

Two-Unit Cold Standby System with the Provision of Preventive Maintenance and Correlated Failures and Repairs

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(Received December 30, 2005)

Abstract: This paper deals with the analysis of a two identical units cold standby system. Assuming the concept of preventive maintenance (P.M.) i.e. when unit is not completely failed but needs a minor repair. The operation of only one unit is sufficient to do the job. When a unit fails totally the standby unit becomes operative instantaneously. Each unit of the system has two modes-Normal (N) and total failure (F). Initially, one unit is operative and the other is kept as cold standby. A single repairman is always available with the system to repair all types of fault. The discipline of repair of failed units is first come first serve (FCFS). However, the P.M. gets the priority over the repair of failed unit. When one of the units is under repair and the other needs the P.M., in such a situation, the repair of the failed unit is discontinued and the repairman starts the P.M. Each repaired unit works as good as new. Also, the failure & repair times of each unit are assumed to be correlated by taking their joint distribution as bivariate exponential with density function as follows:

$$f(x, y) = \alpha\beta(1-r) \exp[-\alpha x - \beta y] I_0(2\sqrt{\alpha\beta rxy}) \quad ; x, y, \alpha, \beta > 0, 0 \leq r \leq 1$$

where $I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$ is the modified Bessel function of type one and

order zero. The distribution of the P.M. and residual repair times of units are taken to be negative exponential with different parameters. Using regenerative point technique in the Marko-renewal process, the following economic measures of system effectiveness are obtained:

- i. The reliability of the system and MTSF.
- ii. Expected up-time of the system in (0, t] and in steady state.
- iii. Expected busy period of the repairman (0, t] and in steady state.
- iv. Net expected profit earned by the system in the interval (0, t] and in steady state.

The behavior of MTSF and cost-benefit analysis of the system have also been studied.

1. Introduction

Two unit redundant system models have been analysed widely in the literature of reliability by various authors including Gopalan and Naidu, Gupta et al. Murari and Goel, Goyal and Murari, Gopalan et al, Gupta and Goel. Gupta and Chaudhary, Mohmoud and Esmail, under the assumption that a failed unit once repaired becomes as good as new. Further, they have also assumed that all the variables representing life times/repair times are mutually independent and uncorrelated. Goel et al, Goel and Gupta, Goel and Shrivastava^{1,2}, Tyagi V. K.³, Gupta R.S.Z. Mumtaz⁴, Quamber, Jhang J.P. and Shele⁵, Guinter, Leoanidas & Kapranis⁶ incorporated the concept of correlation between life time and repair time of a unit and analysed the models pertaining to two unit standby system by taking the joint distribution of failure and repair times as bivariate exponential.

In the present study, we analyse a two identical unit stand by system model, assuming the concept of preventive maintenance (P.M.) i.e. when unit is not completely failed but needs a minor repair and also the concept of correlation is introduced between the failure times and repair times. Initially one unit is operative and the other is kept as cold standby because the operation of only one unit is sufficient to do the job. Each unit of the system has two modes-normal (N) and total failure (F). When a unit fails totally the standby unit becomes operative instantaneously with the help of perfect switching device. When the P.M. is given to any unit, it is kept off-line. Single repairman is always with the system and repair is done on FCFS basis. Priority is always given to P.M. over the repair of failed unit. P.M. always starts a fresh. After repair every unit works as good as new. Time to P.M. and P.M. to complete are taken to be negative exponential with parameters a and b . When the repair of a unit is re-started after interruption, his repair time are assumed to be uncorrelated random variable having the negative exponential distribution with parameter β_2 and β_3 .

The failure and repair times of each unit are jointly distributed having the bivariate exponential density function of the form:

$$f(x, y) = \alpha\beta(1-r) \exp[-\alpha x - \beta y] I_0(2\sqrt{\alpha\beta r x y}) ; x, y, \alpha, \beta > 0, 0 \leq r \leq 1,$$

where $I_0(z) = \sum_{k=0}^{\infty} \frac{(Z/2)^{2k}}{(k!)^2}$ is the modified Bessel function of type one and order zero.

