}}{C_v}, \delta = \frac{k_p}{1 - k_p}.$$

The variation of δ and Z_0 with respect to k_p is given in fig-1 and that of Γ with respect to δ is given in fig-2. Eliminating the temperature from (2.4), (2.6) and (2.8), the internal energy of the mixture may be written as:

$$(2.9) \quad e = \left(\frac{1 - z}{\Gamma - 1} \right) \frac{p}{\rho}.$$

For isentropic change of state of the gas-solid particles mixture and thermodynamic equilibrium condition, the equilibrium sound speed of the mixture is given by:

$$(2.10) \quad a^2 = \frac{\partial p}{\partial \rho} = \frac{\Gamma p}{(1 - Z)\rho}.$$

The well known Rankine-Hugoniot relations across the shock wave may be written as (Ojha & Srivastava¹⁸).

$$(2.11) \quad \frac{p}{p_o} = \frac{2 \Gamma M_o^2 - (\Gamma - 1)}{(\Gamma + 1)},$$

$$(2.12) \quad \frac{\rho}{\rho_o} = \frac{M_o^2 (\Gamma + 1)}{M_o^2 (\Gamma - 1 + 2 Z_o) + 2(1 - Z_o)},$$

$$(2.13) \quad \frac{u}{a_o} = \frac{2(1 - Z_o)}{(\Gamma + 1)} \left(M_o - \frac{1}{M_o} \right),$$

where $M_o = \frac{U}{a_o}$, $a_o = \frac{\Gamma p_o}{\rho_o (1 - Z_o)}$, U is the velocity of the shock wave.

At equilibrium conditions

$$(2.14) \quad \frac{g_o r_o}{r^2} = \frac{1}{\rho} \frac{\partial p}{\partial r},$$

which gives

$$(2.15) \quad p = \frac{g_o r_o \rho_o}{r} \quad \text{if } \rho_o = \text{constant}$$

