

Galactic Motion Relative to CMBR in Two-Fluid FRW Models

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Abstract: In this paper we investigate two-fluid, zero curvature, tilted FRW models. In these models matter content of the universe is represented by a perfect fluid and is comoving with respect to the space-time coordinates. The other fluid is a viscous fluid which represents tilted cosmic microwave background radiation (CMBR). Models are presented describing the cosmic picture before and after the recombination epoch. Apart from presenting physically sound two-fluid models, our investigation aims to model the observed dipole anisotropy of CMBR as uniform streaming motion of the entire galactic content of the observable universe.

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1. Introduction

In the hot big-bang model tracing the entire cosmic history dating back to inflation as a two-fluid description where one fluid represents the matter content and another models the radiation distribution, is fairly accurate as compared to a sequence of two single fluid regimes. In the present epoch the observable universe is composed of highly isotropic background radiation in the microwave region (CMBR) and the matter in the form of galactic concentrations obeying the Hubble flow. The dipole anisotropy¹ of the CMBR is interpreted as a result of the motion of the local group relative to the Hubble flow^{2,3}. An alternative interpretation is that relative to the rest frame of CMBR entire galactic content of the observable universe has a uniform streaming velocity implying that the dipole anisotropy is cosmological in origin^{4,5,6}. Such an interpretation gives rise to a tilted universe. Since the universe at large is spatially homogeneous and isotropic, the background geometry is close to that of a Robertson-Walker space-time with zero or constant (positive or negative) spatial curvature. It is generally argued that a maximally symmetric (isotropic and homogeneous) space-time, such as Robertson-Walker space-time, is consistently manifested only by a maximally symmetric energy-momentum tensor. On the contrary a critical examination of Einstein field equations along with Bianchi identities (conservation laws) without any precondition reveals that even a non maximally symmetric

energy-momentum tensor can give rise to a maximally symmetric space-time. In fact, it has been shown that FRW cosmologies do not necessarily represent perfect fluid solutions but can also be exact solutions of the field equations for a viscous fluid in a frame non comoving to the coordinates of the R-W metric^{7,8}.

Our object is to present two-fluid models with the background geometry that of R-W space-time. In these models we assume that matter content is a perfect fluid comoving with the space-time coordinates. CMBR is modeled as a viscous fluid moving axially relative to the matter content. It may be mentioned here that in two-fluid FRW models⁸ CMBR is comoving with the space-time coordinates whereas the matter distribution is taken to be tilted. We follow the opposite track based on two counts. Matter is presently dominant cosmic component distributed in an isotropic and homogeneous manner so as to fit in the FRW frame work. Secondly tiny anisotropies are observed everywhere in CMBR which makes an adequate case for the CMBR to be represented by a viscous fluid.

2. Field Equations

The zero curvature Robertson-Walker line elements in the axial coordinates is given by

$$(2.1) \quad ds^2 = c^2 dt^2 - R^2(t)(dx^2 + dy^2 + dz^2),$$

where coordinates are comoving with the matter that is, the four-velocity of the matter is given by $v_i = (1, 0, 0, 0)$. Equations of general relativity are

$$(2.2) \quad R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -\frac{8\pi G}{c^4}T_{ij},$$

where

$$(2.3) \quad T_{ij} = (\rho_m + p_m)v_i v_j - p_m g_{ij} + \frac{p_r}{3}(4u_i u_j - g_{ij}) - 2\eta\sigma_{ij} + q_i u_j + q_j u_i.$$

Here the non tensor suffixes m and r denote fluid parameters of matter and radiation respectively. u_i is the time-like unit four-velocity of tilted CMBR and has a non-zero spatial component

$$(2.4) \quad u_i = (\alpha, 0, 0, -\beta R), \quad \alpha^2 - \beta^2 = 1.$$

βR accounts for the deviation of CMBR from the Hubble flow. The physical and kinematical quantities associated with the model are functions of t alone. η is the shear viscosity coefficient of CMBR and σ_{ij} the shear tensor:

$$(2.5) \quad \sigma_{ij} = \frac{1}{2}[u_{i;\gamma}H_j^\gamma + u_{j;\gamma}H_i^\gamma] - \frac{1}{3}u_{;\gamma}^\gamma H_{ij}, \quad H_{ij} = u_i u_j - g_{ij}.$$

The heat conduction vector q_i satisfies the condition

$$(2.6) \quad q_i u^i = 0.$$

which, in view of (2.4), gives

$$(2.7) \quad q_i = Q(-\beta, 0, 0, \alpha R), \quad Q^2 = q_i q^i.$$

The independent field equations of general relativity are

$$(2.8) \quad \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}^2}{R^2} - \wedge = -\frac{8\pi\pi}{c^2} \left(c^2 \frac{\rho_r}{3} + p_m + \frac{2\eta\beta\dot{\beta}}{3c\sqrt{1+\beta^2}} \right),$$

$$(2.9) \quad \frac{-3\ddot{R}}{R^2} + \wedge = -\frac{8\pi G}{c^2} \left(c^2 \frac{\rho_r}{3} (3+4\beta^2) + c^2 \rho_m + \frac{4\eta\beta\dot{\beta}(3+2\beta^2)}{3c\sqrt{1+\beta^2}} \right),$$

$$(2.10) \quad \rho_r = -\frac{\eta\dot{\beta}(3+4\beta^2)}{2c^3\beta\sqrt{1+\beta^2}}.$$

Thus, we are left with a set of three independent equations in variables $\rho_r, \rho_m, p_m, \beta$ and η along with the scale factor R . Thus to integrate the field equations into a determinate solution we require three additional relations. One of these is the equation of state of matter for which we shall assume the γ -law

$$(2.11) \quad p_m = (\gamma - 1)\rho_m c^2.$$

Apart from the equation of state we shall make assumptions for either two of R, β and η . The resulting models are presented in the following section.