2. Notation and States of The System

No	:	Unit in N-mode and operative.
Ns	:	Unit in N-mode and cold-standby.
Npm	:	Unit in N-mode and under P.M.
Nwpm	:	Unit in N-mode and waiting for P.M.
Fr	:	Unit in F- mode and under repair.
Fwr'	:	Unit in F-mode and waiting for restarted repair.
F _R	:	Unit in F-mode and under restarted repair and repair is

continued from the earlier state

a, b	:	constant rate of time to P.M. and P.M. to complete.
Y_2, Y_3	:	constant rate of residual repair time.
X_1, Y_1	:	Random variable denoting the failure and repair times.
$f(x, y)$	:	Joint p.d.f. of $f(x_1, y_1)$.
$g_1(x), G_1(x)$	:	Marginal p.d.f. and c.d.f. of random variable X_1 i.e. $g_1(x) = \alpha_1 (1-r) \exp [-\alpha_1 (1-r) x]$
$k_1(y), K_1(y)$	:	Marginal p.d.f. and c.d.f. of random variable Y_1 i.e. $k_1(y) = \beta_1 (1-r) \exp [-\beta_1 (1-r) y]$
$k_1(y x)$	:	Conditional p.d.f. of y_1 given $X_1 = x$ $= \beta_1 \exp [-\beta_1 y + \alpha_1 r x] I_0 \left[2\sqrt{\alpha_1 \beta_1 r x y} \right]$
$K_1(y x)$	:	Conditional c.d.f. of y_1 given $X_1 = x$.

Thus considering the above symbols under the assumption of the system model, the possible states of the system are as follows:

Up States	Down States	Failed States
$S_0 = (N_0, N_s)$	$S_3 = (N_{pm}, N_{wpm})$	$S_5 = (F_r, F_{wr})$
$S_1 = (N_{pm}, N_0)$	$S_4 = (N_{pm}, F_r)$	
$S_2 = (F_r, N_0)$	$S_6 = (F_{wr}, N_{pm})$	
$S_7 = (F_R, N_0)$		

3. Transition Probabilities and Sojourn Times

Evidently, $p_{31} = p_{42} = p_{52} = p_{67} = 1$. Using the conditional probabilities, one can write the following unconditional transition probabilities as:

$$p_{20} = \int p_{20|x} g_1(x) dx = \beta_1' (1-r) [1 - \beta_1' r]^{-1}; \beta_1' = \frac{\beta_1}{\alpha + \beta_1 + \alpha_c (1-r)},$$

$$p_{25} = \alpha_1 (1-r) [1 - \beta_1'] [a + \alpha_1 (1-r)] (1-r\beta_1')^{-1},$$

$$p_{26} = \alpha (1 - \beta_1') [a + \alpha_1 (1-r)] (1-r\beta_1')^{-1},$$

$$\text{s.t. } p_{20} + p_{25} + p_{26} = 1.$$

The other unconditional direct – transition probabilities are:

$$\begin{array}{ll} P_{01} = a [a + \alpha_1 (1-r)]^{-1} & P_{02} = \alpha_1 (1-r) [a + \alpha_1 (1-r)]^{-1} \\ P_{10} = b [a + b + \alpha_1 (1-r)]^{-1} & P_{13} = a [a + b + \alpha_1 (1-r)]^{-1} \\ P_{14} = \alpha_1 (1-r) [a + b + \alpha_1 (1-r)]^{-1} & P_{70} = \beta_3 [a + \beta_3 + \alpha_1 (1-r)]^{-1} \\ P_{75} = \alpha_1 (1-r) [a + \beta_3 + \alpha_1 (1-r)]^{-1} & P_{76} = a [a + \beta_3 + \alpha_1 (1-r)]^{-1} \end{array}$$