plot of kp Vs delta and Z0

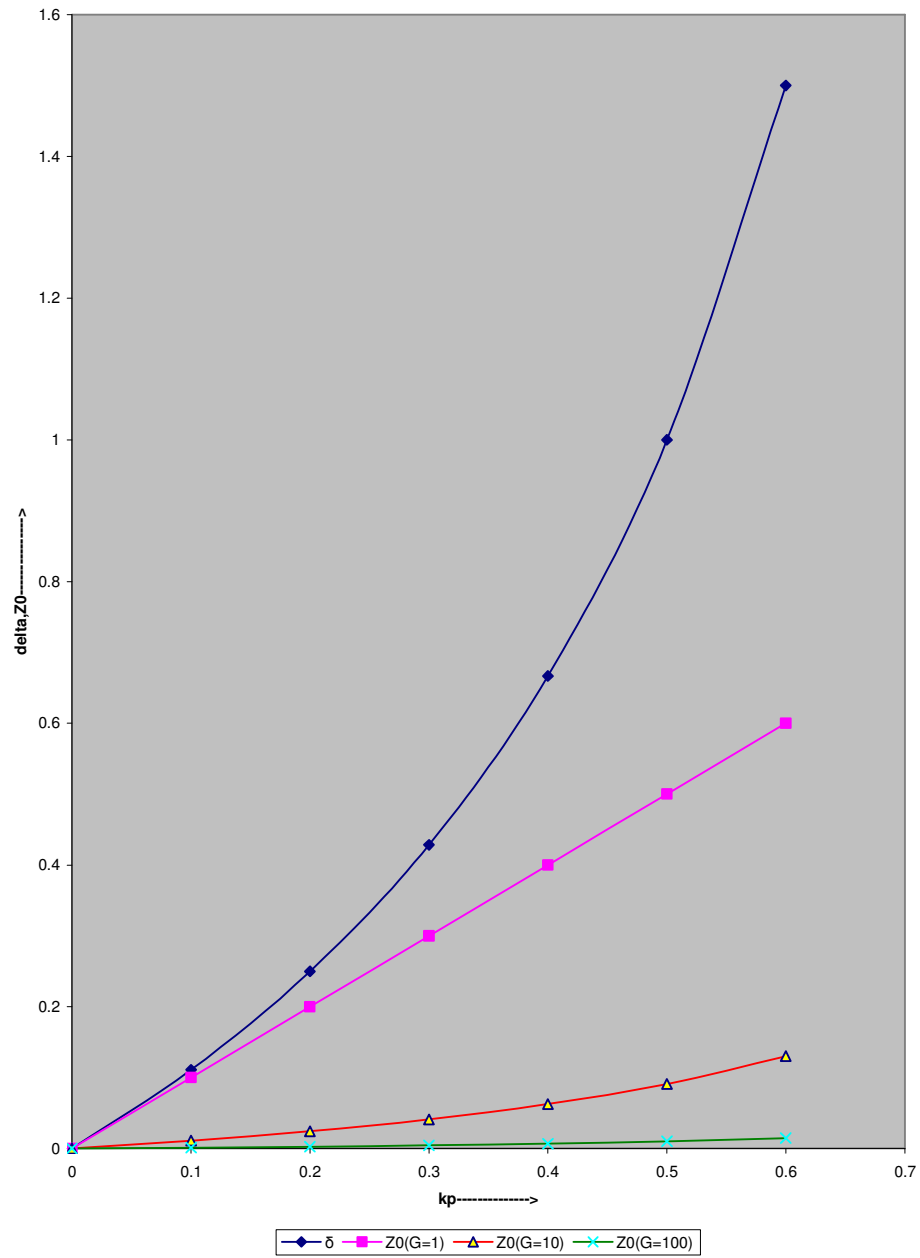


Fig-1

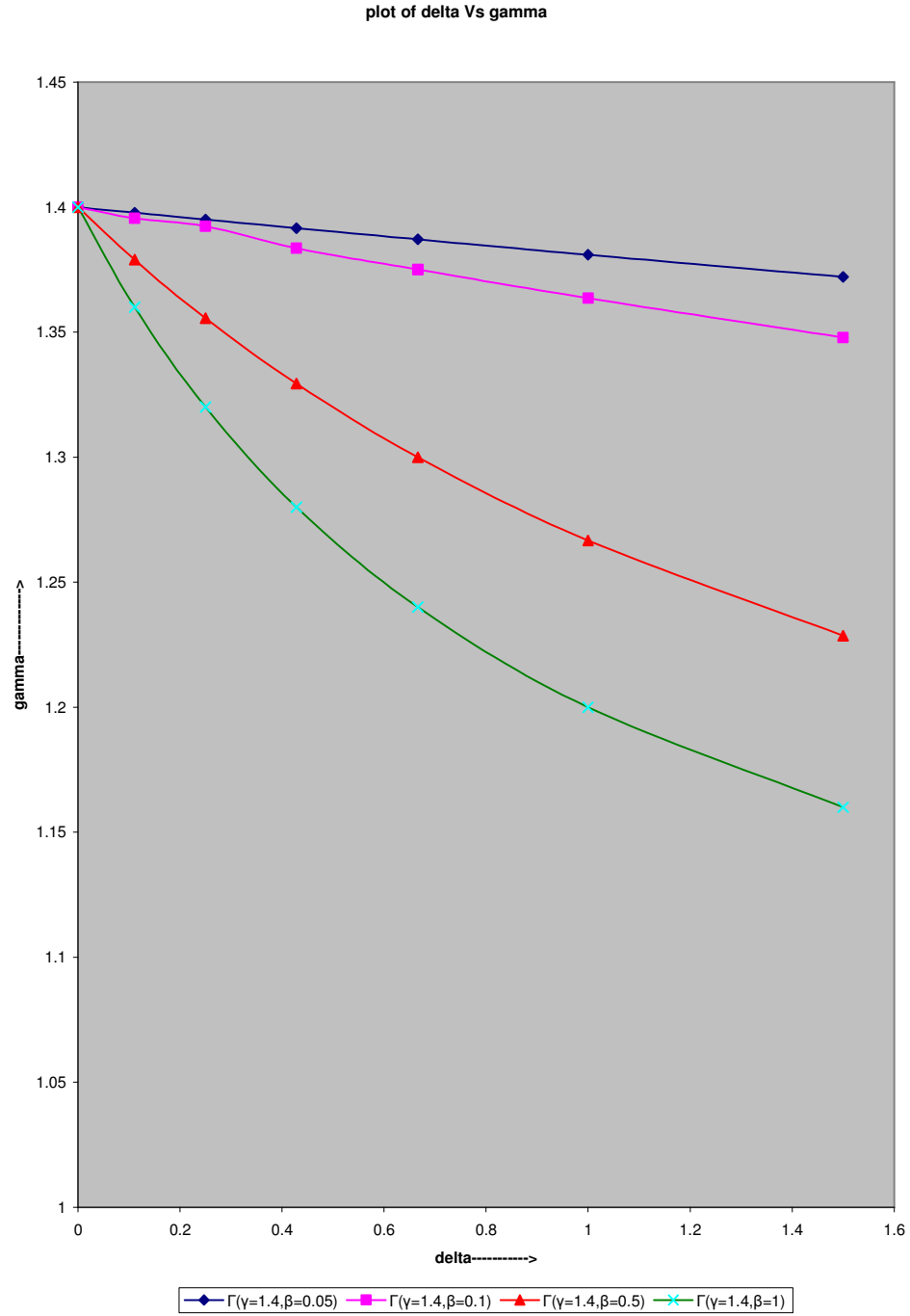


Fig.-2

3. Discussions of the Problem

For diverging shocks, the characteristic form of the system of equations may be written as (Whitham¹).

