

Steady Three -Dimensional Flow and Heat Transfer along an Infinite Vertical Porous Moving Plate in the Presence of Uniform Free Stream and Periodic Suction

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Abstract: The aim of the paper is to investigate steady three-dimensional laminar flow and heat transfer of an incompressible viscous fluid along an infinite hot vertical moving porous plate in the presence of uniform free stream and sinusoidal suction velocity. The governing equations of motion and energy are solved by using the regular perturbation technique. The velocity components, pressure distribution and temperature distribution are derived, discussed numerically and shown through graphs. The expressions of skin- friction coefficient and Nusselt number at the plate are derived, discussed and their numerical values for various values of physical parameters are presented through Table.

1. Introduction

Lachmann¹ reported importance of laminar flow control in the field of aeronautical engineering in view of its applications to reduce drag and hence the vehicle power requirement by a substantial amount. Transition from laminar to turbulent flow which increase drag coefficient, may be prevented by removing the decelerated particles from the boundary layer through the porous or slits of the wall. The effect of different arrangements and configurations of the suction holes and slits on the drag coefficient have been studied extensively both experimentally and theoretically. Out of them some of the investigations were confined to two-dimensional flows discussed by Pai², Schlitching³, Bansal⁴ etc. If variation in the suction velocity distribution is transverse to the potential flow, then the situation of the flow fields will convert to three- dimensional.

Gersten and Gross⁵ studied the effect of a transverse sinusoidal suction velocity distribution on flow and heat transfer over a plane wall. Three-dimensional free convection flow and heat transfer along a porous vertical plate was discussed by Singh et.al.^{6,7} studied three-dimensional fluctuating flow and heat transfer along a plate with suction. Muhuri and Gupta⁸ considered on free convection boundary layer flow on a flat plate due to small fluctuations in surface temperature. Kafousias et. al.⁹ investigated free convection effects on the flow past an accelerated vertical porous plate in an incompressible dissipative fluid with variable suction and injection. Free convective effect on the flow past an infinite vertical porous plate in the presence of constant suction and constant heat flux-I was presented by Sharma¹⁰. Singh¹¹ analyzed three-dimensional viscous flow and heat transfer

along a porous plate. Free convective flow along a vertical plate with constant heat flux was studied by Burak et.al.¹². Das et.al.¹³ discussed transient free convection flow of a viscous incompressible fluid past an infinite vertical plate with periodic heat flux. Steady three-dimensional flow and heat transfer along an infinite porous wall in the presence of heat source, uniform free stream and variable suction have been presented by Sharma and Gupta¹⁴. Soundalgekar et.al.¹⁵ investigated transient free convection flow of a viscous incompressible fluid past an infinite vertical plate with periodic heat flux. Shahnaz¹⁶ studied heat transfer in three-dimensional viscous flow over a porous plate moving with harmonic disturbance.

Aim of the paper is to investigate three-dimensional laminar flow and heat transfer of an incompressible viscous fluid along an infinite hot vertical moving porous plate in the presence of uniform free stream and sinusoidal suction velocity.

2. Formulation of the Problem

Steady three-dimensional laminar flow of an incompressible viscous fluid along an infinite vertical moving porous plate with the velocity U^* , is taken in x^*z^* plane. The direction of the fluid flow is taken along x^* -axis in upwards and y^* -axis is taken along normal to the plate. A sinusoidal suction with mean cross-flow velocity V_0 is applied normal to the surface so that flow of the fluid remains three-dimensional. All physical quantities except body force are taken to be independent of x^* (see figure 1).

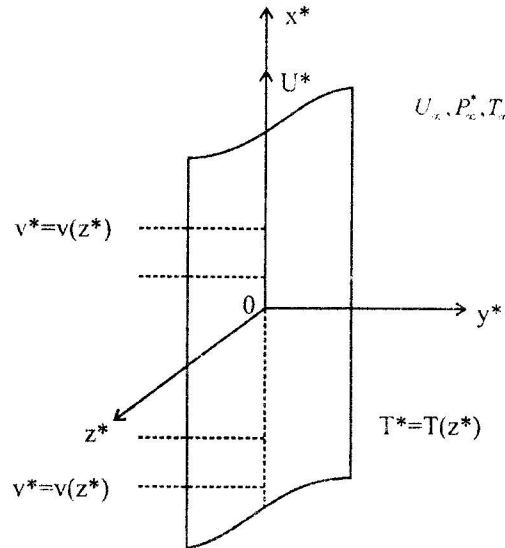


Fig- 1: Physical Model

The governing equations of continuity, motion and energy including the viscous dissipative term are given by

Equation of Continuity

$$(2.1) \quad \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0,$$

Equations of Motion

$$(2.2) \quad v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = g\beta (T^* - T_\infty) + \nu \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right),$$

$$(2.3) \quad v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right),$$

$$(2.4) \quad v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right),$$

Equation of Energy

$$(2.5) \quad \rho C_p \left(v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} \right) = \kappa \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) + 2\mu \left\{ \left(\frac{\partial v^*}{\partial y^*} \right)^2 + \left(\frac{\partial w^*}{\partial z^*} \right)^2 \right\} \\ + \mu \left\{ \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \left(\frac{\partial u^*}{\partial z^*} \right)^2 + \left(\frac{\partial w^*}{\partial y^*} + \frac{\partial v^*}{\partial z^*} \right)^2 \right\},$$

where g is the acceleration due to gravity, β coefficient of volumetric expansion, ν kinematic viscosity, μ coefficient of viscosity, ρ the density of fluid, T_0 reference temperature, T_∞ free stream temperature, T^* temperature of the fluid, p^* pressure, u^* , v^* , w^* are velocity components along x^* , y^* , z^* -axes respectively, C_p specific heat at constant pressure and κ thermal conductivity.