Hence the steady state transition probabilities through non regenerative states are:

$$p_{00}^{(1)} = p_{01}p_{10}, p_{00}^{(2)} = p_{02}p_{20}, p_{01}^{(1,3)} = p_{01}p_{13}, p_{02}^{(2,5)} = p_{02}p_{25}$$

$$p_{02}^{(1,4)} = p_{01}p_{14}, p_{07}^{(2,6)} = p_{02}p_{26}, p_{11}^{(3)} = p_{13}p_{13}, p_{12}^{(4)} = p_{14}, p_{22}^{(5)} = p_{25}$$

$$p_{27}^{(6)} = p_{26}, p_{72}^{(5)} = p_{75}, p_{77}^{(6)} = p_{76}$$

These transition probabilities are seen to satisfy the following relations:

$$p_{00}^{(1)} + p_{00}^{(2)} + p_{01}^{(1,3)} + p_{02}^{(2,5)} + p_{02}^{(1,4)} + p_{07}^{(2,6)} = 1$$

$$p_{10} + p_{11}^{(3)} + p_{12}^{(4)} = 1$$

$$p_{20} + p_{22}^{(5)} + p_{27}^{(6)} = 1$$

$$(3.1) \quad p_{70} + p_{72}^{(5)} + p_{77}^{(6)} = 1$$

Transition Diagram

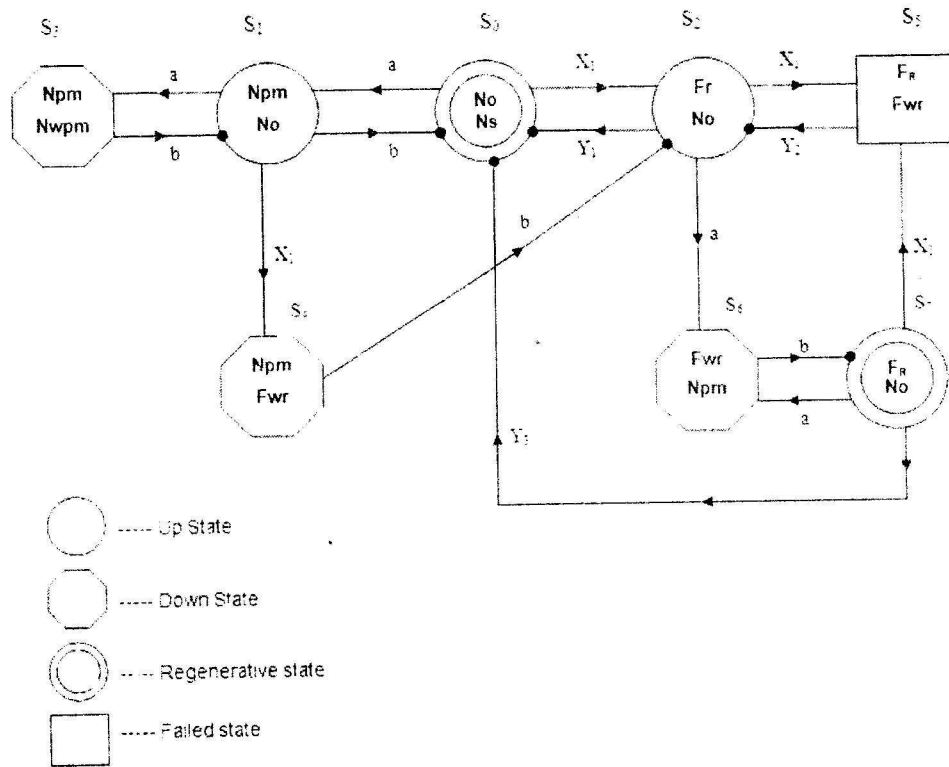


Fig. 1

The sojourn times in various states are:

$$\varphi_0 = [a + \alpha(1-r)]^{-1}, \quad \varphi_1 = [a + b + \alpha(1-r)]^{-1},$$

$$\varphi_2 = \int \varphi_{2,x} g_1(x) dx = [a + \alpha_1(1-r)]^{-1} [1 - \beta_1'(1-r)(1-r\beta_1')^{-1}],$$

where $\varphi_{2,x} = \int \bar{K}_1(u|x) e^{-at} G_1(t) dt$

$$(3.2) \quad \varphi_3 = \varphi_4 = b^{-1}, \varphi_5 = \beta_2^{-1}, \varphi_6 = b^{-1}, \varphi_7 = [a + \beta_3 + \alpha_1(1-r)]^{-1}$$