$$(3.1) \quad dp + \rho a du + \frac{2\rho a^2 u}{(u+a)} \frac{dr}{r} + \frac{\rho a g_o r_o}{(u+a)} \frac{dr}{r^2} = 0.$$

From (2.10) and shock conditions (2.11) to (2.13) we obtain.

$$(3.2) \quad a = \frac{a_o \left((M_o^2 (\Gamma - 1 + 2 Z_o) + 2 (1 - Z_o)) \sqrt{2\Gamma M_o^2 - (\Gamma - 1)} \right)}{M_o (\Gamma + 1) \sqrt{(\Gamma - 1) M_o^2 + 2}}.$$

Further,

$$(3.3) \quad d p = \frac{4\Gamma \rho_o M_o dM_o}{(\Gamma + 1)},$$

$$(3.4) \quad du = \frac{2a_o(1 - Z_o) (M_o^2 + 1)}{(\Gamma + 1)} \frac{dM_o}{M_o},$$

$$(3.5) \quad \rho \alpha = \frac{a_o \rho_o M_o \sqrt{2\Gamma M_o^2 - (\Gamma - 1)}}{\sqrt{(\Gamma - 1) M_o^2 + 2}},$$

$$(3.6) \quad \rho a du = \frac{2a_o^2 (1 - Z_o) \rho_o (M_o^2 + 1) \sqrt{2\Gamma M_o^2 - (\Gamma - 1)}}{(\Gamma + 1) \sqrt{(\Gamma - 1) M_o^2 + 2}} \cdot \frac{dM_o}{M_o},$$

(3.7)

$$\rho a^2 u = \frac{\rho_o \left(2\Gamma M_o^2 - (\Gamma - 1) \right) \left(M_o^2 (\Gamma - 1 + 2 Z_o) + 2 (1 - z_o) \right) a_o^3}{\left((\Gamma - 1) M_o^2 + 2 \right) (\Gamma + 1)} \times \frac{2(1 - Z_o) (M_o^2 - 1)}{M_o (\Gamma + 1)},$$

$$(3.8) \quad u + a = \frac{2(1 - Z_o) a_o (M_o^2 - 1)}{M_o (\Gamma + 1)} + \frac{a_o \left(M_o^2 (\Gamma - 1 + 2 Z_o) + 2 (1 - Z_o) \right) \sqrt{2\Gamma M_o^2 - (\Gamma - 1)}}{M_o (\Gamma + 1) \sqrt{(\Gamma - 1) M_o^2 + 2}},$$

Putting (3.2) – (3.8) in (3.1) and then using (2.15) we may write,

$$(3.9) \quad \left\{ 4\Gamma M_o^2 + 2 \frac{(M_o^2 + 1) M_o \sqrt{2\Gamma M_o^2 - (\Gamma - 1)}}{\sqrt{(\Gamma - 1) M_o^2 + 2}} \right\} \frac{dM_o}{M_o} + \left[\frac{4\Gamma (M_o^2 - 1) \left(2\Gamma M_o^2 - (\Gamma - 1) \right) \left(M_o^2 (\Gamma - 1 + 2 Z_o) + 2 (1 - z_o) \right)}{2 \left((\Gamma - 1) M_o^2 + 2 \right) (1 - Z_o) (M_o^2 - 1) + \left(M_o^2 (\Gamma - 1 + 2 Z_o) + 2 (1 - Z_o) \right) \sqrt{2\Gamma M_o^2 - (\Gamma - 1)} \left((\Gamma - 1) M_o^2 + 2 \right)} \right]$$

$$+ \frac{\frac{(\Gamma+1)^2 M_o^2}{\Gamma} \sqrt{2\Gamma M_o^2 - (\Gamma-1)}}{\sqrt{(\Gamma-1)M_o^2 + 2} \left[2(1-Z_o)(M_o^2-1) \right] + \left(M_o^2(\Gamma-1+2Z_o) + 2(1-Z_o) \right) \sqrt{2\Gamma M_o^2 - (\Gamma-1)}} \left] \frac{dr}{r} = 0$$