The boundary conditions are

$$y^* = 0 : u^* = U^*, v^* = v(z^*) = -V_0 (1 + \epsilon \cos(\pi z^* / l)), w^* = 0,$$

$$T^* = T(z^*) = T_\infty + (T_0 - T_\infty) (1 + \epsilon \cos(\pi z^* / l))$$

$$(2.6) \quad y^* \rightarrow \infty : u^* \rightarrow U_\infty, v^* \rightarrow -V_0, w^* \rightarrow 0, p^* \rightarrow p_\infty^*, T^* \rightarrow T_\infty,$$

where V_0 mean cross flow velocity and l the wave length of the cross-flow suction velocity.

3. Method of Solution

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{l}, z = \frac{z^*}{l}, u = \frac{u^*}{U_\infty}, v = \frac{v^*}{V_0}, w = \frac{w^*}{V_0}, p = \frac{p^*}{\left(\frac{\mu V_0}{l} \right)}, p_\infty = \frac{p_\infty^*}{\left(\frac{\mu V_0}{l} \right)},$$

$$\theta = \frac{T^* - T_\infty}{T_o - T_\infty}, \text{ Re} = \frac{V_o l}{\nu}, \text{ Pr} = \frac{\mu C_p}{\kappa},$$

$$\text{Ec} = \frac{U_\infty^2}{C_p (T_o - T_\infty)}, \text{ Gr} = \frac{g \beta l^2 (T_o - T_\infty)}{\nu U_\infty},$$

$$(3.1) \quad \lambda = \frac{\nu_o}{\ell U_\infty} \text{ and } U = \frac{U^*}{U_\infty}.$$

into the equations (2.1) to (2.5), we get

$$(3.2) \quad \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

$$(3.3) \quad \text{Re} \left(v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \text{Gr} \theta + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2},$$

$$(3.4) \quad \text{Re} \left(v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2},$$

$$(3.5) \quad \text{Re} \left(v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2},$$

$$(3.6) \quad \text{Re Pr} \left(v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) = \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + 2 \text{Pr Ec Re}^2 \lambda^2 \left\{ \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right\}$$

$$+ \text{Pr Ec Re}^2 \lambda^2 \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \text{Pr Ec} \left\{ \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 \right\},$$

where Re the cross- flow Reynolds number, Pr the Prandtl number, Ec the Eckert number and Gr the Grashof number.

The corresponding boundary conditions in non-dimensional form are

$$y = 0 : u = U, v = -(1 + \varepsilon \cos \pi z), w = 0, \theta = 1 + \varepsilon \cos \pi z$$

$$(3.7) \quad y \rightarrow \infty : u \rightarrow 1, v \rightarrow -1, w \rightarrow 0, p \rightarrow p_\infty, \theta \rightarrow 0.$$

In view of the boundary conditions (3.7), the physical quantities are assumed as given by

$$(3.8) \quad F(y, z) = F_0(y) + \varepsilon F_1(y, z),$$

where F stands for any of u, v, w, p or θ .

Here zeroth-order and first-order stand for two-dimensional flow and three-dimensional flow, respectively.

Substituting (3.8) into the equations (3.2) to (3.6) and equating the terms $O(\epsilon)$, we get

Zeroth-order Equations

$$(3.9) \quad v_o' = 0 \Rightarrow v_o = -1,$$

$$(3.10) \quad u_o'' + \text{Re } u_o' = -Gr \theta_o,$$

$$(3.11) \quad \theta_o'' + \text{Pr Re } \theta_o' = -\text{Pr Ec } u_o'^2.$$

where prime denotes differentiation with respect to y .

First-order Equations

$$(3.12) \quad \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0,$$

$$(3.13) \quad \text{Re} \left(v_1 \frac{du_o}{dy} - \frac{\partial u_1}{\partial y} \right) = \frac{\partial^2 u_1}{\partial z^2} + \frac{\partial^2 u_1}{\partial y^2} + Gr \theta_1,$$

$$(3.14) \quad -\text{Re} \frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2},$$

$$(3.15) \quad -\text{Re} \frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2},$$

$$(3.16) \quad \text{Pr Re} \left(v_1 \frac{d\theta_o}{dy} - \frac{\partial \theta_1}{\partial y} \right) = \frac{\partial^2 \theta_1}{\partial z^2} + \frac{\partial^2 \theta_1}{\partial y^2} + 2 \text{Pr Ec} \frac{du_o}{dy} \frac{\partial u_1}{\partial y}.$$

The corresponding boundary conditions are

$$y = 0 : u_0 = U, u_1 = 0, v_1 = -\cos \pi z, w_1 = 0, \theta_0 = 1, \theta_1 = \cos \pi z,$$

$$(3.17) \quad y \rightarrow \infty : u_0 \rightarrow 1, u_1 \rightarrow 0, v_1 \rightarrow 0, w_1 \rightarrow 0, p_1 \rightarrow 0, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0.$$

Mean Flow and Mean Temperature

Equations (3.10) and (3.11) are coupled second order differential equations. Since for incompressible fluids, the Eckert number Ec is very small, therefore u_0 and θ_0 can be expanded in the powers of Ec as given by

$$(3.18) \quad g = g_o + \text{Ec } g_1 + O(\text{Ec}^2),$$

where g stands for any u_0 or θ_0 .