4. Analysis of Reliability and MTSF

Assuming the failed states and down states as absorbing, by renewal process arguments one has:

$$(4.1) \quad R_0(t) = M_0(t) + q_{00}^{(1)}(t) \odot R_0(t) + q_{00}^{(2)}(t) \odot R_0(t),$$

where $M_0(t) = V_0(t) + Q_{01}(t) \odot V_1(t) + Q_{02}(t) \odot V_2(t)$

and $V_0(t) = p$ [system started from state S_0 does not transits to any other state till time t]

Taking the Laplace transformation (L.T.) of relation (4.1) one can obtain:

$$(4.2) \quad R_0^*(s) = \frac{M_0^*(s)}{1 - q_{00}^{*(1)}(s) - q_{00}^{*(2)}(s)} = \frac{N_1(s)}{D_1(s)}$$

Taking the inverse Laplace transformation (I. L.T.) of (4.2), one can get the reliability of the system, when the system initially starts from S_0 .

To get the MTSF, we have

$$(4.3) \quad E(T_0) = \int_0^\infty R_0(t) dt = \lim_{s \rightarrow 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} = \frac{N_2}{D_2},$$

where $N_2 = \varphi_0 + p_{01} \varphi_1 + p_{02} \varphi_2$ and $D_2 = 1 - p_{01} p_{10} - p_{02} p_{20}$.

5. Availability Analysis

With the usual definitions of $Z_i(t)$ and $V_i(t)$, we have

$$Z_0(t) = V_0(t) + Q_{01}(t) \odot V_1(t) + Q_{02}(t) \odot V_2(t),$$

$$Z_1(t) = V_1(t), \quad Z_2(t) = V_2(t), \quad Z_7(t) = V_7(t),$$

with $V_0(t) = \exp[-\{a + \alpha_1(1-r)\}t]$; $V_1(t) = \exp[-\{a + b + \alpha_1(1-r)\}t]$,

$$V_2(t) = \int \exp[-\{a + \alpha_1(1-r)\}t] \bar{K}_1(y|x) g_1(x) dx,$$

$$V_7(t) = \exp[-\{a + \beta_3 + \alpha_1(1-r)\}t].$$

By simple probabilistic arguments we observe that

$$\begin{aligned} A_0(t) &= Z_0(t) + \{q_{00}^{(1)}(t) + q_{00}^{(2)}(t)\} \odot A_0(t) + q_{01}^{(1,3)}(t) \odot A_1(t) + \{q_{02}^{(2,5)}(t) + q_{02}^{(1,4)}(t)\} \odot \\ &\quad A_2(t) + q_{07}^{(2,6)}(t) \odot A_7(t), \\ A_1(t) &= Z_1(t) + q_{10}(t) \odot A_0(t) + q_{11}^{(3)}(t) \odot A_1(t) + q_{12}^{(4)}(t) \odot A_2(t), \\ A_2(t) &= Z_2(t) + q_{20}(t) \odot A_0(t) + q_{22}^{(5)}(t) \odot A_2(t) + q_{27}^{(6)}(t) \odot A_7(t), \\ (5.1) \quad A_7(t) &= Z_7(t) + q_{70}(t) \odot A_0(t) + q_{72}^{(5)}(t) \odot A_2(t) + q_{77}^{(6)}(t) \odot A_7(t). \end{aligned}$$