or

$$(3.10) \quad \frac{dM_o}{M_o} = -K(M_o) \frac{dr}{r}.$$

where

(3.11)

$$K(M_o) = - \frac{4\Gamma(M_o^2-1) \left(2\Gamma M_o^2 - (\Gamma-1) \right) \left(M_o^2(\Gamma-1+2Z_o) + 2(1-Z_o) \right)}{2(1-Z_o)(M_o^2-1) \sqrt{(\Gamma-1)M_o^2 + 2} + \left(M_o^2(\Gamma-1+2Z_o) + 2(1-Z_o) \right) \sqrt{2\Gamma M_o^2 - (\Gamma-1)}} \\ + \frac{\frac{M_o^2(\Gamma+1)^2 \sqrt{2\Gamma M_o^2 - (\Gamma-1)}}{\Gamma}}{4\Gamma M_o^2 \sqrt{(\Gamma-1)M_o^2 + 2} + 2M_o(M_o^2+1) \sqrt{2\Gamma M_o^2 - (\Gamma-1)}}.$$

The law of propagation of shock waves can be determined from the numerical integration of (3.10). For certain constant value of $K(M_o)$, integration of (3.10) gives,

$$(3.12) \quad M_o \propto r^{-K(M_o)}.$$

The variation of $K(M_o)$ with respect to M_o with gravity and without gravity are given in fig-3, 4, 5 and 6.

The variation of pressure, density and velocity can be determined, from (2.11), (2.12) and (2.13) behind the shock wave.

The problem may be discussed from the simplified form of the resulting equation for the limiting cases of very weak and very strong shock waves by taking $M_o = 1 + \alpha$.

For weak shocks, $\alpha \ll 1$. In this case,

$$(3.13) \quad p = p_o \left(1 + \frac{4\alpha\Gamma}{\Gamma+1} \right),$$

$$(3.14) \quad \rho = \rho_o \left(1 + \frac{4\alpha(1-Z_o)}{\Gamma+1} \right),$$

$$(3.15) \quad u = \frac{4a_o(1-Z_o)\alpha}{(\Gamma+1)},$$

and

$$(3.16) \quad a = a_o \left(1 + \frac{2\alpha(\Gamma-1+2Z_o)}{\Gamma+1} \right),$$