Substituting (3.18) into the equations (3.10) and (3.11), and equating like powers of Ec , we get

$$(3.19) \quad u_{o0}'' + \text{Re } u_{o0}' = -Gr \theta_{o0},$$

$$(3.20) \quad u_{01}'' + \text{Re } u_{01}' = -Gr \theta_{01},$$

$$(3.21) \quad \theta_{00}'' + \text{Pr Re } \theta_{00}' = 0,$$

$$(3.22) \quad \theta_{01}'' + \text{Pr Re } \theta_{01}' = -\text{Pr} \left(u_{00}' \right)^2.$$

where dash denotes differentiation with respect to y .

Now, the corresponding boundary conditions are reduced to

$$(3.23) \quad \begin{aligned} y = 0 : u_{00} = U, u_{01} = 0, \theta_{00} = 1, \theta_{01} = 0, \\ y \rightarrow \infty : u_{01} \rightarrow 1, u_{01} \rightarrow 0, \theta_{00} \rightarrow 0, \theta_{01} \rightarrow 0 \end{aligned}$$

The equations (3.19) to (3.22) are ordinary second order differential equations and solved under the boundary conditions (3.23). The expressions of $u_{00}(y)$, $u_{01}(y)$, $\theta_{00}(y)$ and $\theta_{01}(y)$ are known and given by

$$(3.24) \quad u_{00} = 1 - A_1 e^{-\text{Pr Re } y} + A_2 e^{-\text{Re } y},$$

$$(3.25) \quad u_{01} = A_{11} e^{-\text{Re } y} - A_7 e^{-\text{Pr Re } y} + A_8 e^{-2\text{Pr Re } y} + A_9 e^{-2\text{Re } y} - A_{10} e^{-(1+\text{Pr})\text{Re } y},$$

$$(3.26) \quad \theta_{00} = e^{-\text{Pr Re } y},$$

$$(3.27) \quad \theta_{01} = A_6 e^{-\text{Pr Re } y} - A_3 e^{-2\text{Pr Re } y} - A_4 e^{-2\text{Re } y} + A_5 e^{-(1+\text{Pr})\text{Re } y}.$$

Here A_1 to A_{11} are constants and their expressions are given in Appendix.

Cross-flow Solution

It is observed that the equations (3.12), (3.14) and (3.15) are independent of main flow u_1 and temperature distribution θ_1 . In view of the boundary conditions (3.17) and equation of continuity (3.12), it is assumed that

$$(3.28) \quad \begin{aligned} v_1(y, z) &= \pi v_{11}(y) \cos \pi z, w_1(y, z) \\ &= -v_{11}'(y) \sin \pi z, p_1(y, z) = p_{11}(y) \cos \pi z \end{aligned}$$

Here prime denotes differentiation with respect to y . Substituting (3.28) into the equations (3.14) and (3.15), resulting a pair of ordinary differential equations given by

$$(3.29) \quad v_{11}'' + \text{Re } v_{11}' - \pi^2 v_{11} = \frac{1}{\pi} p_{11}',$$

$$(3.30) \quad v_{11}''' + \text{Re } v_{11}'' - \pi^2 v_{11}' = \pi p_{11},$$

where dash denotes differentiation with respect to y .

The corresponding boundary conditions are

$$y = 0 : v_{11} = -1/\pi, v_{11}' = 0,$$

$$(3.31) \quad y \rightarrow \infty : v_{11} \rightarrow 0, v'_{11} \rightarrow 0.$$

The equations (3.29) and (3.30) are solved under the boundary conditions (3.31), the expression of $v_{11}(y)$ is known and given by

$$(3.32) \quad v_{11}(y) = \frac{1}{\pi(\lambda - \pi)} (\pi e^{-\lambda y} - \lambda e^{-\pi y}).$$

Hence the expressions for velocity components v_1 , w_1 and pressure p_1 are known and given by

$$(3.33) \quad v_1(y, z) = \frac{1}{(\pi - \lambda)} (e^{-\pi y} - e^{-\lambda y}) \cos \pi z,$$

$$(3.34) \quad w_1(y, z) = \frac{-\lambda}{\pi - \lambda} (e^{-\pi y} - e^{-\lambda y}) \sin \pi z,$$

$$(3.35) \quad p_1(y, z) = \frac{\text{Re } \lambda}{(\pi - \lambda)} e^{-\pi y} \cos \pi z,$$

$$\text{where } \lambda = \frac{\text{Re}}{2} + \sqrt{\frac{\text{Re}^2}{4} + \pi^2}.$$

Main Flow and Temperature Distribution

Equations (3.13) and (3.16) are coupled second order differential equations. Since the Eckert number Ec is very small for incompressible fluids, therefore u_1 and θ_1 can be expanded in the powers of Ec as given by

$$(3.36) \quad h = h_0 + Ec h_1 + O(Ec^2)$$

where h stands for u_1 or θ_1 .