Taking the Laplace transformation of the above set of equations, one can obtain $A_0^*(s)$. Using this result, the steady state availability of the system is:

$$(5.2) \quad A_0 = A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = N_2 [D_2]^{-1}$$

where

$$\begin{aligned} (5.3) \quad N_2 &= \{(\varphi_0 + p_{01}\varphi_1 + p_{02}\varphi_2)(1 - p_{11}^{(3)}) + \varphi_1 p_{01}^{(1,3)}\} \{p_{20}p_{72}^{(5)} + p_{70}(1 - p_{22}^{(5)})\} \\ &\quad + \{\varphi_2(1 - p_{77}^{(6)}) + (\varphi_7 p_{27}^{(6)})\} \{(1 - p_{11}^{(3)})(1 - p_{00}^{(1)} - p_{00}^{(2)}) - p_{10}p_{01}^{(1,3)}\} \\ &\quad + p_{07}^{(2,6)}(1 - p_{11}^{(3)})(\varphi_7 p_{20} - \varphi_2 p_{70}), \end{aligned}$$

$$(5.4) \quad D_2 = \varphi_0 + p_{01}\varphi_1 + p_{02}\varphi_2 + p_{01}(\varphi_3 p_{13} + \varphi_4 p_{14}) + p_{02}(\varphi_5 p_{25} + \varphi_6 p_{26}).$$

The expected up-time of the system in $(0, t]$ is

$$\mu_{up}(t) = \int_0^t A_0(u) du,$$

so that

$$\mu_{up}^*(s) = s^{-1} A_0^*(s).$$

6. Busy Period Analysis

$$\begin{aligned} B_0(t) &= \{q_{00}^{(1)}(t) + q_{00}^{(2)}(t)\} \odot B_0(t) + q_{01}^{(1,3)}(t) \odot B_1(t) + q_{02}^{(1,4)}(t) + q_{02}^{(2,5)}(t) \odot \\ &\quad B_2(t) + q_{07}^{(2,6)}(t) \odot B_7(t) \\ B_1(t) &= W_1(t) + q_{10}(t) \odot B_0(t) + q_{11}^{(3)}(t) \odot B_1(t) + q_{12}^{(4)}(t) \odot B_2(t) \end{aligned}$$

$$B_2(t) = W_2(t) + q_{20}(t) \odot B_0(t) + q_{22}^{(5)}(t) \odot B_2(t) + q_{27}^{(6)}(t) \odot B_2(t)$$

$$(6.1) \quad B_7(t) = W_7(t) + q_{70}(t) \odot B_0(t) + q_{72}^{(5)}(t) \odot B_2(t) + q_{77}^{(6)}(t) \odot B_7(t)$$

with

$$W_1(t) = V_1(t) + q_{13}(t) \odot V_3(t) + q_{14}(t) \odot V_4(t),$$

$$W_2(t) = V_2(t) + q_{25}(t) \odot V_5(t) + q_{26}(t) \odot V_6(t),$$

$$W_7(t) = V_7(t) + q_{76}(t) \odot V_6(t) + q_{75}(t) \odot V_5(t).$$

Taking the L.T. of the above set of equations (6.1) one can obtain $B_0^*(s)$ and the steady state probability that the repairman is busy is given by

$$(6.2) \quad B_0 = \lim_{t \rightarrow 0} B_0(t) = \lim_{s \rightarrow 0} s B_0^*(s) = N_3(o) [D_2'(0)]^{-1} = N_3 [D_3]^{-1},$$

where

$$(6.3) \quad N_3 = (\phi_1 + p_{13}\phi_3 + p_{14}\phi_4) p_{01}^{(1,3)} [(1 - p_{77}^{(6)}) (1 - p_{22}^{(5)}) - p_{27}^{(6)} p_{72}^{(5)}]$$

$$+ (\phi_2 + p_{25}\phi_5 + p_{26}\phi_6) [(1 - p_{77}^{(6)}) \{ p_{01}^{(1,3)} p_{12}^{(4)} + (p_{02}^{(1,4)} + p_{02}^{(2,5)}) (1 - p_{11}^{(3)}) \}]$$

$$+ p_{07}^{(2,6)} p_{72}^{(5)} + (1 - p_{11}^{(3)})] + (\phi_7 + p_{76}\phi_6 + p_{75}\phi_5) [p_{01}^{(1,3)} p_{12}^{(4)} p_{27}^{(6)}]$$

$$+ p_{27}^{(6)} (p_{02}^{(1,4)} + p_{02}^{(2,5)}) (1 - p_{11}^{(3)}) + p_{07}^{(2,6)} (1 - p_{11}^{(3)}) (1 - p_{22}^{(5)})]$$

and D_2 is same as in equation (5.4).