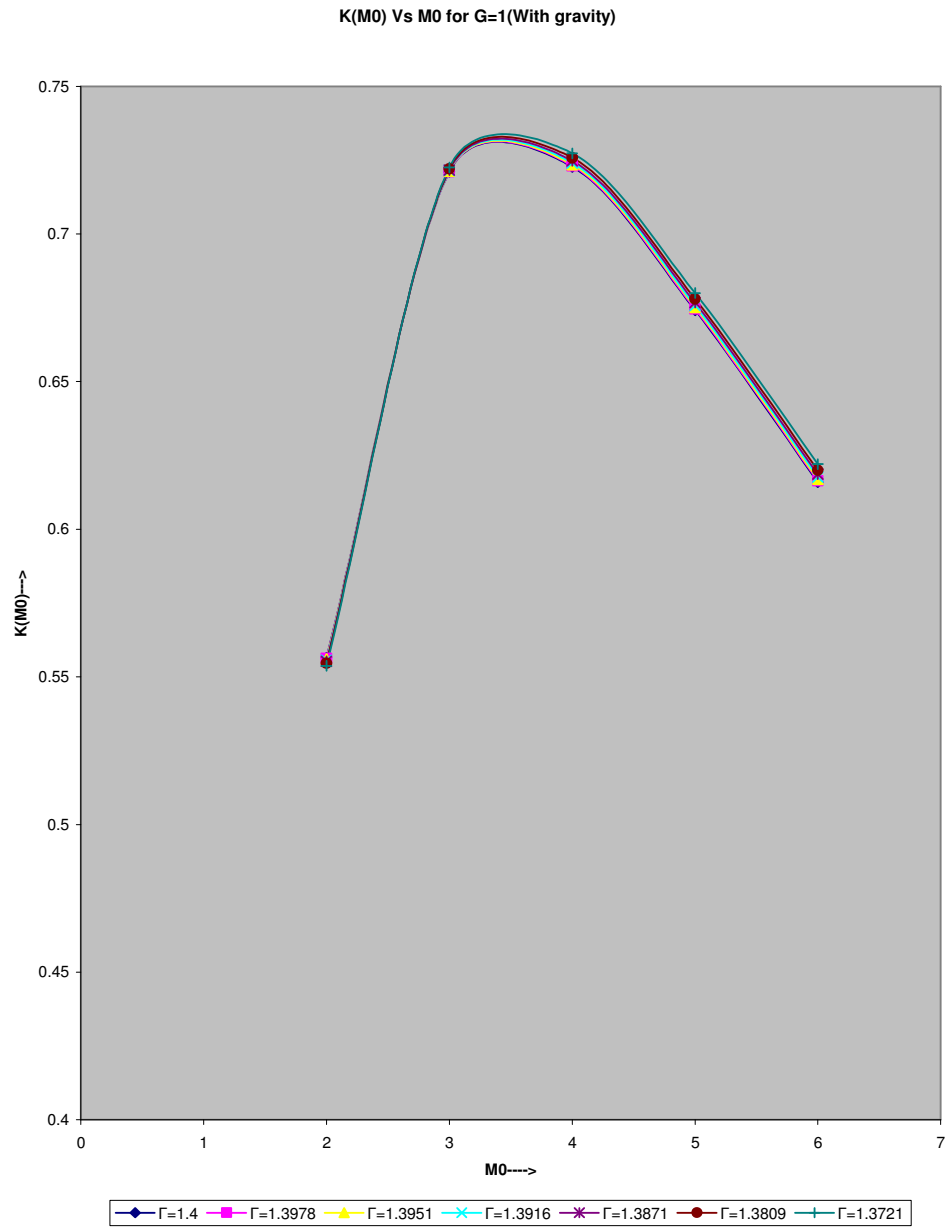


Figure - 3

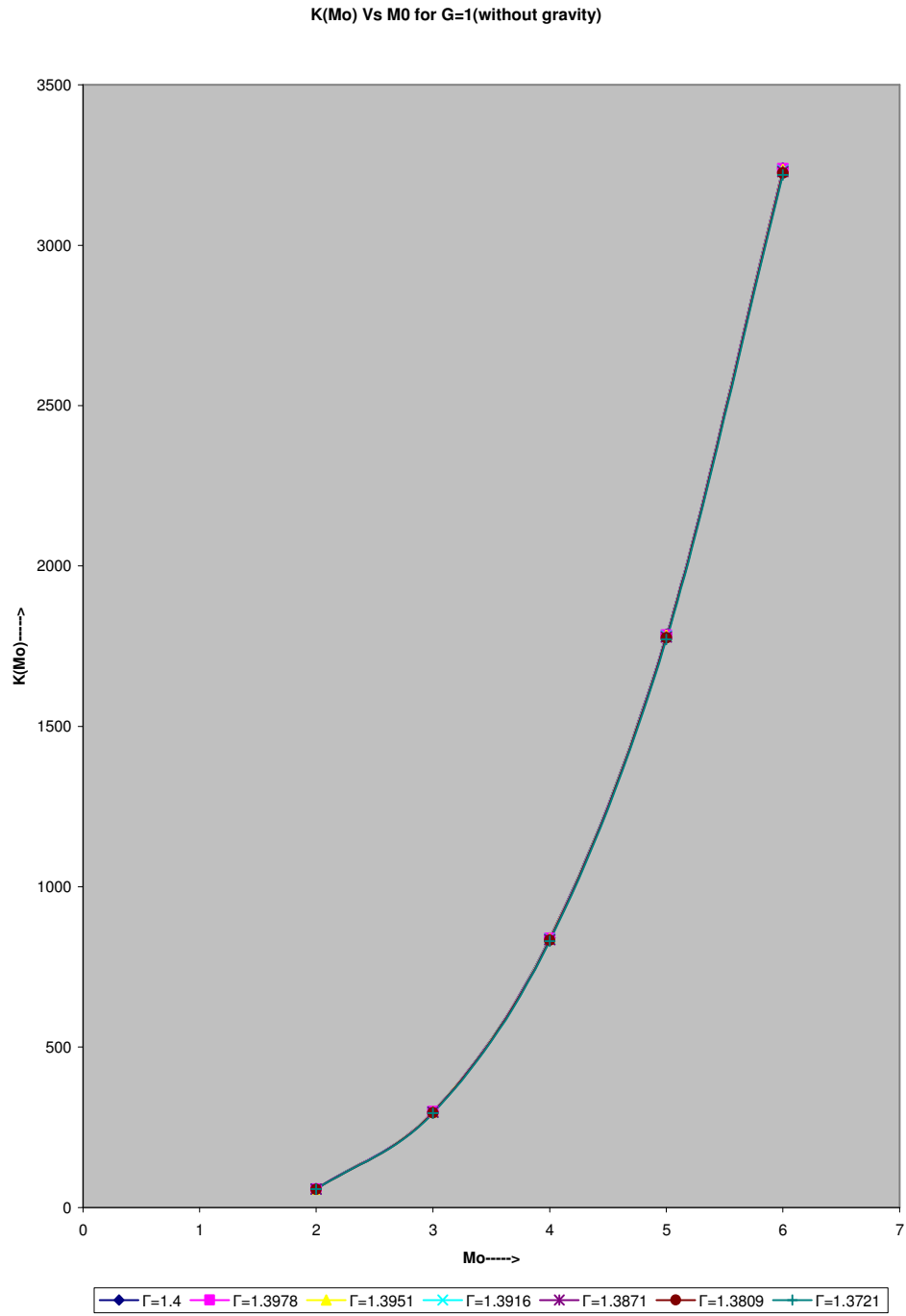


Figure - 4

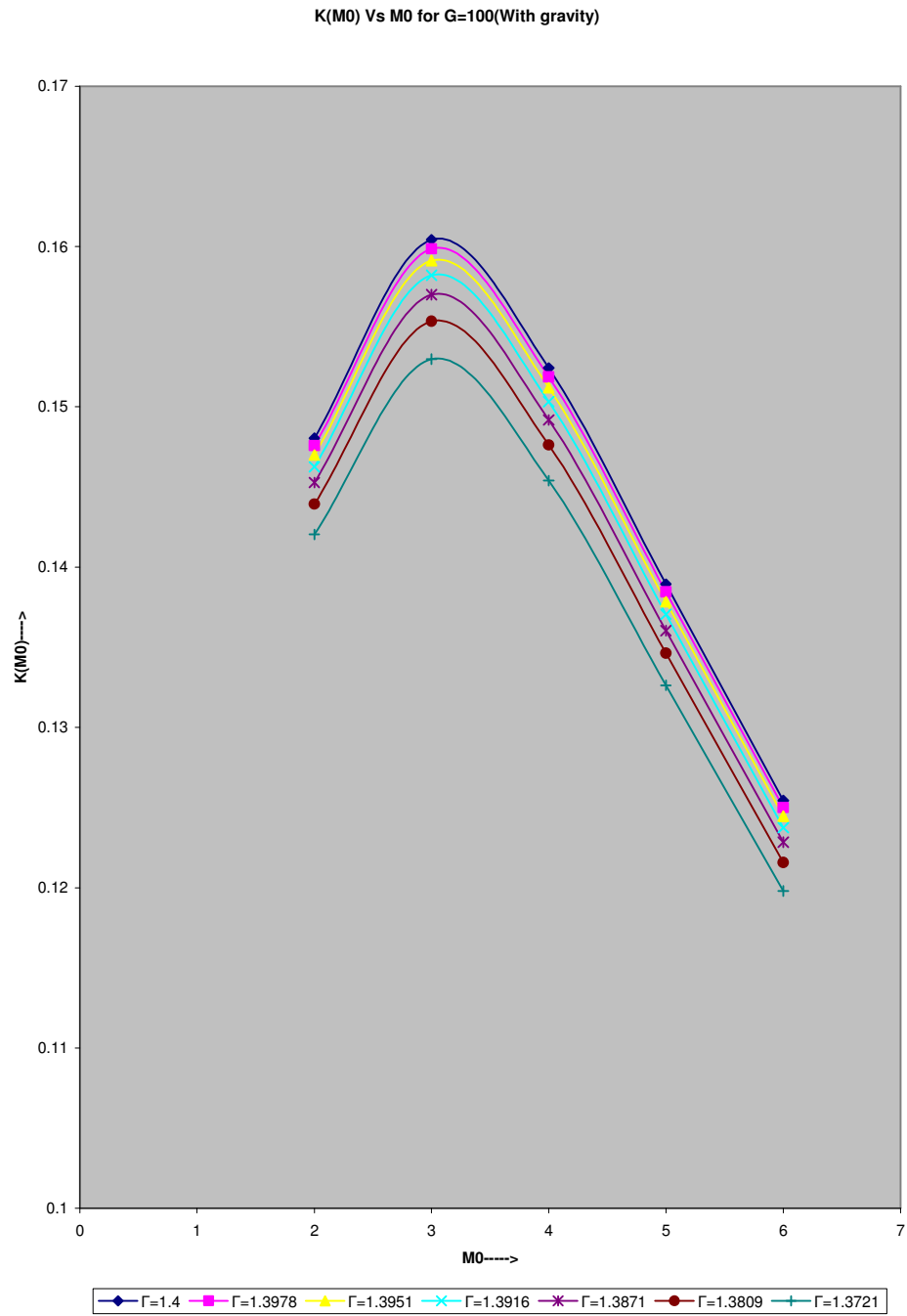


Figure - 5