Substituting (3.36) into the equations (3.13) and (3.16) and equating the like powers of Ec , we get

$$(3.37) \quad \frac{\partial^2 u_{10}}{\partial y^2} + \frac{\partial^2 u_{10}}{\partial z^2} = -Gr \theta_{10} + \text{Re} \left(-\frac{\partial u_{10}}{\partial y} + v_1 \frac{du_{00}}{dy} \right),$$

$$(3.38) \quad \frac{\partial^2 u_{11}}{\partial y^2} + \frac{\partial^2 u_{11}}{\partial z^2} = -Gr \theta_{11} + \text{Re} \left(-\frac{\partial u_{11}}{\partial y} + v_1 \frac{du_{01}}{dy} \right),$$

$$(3.39) \quad \frac{\partial^2 \theta_{10}}{\partial y^2} + \frac{\partial^2 \theta_{10}}{\partial z^2} = \text{Pr Re} \left(-\frac{\partial \theta_{10}}{\partial y} + v_1 \frac{d\theta_{00}}{dy} \right),$$

$$(3.40) \quad \frac{\partial^2 \theta_{11}}{\partial y^2} + \frac{\partial^2 \theta_{11}}{\partial z^2} = -2 \text{Pr} \frac{du_{00}}{dy} \frac{\partial u_{10}}{\partial y} + \text{Pr Re} \left(-\frac{\partial \theta_{11}}{\partial y} + v_1 \frac{d\theta_{01}}{dy} \right).$$

The corresponding boundary conditions are

$$(3.41) \quad \begin{aligned} y = 0 : u_{10} = 0, u_{11} = 0, \theta_{10} = \cos \pi z, \theta_{11} = 0, \\ y \rightarrow \infty : u_{10} \rightarrow 0, u_{11} \rightarrow 0, \theta_{10} \rightarrow 0, \theta_{11} \rightarrow 0. \end{aligned}$$

Now, to reduce the partial differential equations (3.37) to (3.40) into ordinary differential equations, applying variables separation technique as given by

$$(3.42) \quad \begin{aligned} u_{10}(y, z) = u_{20}(y) \cos \pi z, u_{11}(y, z) = u_{21}(y) \cos \pi z, \\ \theta_{10}(y, z) = \theta_{20}(y) \cos \pi z \text{ \& } \theta_{11}(y, z) = \theta_{21}(y) \cos \pi z. \end{aligned}$$

Using (3.42) into the equations (3.37) to (3.40), we get

$$(3.43) \quad u_{20}'' + \text{Re } u_{20}' - \pi^2 u_{20} = -Gr \theta_{20} + \pi \text{Re } v_{11} u_{00}',$$

$$(3.44) \quad u_{21}'' + \text{Re } u_{21}' - \pi^2 u_{21} = -Gr \theta_{21} + \pi \text{Re } v_{11} u_{00}',$$

$$(3.45) \quad \theta_{20}'' + \text{Pr Re } \theta_{20}' - \pi^2 \theta_{20} = \pi \text{Pr Re } v_{11} \theta_{00}',$$

$$(3.46) \quad \theta_{21}'' + \text{Pr Re } \theta_{21}' - \pi^2 \theta_{21} = \pi \text{Pr Re } v_{11} \theta_{01}' - 2 \text{Pr } u_{00}' u_{20}'$$

where dash denotes differentiation with respect to y .

Now, the corresponding boundary conditions are

$$(3.47) \quad \begin{aligned} y = 0 : u_{20} = 0, u_{21} = 0, \theta_{20} = 1, \theta_{21} = 0, \\ y \rightarrow \infty : u_{20} \rightarrow 0, u_{21} \rightarrow 0, \theta_{20} \rightarrow 0, \theta_{21} \rightarrow 0. \end{aligned}$$

The equations (3.43) to (3.46) are solved under the boundary conditions (3.47), the solutions of u_{20} , u_{21} , θ_{20} and θ_{21} are known and given by

$$(3.48) \quad u_{20} = A_{18} e^{-\lambda y} - A_{15} e^{-my} + A_{16} e^{-(\lambda + \text{Pr})y} - A_{17} e^{-(\pi + \text{Pr Re})y},$$

$$(3.49) \quad \begin{aligned} u_{21} = & A_{33} e^{-(\lambda + 2 \text{Pr Re})y} - A_{34} e^{-(\lambda + 2 \text{Re})y} + A_{35} e^{-(\lambda + \text{Re} + \text{Pr Re})y} \\ & + A_{36} e^{-(\lambda + \text{Pr Re})y} + A_{37} e^{-(\pi + 2 \text{Re})y} - A_{38} e^{-(\pi + \text{Re} + \text{Pr Re})y} \\ & - A_{39} e^{-(\pi + \text{Pr Re})y} + A_{40} e^{-(m + \text{Pr Re})y} - A_{41} e^{-(m + \text{Re})y} \\ & - A_{42} e^{-(\lambda + \text{Pr} + \text{Pr Re})y} + A_{43} e^{-(\lambda + \text{Pr} + \text{Re})y} + A_{44} e^{-(\lambda + \text{Re})y} \\ & - A_{45} e^{-my} + A_{46} e^{-(\pi + 2 \text{Pr Re})y} + A_{47} e^{-(\pi + \text{Re})y} + A_{48} e^{-\lambda y}, \end{aligned}$$

$$(3.50) \quad \theta_{20} = A_{14} e^{-my} - A_{12} e^{-(\lambda + \text{Pr Re})y} + A_{13} e^{-(\pi + \text{Pr Re})y},$$

$$(3.51) \quad \begin{aligned} \theta_{21} = & A_{19} e^{-(\lambda + 2 \text{Pr Re})y} + A_{20} e^{-(\lambda + 2 \text{Re})y} - A_{21} e^{-(\lambda + \text{Re} + \text{Pr Re})y} \\ & - A_{22} e^{-(\lambda + \text{Pr Re})y} + A_{23} e^{-(\pi + 2 \text{Pr Re})y} - A_{24} e^{-(\pi + 2 \text{Re})y} \end{aligned}$$

$$\begin{aligned}
& + A_{25} e^{-(\pi + \text{Re} + \text{Pr Re})y} + A_{26} e^{-(\pi + \text{Pr Re})y} - A_{27} e^{-(m + \text{Pr Re})y} \\
& + A_{28} e^{-(\lambda + \text{Pr} + \text{Pr Re})y} + A_{29} e^{-(m + \text{Re})y} - A_{30} e^{-(\lambda + \text{Pr} + \text{Re})y} \\
& - A_{31} e^{-(\lambda + \text{Re})y} + A_{32} e^{-my},
\end{aligned}$$

where to A_{12} to A_{48} are constants and their expressions are given in Appendix.