The expected busy period of the repairman in $(0, t]$ is

$$\mu_b(t) = \int_0^t B_0(u) dt,$$

so that

$$\mu_b^*(s) = s^{-1} B_0^*(s).$$

7. Cost Analysis

We obtain the profit function of the system, by considering the expected up-time of the system and the busy period of the repairman in $(0, t]$. The expected net gain incurred in $(0, t]$ is

$$(7.1) \quad P(t) = \text{expected revenue in } (0, t] - \text{expected repair cost in } (0, t]$$

$$= C_1 \mu_{up}(t) - C_2 \mu_b(t),$$

where C_1 is the revenue per unit up-time and C_2 is the repair cost per unit time. Therefore, the expected gain per unit time is $P(t) = P(t)/t$.

8. Particular Case

In particular, when failure and repair times are independent, i.e. $f(x, y) = g(x)h(y)$ (or $r = 0$) then all the states will become regenerative. The following relations among various probabilities are also observed:

$$Q_{ij}^{(k)}(t) = Q_{ik}(t) \otimes Q_{kj}(t),$$

$$\text{so that } \tilde{Q}_{ij}^{(k)}(s) = \tilde{Q}_{ik}(s) \otimes \tilde{Q}_{kj}(s).$$

$$\text{Also } \tilde{Q}_{ij}^{(K)}(s)|_{s=0} = P_{ij}^{(K)} = P_{iK} P_{Kj},$$

$$\text{and } m_{ij}^{(k)} = -\frac{d}{ds} Q_{ij}^{(k)}(s) \Big|_{s=0} = -\frac{d}{ds} [\tilde{Q}_{ik}(s) \tilde{Q}_{kj}(s)] \Big|_{s=0} = m_{ik} p_{kj} + m_{kj} p_{ik}.$$

Using the above relations, we have

$$E(T) = N_1^P [D_1^P]^{-1}$$

$$\text{and } R_0^P = N_2^P [D_2^P]^{-1},$$

then

$$A_0^P = N_3^P [D_3^P]^{-1},$$

$$B_0^P = N_4^P [D_3^P]^{-1},$$

where

$$N_3^P = p_{20}[(\varphi_0 + p_{01}\varphi_1 + p_{02}\varphi_2)(1 - p_{11}^{(3)}) + \varphi_1 p_{01}^{(1,3)}] \\ + \varphi_2[(1 - p_{00}^{(1)} - p_{00}^{(2)})(1 - p_{11}^{(3)}) - p_{10} p_{01}^{(1,3)}],$$

$$D_3^P = \varphi_0 + p_{01}\varphi_1 + p_{02}\varphi_2 + p_{01}p_{13}\varphi_3 + \varphi_4(p_{01}p_{14} + p_{02}p_{24}) + p_{02}p_{25}\varphi_5,$$

$$N_4^P = (\varphi_1 + p_{13}\varphi_3 + p_{14}\varphi_4)p_{01}^{(1,3)}p_{20} + (\varphi_2 + p_{25}\varphi_5 + p_{24}\varphi_4)$$

$$[p_{01}^{(1,3)}p_{12}^{(4)} + (1 - p_{11}^{(3)})(p_{02}^{(2,4)} + p_{02}^{(1,4)} + p_{02}^{(2,5)})].$$

9. Graphical Study

For studying the impact of correlation(r) between failure and repair times on MTSF and availability, the MTSF and the availability are plotted with respect to correlation coefficient (r). In fig 2 and fig 3 only positive values of r are considered as the joint pdf of x & y is true only for $0 \leq r \leq 1$.