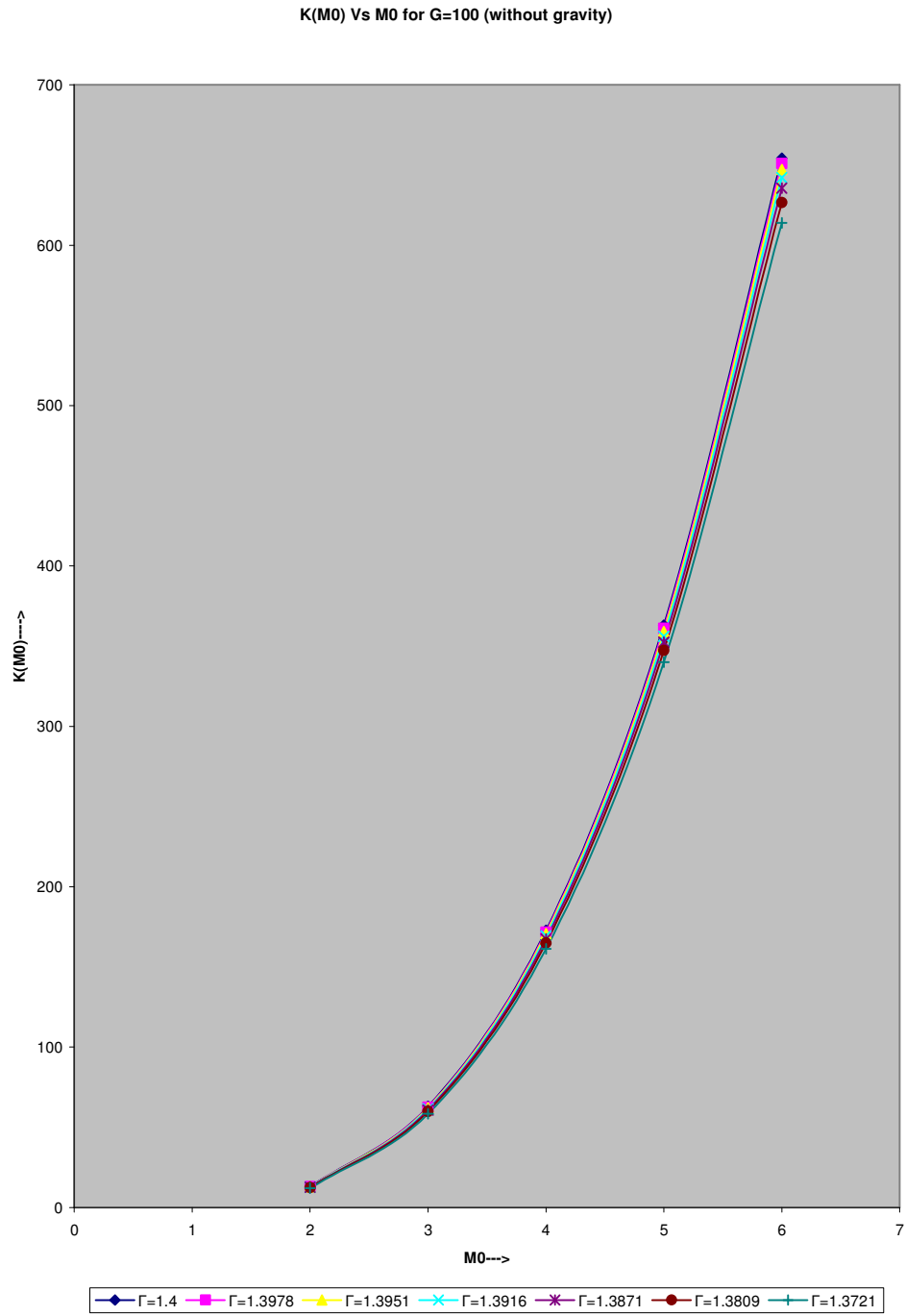


Figure - 6

$$(3.17) \quad dp = \frac{4\Gamma p_o}{(\Gamma+1)} d\alpha,$$

$$(3.18) \quad du = \frac{4a_o (1-Z_o) d\alpha}{(\Gamma+1)},$$

$$(3.19) \quad \rho a = \rho_o a_o (1+2\alpha),$$

$$(3.20) \quad \rho a^2 u = \frac{4\rho_o a_o^3 (1-Z_o)\alpha}{(\Gamma+1)}.$$

Putting these values in equation (3.1) we get.