Finally, the main velocity and the temperature distribution are obtained in the following form

$$(3.52) \quad u(y, z, t) = u_0(y) + \epsilon u_1(y, z, t) \text{ and } \theta(y, z, t) = \theta_0(y) + \epsilon \theta_1(y, z, t),$$

where

$$u_1 = u_{20}(y) \cos \pi z + Ec u_{21}(y) \cos \pi z$$

$$\text{and } \theta_1 = \theta_{20}(y) \cos \pi z + Ec \theta_{21}(y) \cos \pi z.$$

4. Skin- Friction

Skin- friction coefficient at the plate is given by

$$(4.1) \quad C_f = -\frac{\tau_{x^*y^*}}{\rho V_0 U_\infty} = -\frac{1}{\text{Re}} \left(\frac{\partial u}{\partial y} \right)_{y=0},$$

where $\tau_{x^*y^*} = \mu \left(\frac{\partial u^*}{\partial y^*} \right)_{y^*=0}$ is the shear stress at the plate.

Hence, the expression of skin-friction coefficient at the plate is given by

$$\begin{aligned}
(4.2) \quad C_f = & -\frac{1}{\text{Re}} \left[\text{Pr Re } A_1 - \text{Re } A_2 + Ec(\text{Pr Re } A_7 - \text{Pr Re } A_8 - 2 \text{Re } A_9 \right. \\
& + (1 + \text{Pr}) \text{Re } A_{10} - \text{Re } A_{11}) + \epsilon \{ m A_{15} - (\lambda + \text{Pr}) A_{16} + (\pi + \text{Pr Re}) A_{17} \\
& - \lambda A_{18} \} \cos \pi z - \epsilon Ec \{ A_{33} (\lambda + 2 \text{Pr Re}) + (\lambda + 2 \text{Re}) A_{34} \\
& + (\lambda + \text{Re} + \text{Pr Re}) A_{35} + (\lambda + \text{Pr Re}) A_{36} + (\pi + 2 \text{Re}) A_{37} + (\pi + \text{Re} \\
& + \text{Pr Re}) A_{38} + (\pi + \text{Pr Re}) A_{39} + (m + \text{Pr Re}) A_{40} - (m + \text{Re}) A_{41} \\
& - (\lambda + \text{Pr} + \text{Pr Re}) A_{42} + (\lambda + \text{Pr} + \text{Re}) A_{43} + (\lambda + \text{Re}) A_{44} - m A_{45} \\
& \left. + (\pi + 2 \text{Pr Re}) A_{46} + (\pi + \text{Re}) A_{47} + \lambda A_{48} \} \cos \pi z \right].
\end{aligned}$$

5. Nusselt Number

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$(5.1) \quad Nu = \frac{\ell q}{\kappa(T_o - T_\infty)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0},$$

$$\text{where } q = -\kappa \left(\frac{\partial T^*}{\partial y^*} \right)_{y^*=0}.$$

Hence, the expression of the Nusselt number at the plate is given by

$$(5.2) \quad Nu = \text{Pr Re} - \text{Ec} \{ 2 \text{Pr Re } A_3 + 2 \text{Re } A_4 - (1 + \text{Pr}) \text{Re } A_5 - \text{Pr Re } A_6 \} \\ - \epsilon \{ [(\lambda + \text{Pr Re}) A_{12} - (\pi + \text{Pr Re}) A_{13} - m A_{14} + \text{Ec} \{ -(\lambda + 2 \text{Pr Re}) A_{19} \\ - (\lambda + 2 \text{Re}) A_{20} + (\lambda + \text{Re} + \text{Pr Re}) A_{21} - (\lambda + \text{Pr Re}) A_{22} + (\pi + 2 \text{Pr Re}) A_{23} \\ + (\pi + 2 \text{Re}) A_{24} - (\pi + \text{Re} + \text{Pr Re}) A_{25} - (\pi + \text{Pr Re}) A_{26} + (m + \text{Pr Re}) A_{27} \\ - (\lambda + \text{Pr} + \text{Pr Re}) A_{28} - (m + \text{Re}) A_{29} + (\lambda + \text{Pr} + \text{Re}) A_{30} - m A_{32} \} \cos \pi z \}.$$

Table-1: Values of C_f and Nu at the plate for various values of physical parameter

Pr	Gr	Re	Ec	α	z	C_f	Nu
0.71	5	0.5	0.01	0.5	0	-56.11651	-1.02083
7	5	0.5	0.01	0.5	0	-3.40941	3.52232
0.71	10	0.5	0.01	0.5	0	-275.68368	-5.18434
0.71	5	1	0.01	0.5	0	-7.99395	0.56258
0.71	5	0.5	0.02	0.5	0	-83.54765	-2.43019
0.71	5	0.5	0.01	2	0	-54.23325	-0.92735
0.71	5	0.5	0.01	0.5	1	-55.79853	0.09120

6. Results and Discussion

It is observed from figure 2 that the fluid velocity increases due to increase in the Grashof number, the Eckert number or plate velocity, while it decreases due to increase in the cross-flow Reynolds number or the Prandtl number. It is noted from figure 3 that the fluid temperature increases due to increase in the Grashof number or the Eckert number, while it decreases due to increase in the plate velocity, cross-flow Reynolds number or the Prandtl number.