From fig 2, it is clear that the MTSF increases as r increases in all three curves for different values of failure and repair rates as given in table 1. In curve III, MTSF increases rapidly as on increasing repair rates but in curve II, MTSF increases slowly as compare to curve III on decreasing the repair rate. In curve I, MTSF increases very slowly with increasing the failure rate. Same case arises in case of availability i.e. in Fig -3. From both the graphs it can be observed that the MTSF and availability are increasing function of r .

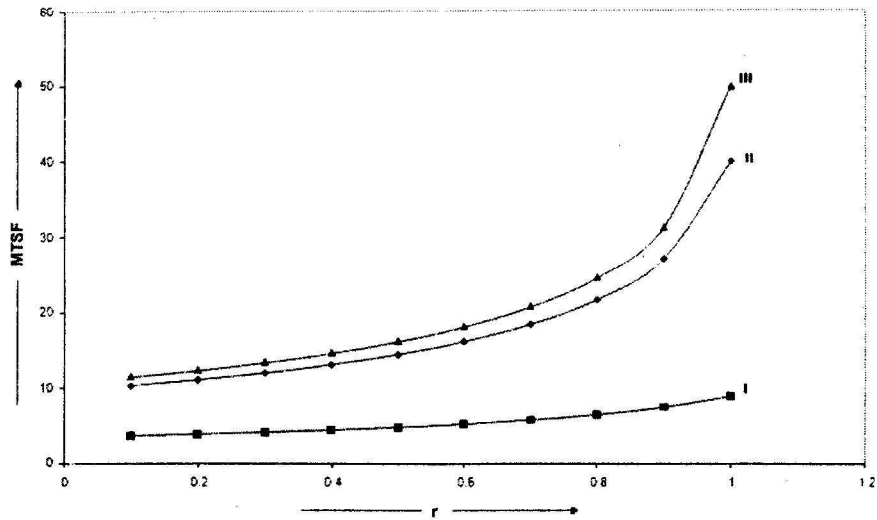
BEHAVIOUR OF MTSF WITH RESPECT TO r 

Fig. 2

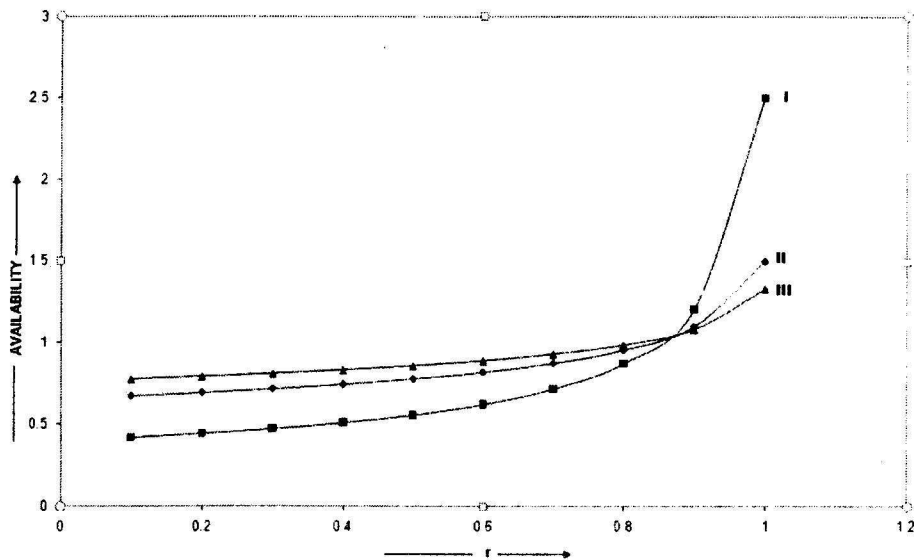
BEHAVIOUR OF AVAILABILITY WITH RESPECT TO r 

Fig. 3

Table 1 – Numerical values of Failure Rates & Repair Rates for MTSF & Availability Graphs against correlation coefficient

	Failure Rates		Repair Rates			
	α_1	a	b	β_1	β_2	β_3
Curve I	.4	.3	.2	.3	.3	.4
Curve II	.2	.1	.2	.3	.3	.4
Curve III	.2	.1	.3	.4	.4	.5

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