$$(3.21) \quad d\alpha + \left(\alpha + \frac{(1-\alpha)(\Gamma+1)}{8\Gamma} \right) \frac{dr}{r} = 0,$$

or

$$\frac{d\alpha}{\left(\alpha + \frac{\Gamma+1}{7\Gamma-1} \right)} = - \left(\frac{7\Gamma-1}{8\Gamma} \right) \frac{dr}{r}.$$

Integrating (3.21), we have,

$$(3.22) \quad \alpha + \frac{\Gamma+1}{7\Gamma-1} = A r^{-\left(\frac{7\Gamma-1}{8\Gamma} \right)} \quad A = \text{Constant}.$$

In absence of gravitation,

$$(3.23) \quad \alpha = \frac{B}{r}, \quad B = \text{Constant}.$$

Variation in pressure, density and velocity can be determined from (3.13), (3.14) and (3.15).

For strong shock $U \gg a_o$. In this case

$$(3.24) \quad p = \frac{2}{\Gamma+1} \rho_o U^2 (1-Z_o),$$

$$(3.25) \quad \rho = \rho_o \left(\frac{\Gamma+1}{\Gamma-1+2Z_o} \right),$$

$$(3.26) \quad u = \frac{2U(1-Z_o)}{(\Gamma+1)},$$

and

$$(3.27) \quad a = \frac{U(\Gamma-1+2Z_o)}{(\Gamma+1)} \sqrt{\frac{2\Gamma}{\Gamma-1}},$$

$$(3.28) \quad \rho a = \rho_o U \sqrt{\frac{2\Gamma}{\Gamma-1}},$$

$$(3.29) \quad \rho a du = \frac{2\rho_o U (1-Z_o)}{(\Gamma+1)} \sqrt{\frac{2\Gamma}{\Gamma-1}} dU ,$$

$$(3.30) \quad \rho a^2 u = \frac{4\rho_o \Gamma U^3 (1-Z_o) (\Gamma-1+2Z_o)}{(\Gamma+1)^2 (\Gamma-1)} .$$

Putting (3.24) – (3.30) in (3.1) we get

$$(3.31) \quad \left(2 + \sqrt{\frac{2\Gamma}{\Gamma-1}} \right) 2UdU + \left\{ \frac{8U^2 \Gamma (\Gamma-1+2Z_o)}{(\Gamma-1) \left\{ 2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}} \right\}} \right. \\ \left. + \frac{2\sqrt{\frac{2\Gamma}{\Gamma-1}} \frac{(\Gamma+1)}{\Gamma} U^2}{2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}}} \right\} \frac{dr}{r} = 0 ,$$

$$(3.32) \quad \frac{dU^2}{U^2} = - \frac{\frac{8\Gamma(\Gamma-1+2Z_o)}{(\Gamma-1) \left\{ 2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}} \right\}} + \frac{2(\Gamma+1) \sqrt{\frac{2}{\Gamma(\Gamma-1)}}}{2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}}} \frac{dr}{r}}{\left(2 + \sqrt{\frac{2\Gamma}{\Gamma-1}} \right)} .$$

Integration of (3.32) will give law of variation of shock velocity as,

$$(3.33) \quad U = C r^{-\frac{d}{2}}, \quad C = \text{Constant},$$

where

$$(3.34) \quad d = \frac{\left\{ \frac{8\Gamma(\Gamma-1+2Z_o)}{(\Gamma-1) \left\{ 2 + \sqrt{\frac{2\Gamma}{\Gamma-1}} \right\} \left\{ 2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}} \right\}} + \frac{2(\Gamma+1) \sqrt{\frac{2}{\Gamma(\Gamma-1)}}}{2(1-Z_o) + (\Gamma-1+2Z_o) \sqrt{\frac{2\Gamma}{\Gamma-1}}} \right\}}{\left(2 + \sqrt{\frac{2\Gamma}{\Gamma-1}} \right)} .$$

Variation of p, ρ and u can be determined from (3.24), (3.25) and (3.26).

In absence of solid particles, d takes the form.

$$(3.35) \quad d = \frac{\left\{ \frac{8\gamma}{2 + \sqrt{2\gamma(\gamma-1)}} + \frac{2(\gamma+1)\sqrt{\frac{2}{\gamma(\gamma-1)}}}{2 + (\gamma-1)\sqrt{\frac{2\gamma}{\gamma-1}}} \right\}}{\left(2 + \sqrt{\frac{2\gamma}{\gamma-1}} \right)}.$$

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