It is observed from Table-1 that the skin-friction coefficient at the plate increases due to increase in the Prandtl number, the cross-flow Reynolds number or plate velocity, while it decreases due to increase in the Grashof number or the Eckert number. It is also noted from Table-1 that the Nusselt number at the plate increases due to increase in the Prandtl number, the cross-flow Reynolds number or plate velocity, while it decreases due to increase in the Grashof number or the Eckert number.

7. Conclusions

1. Fluid velocity increases due to increase in the buoyancy force i.e. density variation in the neighbourhood of the plate and decreases due to increase in the Prandtl number or more fluid withdrawn through the plate which are in full agreement with physical phenomena.
2. Fluid temperature increases due to increase in the buoyancy force or addition of heat due to viscous dissipation, while it decreases due to more fluid withdrawn through the plate.
3. Skin-friction coefficient at the plate increases due to more fluid withdrawn through the plate, while it decreases due to increase in the buoyancy force or addition of heat due to viscous dissipation.
4. Nusselt number at the plate increases due to more fluid withdrawn through the plate, while it decreases due to increase in the buoyancy force or addition of heat due to viscous dissipation.

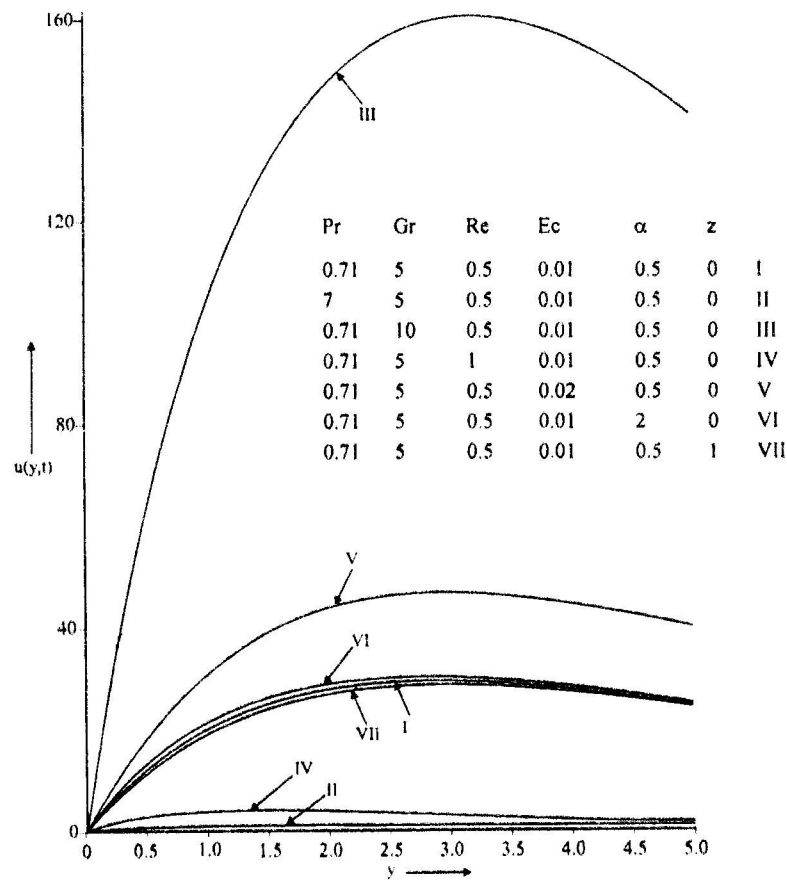


Fig- 2: Velocity distribution versus y

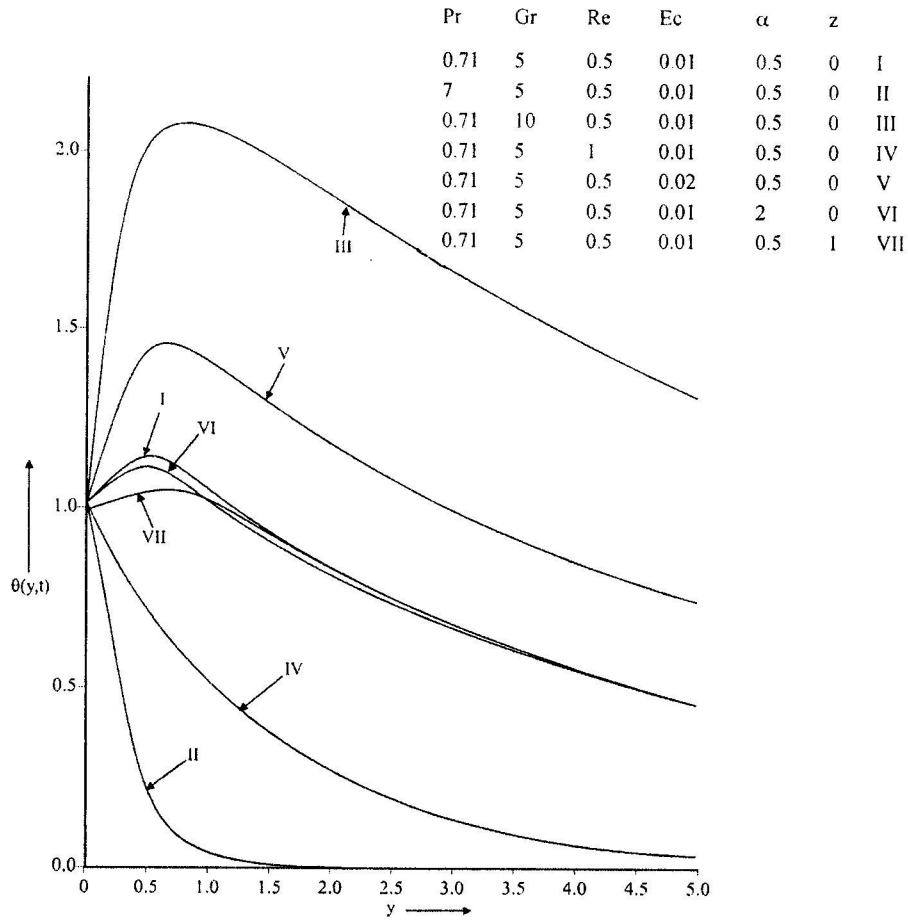


Fig- 3: Temperature distribution versus y

Appendix

$$A_1 = \frac{Gr}{D_1}, \quad A_2 = A_1 + U - 1, \quad A_3 = \frac{A_1^2 Pr}{2}, \quad A_4 = \frac{A_2^2 Pr}{2(2 - Pr)},$$

$$A_5 = \frac{2A_1 A_2 Pr^2}{Pr + 1}, \quad A_6 = A_3 + A_4 - A_5, \quad A_7 = \frac{Gr A_6}{D_1}, \quad A_8 = \frac{Gr A_3}{D_2},$$

$$A_9 = \frac{Gr A_4}{2Re^2}, \quad A_{10} = \frac{Gr A_5}{Pr Re^2 (Pr + 1)}, \quad A_{11} = A_7 - A_8 - A_9 + A_{10},$$

$$A_{12} = \frac{\pi Pr^2 Re^2}{D_3}, \quad A_{13} = \frac{Pr Re \lambda}{\pi(\lambda - \pi)}, \quad A_{14} = 1 + A_{12} - A_{13},$$

$$\begin{aligned}
A_{15} &= \frac{Gr A_{14}}{Re m(Pr-1)}, \quad A_{16} = \frac{Gr A_{12}}{D_4}, \quad A_{17} = \frac{Gr A_{13}}{D_5}, \quad A_{18} = A_{15} - A_{16} + A_{17}, \\
A_{19} &= \frac{2 Pr^2 Re^2 \pi A_3}{D_6}, \quad A_{20} = \frac{2 Pr^2 Re^2 \pi A_4}{D_7}, \quad A_{21} = \frac{Pr Re^2 (1+Pr) \pi A_5}{D_8}, \\
A_{22} &= \frac{1}{D_9} \left\{ \frac{Pr^2 Re^2 A_6 \pi}{(\lambda - \pi)} - 2 Pr^2 Re \lambda A_1 A_{18} \right\}, \\
A_{23} &= \frac{1}{D_{10}} \left\{ \frac{2 Pr^2 Re^2 \lambda A_3}{(\lambda - \pi)} + 2 Pr^2 Re (\pi + Pr Re) A_1 A_{17} \right\}, \quad A_{24} = \frac{2 Pr^2 Re^2 \lambda A_4}{D_{11}}, \\
A_{25} &= \left\{ \frac{Pr Re^2 (1+Pr) \lambda A_5}{(\lambda - \pi)} + 2 Pr Re (\pi + Pr Re) A_2 A_{17} \right\} \frac{1}{D_{12}}, \\
A_{26} &= \frac{Pr Re A_6 \lambda}{\pi(\lambda - \pi)}, \quad A_{27} = Pr A_1 A_{15}, \quad A_{28} = \frac{2 Pr^2 Re (\lambda + Pr) A_1 A_{16}}{D_{13}}, \\
A_{29} &= \frac{2 Pr Re m A_2 A_{15}}{D_{14}}, \quad A_{30} = \frac{2 Pr Re (\lambda + Pr) A_2 A_{16}}{D_{13}}, \quad A_{31} = \frac{2 Pr Re \lambda A_2 A_{18}}{D_{12}}, \\
A_{32} &= -A_{19} - A_{20} + A_{21} + A_{22} + A_{23} + A_{24} - A_{25} - A_{26} + A_{27} - A_{28} - A_{29} + A_{30} + A_{31} \\
A_{33} &= \left\{ Gr A_{19} + \frac{2 Pr Re^2 \pi A_8}{(\lambda - \pi)} \right\} \frac{1}{D_{17}}, \quad A_{34} = \left\{ Gr A_{20} + \frac{2 Re^2 \pi A_9}{(\lambda - \pi)} \right\} \frac{1}{D_{18}}, \\
A_{35} &= \left\{ Gr A_{21} + \frac{Re^2 (1+Pr) \pi A_{10}}{(\lambda - \pi)} \right\} \frac{1}{D_{19}}, \quad A_{36} = \left\{ Gr A_{22} + \frac{Pr Re^2 \pi A_7}{(\lambda - \pi)} \right\} \frac{1}{D_{20}}, \\
A_{37} &= \left\{ Gr A_{24} + \frac{2 Re^2 \lambda A_9}{(\lambda - \pi)} \right\} \frac{1}{D_{21}}, \quad A_{38} = \left\{ Gr A_{25} + \frac{Re^2 (1+Pr) \lambda A_{10}}{(\lambda - \pi)} \right\} \frac{1}{D_{21}}, \\
A_{39} &= \left\{ Gr A_{26} + \frac{Pr Re^2 \lambda A_7}{(\lambda - \pi)} \right\} \frac{1}{D_{23}}, \quad A_{40} = \frac{Gr A_{27}}{D_{24}}, \quad A_{41} = \frac{Gr A_{29}}{D_{25}}, \\
A_{42} &= \frac{Gr A_{28}}{D_{26}}, \quad A_{43} = \frac{Gr A_{30}}{D_{27}}, \quad A_{44} = \left\{ Gr A_{31} - \frac{Re^2 \pi A_{11}}{(\lambda - \pi)} \right\} \frac{1}{D_{28}}, \\
A_{45} &= \frac{Gr A_{32}}{m^2 - Re m - \pi^2}, \quad A_{46} = \frac{2 Pr Re^2 \lambda A_8}{D_{29}}, \quad A_{47} = \frac{Re \lambda A_{11}}{\pi(\lambda - \pi)},
\end{aligned}$$

$$\begin{aligned}
A_{48} &= A_{33} + A_{34} - A_{35} - A_{36} - A_{37} + A_{38} \\
&\quad + A_{39} - A_{40} + A_{41} + A_{42} - A_{43} - A_{44} + A_{45} - A_{46} - A_{47}, \\
D_1 &= \text{Pr}^2 \text{Re}^2 - \text{Pr Re}^2, \quad D_2 = 4 \text{Pr}^2 \text{Re}^2 - 2 \text{Pr Re}^2, \quad D_3 = (\lambda^2 + \lambda \text{Pr Re} - \pi^2)(\lambda - \pi), \\
D_4 &= \text{Pr}^2 + 2\lambda \text{Re}^2 - \text{Pr Re}, \quad D_5 = (\pi + \text{Pr Re})^2 - \text{Re}(\pi + \text{Pr Re}) - \pi^2, \\
D_6 &= (\lambda - \pi)\{(\lambda + 2 \text{Pr Re})^2 - \text{Pr Re}(\lambda + 2 \text{Pr Re}) - \pi^2\}, \\
D_7 &= (\lambda - \pi)\{(\lambda + 2 \text{Re})^2 - \text{Pr Re}(\lambda + 2 \text{Re}) - \pi^2\}, \\
D_8 &= (\lambda - \pi)\{(\lambda + \text{Re} + \text{Pr Re})^2 - \text{Pr Re}(\lambda + \text{Re} + \text{Pr Re}) - \pi^2\}, \\
D_9 &= (\lambda + \text{Pr Re})^2 - \text{Pr Re}(\lambda + \text{Pr Re}) - \pi^2, \\
D_{10} &= (\pi + 2 \text{Pr Re})^2 - \text{Pr Re}(\pi + 2 \text{Pr Re}) - \pi^2, \\
D_{11} &= (\pi + 2 \text{Re})^2 - \text{Pr Re}(\pi + 2 \text{Re}) - \pi^2, \\
D_{12} &= (\pi + \text{Re} + \text{Pr Re}) - \text{Pr Re}(\pi + \text{Re} + \text{Pr Re}) - \pi^2, \\
D_{13} &= (\lambda + \text{Pr} + \text{Pr Re}) - \text{Pr Re}(\lambda + \text{Pr} + \text{Pr Re}) - \pi^2, \\
D_{14} &= (m + \text{Re}) - \text{Pr Re}(m + \text{Re}) - \pi^2, \quad D_{15} = (\lambda + \text{Re} + \text{Pr}) - \text{Pr Re}(\lambda + \text{Re} + \text{Pr}) - \pi^2, \\
D_{16} &= (\lambda + \text{Re}) - \text{Pr Re}(\lambda + \text{Re}) - \pi^2, \quad D_{17} = (\lambda + 2 \text{Re}) - \text{Pr Re}(\lambda + 2 \text{Re}) - \pi^2, \\
D_{18} &= (\lambda + 2 \text{Pr Re}) - \text{Pr Re}(\lambda + 2 \text{Pr Re}) - \pi^2, \\
D_{19} &= (\lambda + \text{Re} + \text{Pr Re}) - \text{Re}(\lambda + \text{Re} + \text{Pr Re}) - \pi^2, \\
D_{20} &= (\lambda + \text{Pr Re}) - \text{Re}(\lambda + \text{Re}) - \pi^2, \\
D_{21} &= (\pi + 2 \text{Re}) - \text{Re}(\pi + 2 \text{Re}) - \pi^2, \\
D_{22} &= (\pi + \text{Re} + \text{Pr Re}) - \text{Re}(\pi + \text{Re} + \text{Pr Re}) - \pi^2, \\
D_{23} &= (\pi + \text{Pr Re}) - \text{Re}(\pi + \text{Pr Re}) - \pi^2, \quad D_{24} = (m + \text{Pr Re}) - \text{Re}(m + \text{Pr Re}) - \pi^2, \\
D_{25} &= (m + \text{Re}) - \text{Re}(m + \text{Re}) - \pi^2, \quad D_{26} = (\lambda + \text{Pr} + \text{Pr Re}) - \text{Re}(\lambda + \text{Pr} + \text{Pr Re}) - \pi^2, \\
D_{27} &= (\lambda + \text{Pr} + \text{Re}) - \text{Re}(\lambda + \text{Pr} + \text{Re}) - \pi^2, \quad D_{28} = (\lambda + \text{Re}) - \text{Re}(\lambda + \text{Re}) - \pi^2, \\
D_{29} &= \left\{ \pi + 2 \text{Pr Re} - \text{Re}(\pi + 2 \text{Pr Re}) - \pi^2 \right\} (\lambda - \pi), \quad m = \frac{\text{Pr Re} + \sqrt{\text{Pr}^2 \text{Re}^2 + 4\pi^2}}{2}.
\end{aligned}$$

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