3. Cosmological Models with Interacting Radiation and Matter

Model (i) Here we assume the power law form for R and η

$$(3.1) \quad \text{a) } R = t^m, \quad \text{b) } \eta = t^{-n},$$

where m and n are positive numbers. From (2.8), (2.9), (2.10), (2.11) and (3.1), we get a differential equation in β

$$(3.2) \quad \frac{\eta(4-3\gamma)\dot{\beta}}{2c\beta\sqrt{1+\beta^2}} - \frac{c^2}{8\pi G t^2} (3\gamma m^2 - 2m - \wedge t^2) = 0,$$

provided $\gamma \neq 4/3$. In view of (3.1b), the equation (3.2) integrates into

$$(3.3) \quad \frac{(4-3\gamma)}{2} \ln \frac{\beta}{1+\sqrt{1+\beta^2}} - \frac{c^2 m(3\gamma m - 2)t^{n-1}}{8\pi G(n-1)} + \frac{c^2 \wedge t^{n+1}}{8\pi G(n+1)} - \left(\frac{4-3\gamma}{2}\right) \ln a = 0,$$

where 'a' is the constant of integration. Solving for β , we get

$$(3.4) \quad \beta = \frac{2a \exp \frac{c^2}{(4-3\gamma)} \left[\frac{(3\gamma m - 2)m t^{n-1}}{4\pi G(n-1)} - \frac{\wedge t^{n+1}}{4\pi G(n+1)} \right]}{1 - a^2 \exp \frac{c^2}{(4-3\gamma)} \left[\frac{(3\gamma m - 2)m t^{n-1}}{2\pi G(n-1)} - \frac{\wedge t^{n+1}}{2\pi G(n+1)} \right]}.$$

We note that $\beta > 0$ for all t and decreases gradually from a finite value at $t = 0$ to zero as $t \rightarrow \infty$, provided $m < 2/3\gamma$, $n > 1$, $0 < a < 1$. We further note that $\beta R \rightarrow 0$ as t approaches 0 and ∞ . Thus, if βR increases from zero to its maximum, then tapers off to zero as $t \rightarrow \infty$. The kinematical behavior of the model is reasonable. Since initially photons and matter were in a bound state, there could be no relative motion.

From (2.7), (2.8), (2.9), (2.10), (2.11), (3.1) and (3.4), the physical parameters are given by

$$(3.5) \quad \begin{aligned} \text{a) } \rho_m &= \frac{6m(2m-1) - 4\Lambda t^2}{8\pi G(4-3\gamma)t^2}, & \text{b) } \rho_r &= \frac{[2m - 3\gamma m^2 + \Lambda \gamma t^2](3 + 3b^4 + 10b^2)}{8\pi G(4-3\gamma)t^2(1-b^2)^2}, \\ \text{c) } Q &= \frac{c^2}{\pi G t^2} \left(\frac{2m - 3\gamma m^2 + \Lambda \gamma t^2}{(4-3\gamma)} \right) \frac{b(1+b^2)}{(1-b^2)^2}, \end{aligned}$$

where

$$(3.6) \quad b \equiv a \exp \frac{c^2}{(4-3\gamma)4\pi G} \left[\frac{(3\gamma m^2 - 2m)t^{n-1}}{(n-1)} - \frac{\Lambda \gamma t^{n+1}}{(n+1)} \right].$$

As a particular case of this model, we assume $\Lambda = 0$. Equation (3.4) assumes a simple form

$$(3.7) \quad \beta = \frac{2ae^{b_1 t^{n-1}}}{1 - a^2 e^{2b_1 t^{n-1}}}, \quad b_1 \equiv -\frac{c^2(2-3\gamma m)m}{4\pi G(n-1)(4-3\gamma)}.$$

From (2.7), (2.8), (2.9), (2.10), (2.11), (3.1) and (3.7), we obtain

$$(3.8) \quad \begin{aligned} \text{a) } \rho_m &= \frac{3m(2m-1)}{4\pi G(4-3\gamma)t^2}, & \text{b) } \rho_r &= \frac{(2m-3\gamma m^2) \left[3 + 3a^4 e^{4b_1 t^{n-1}} + 10a^2 e^{2b_1 t^{n-1}} \right]}{8\pi G(4-3\gamma)t^2(1-a^2 e^{2b_1 t^{n-1}})^2}, \\ \text{c) } \beta R &= \frac{2at^m e^{b_1 t^{n-1}}}{1 - a^2 e^{2b_1 t^{n-1}}}, & \text{d) } Q &= \frac{ac^2(2m-3\gamma m^2)}{\pi G t^2(4-3\gamma)} e^{b_1 t^{n-1}} \frac{(1 + a^2 e^{2b_1 t^{n-1}})}{(1 - a^2 e^{2b_1 t^{n-1}})^2}. \end{aligned}$$

We note that $\rho_m \geq 0$ for $m \geq 1/2$; $\rho_r > 0$ for $m < 2/3\gamma$. ρ_r , Q and η are always monotonically decreasing function of time, whereas ρ_m / ρ_r is monotonically increasing.

Model (ii) Here we assume power law for R and β

$$(3.9) \quad R = t^m, \quad \beta = \beta_0 R_0^n R^{-n}.$$

The above assumptions imply that we are not only assuming the scale factor $R(t)$ but also βR , the deviation of radiation from the Hubble flow. From equations (2.7), (2.8), (2.9), (2.10), (2.11) and (3.9), we obtain

$$(3.10) \quad \rho_m = \frac{6m(2m-1) - 4\Lambda t^2}{8\pi G(4-3\gamma)t^2},$$

$$(3.11) \quad \rho_r = \frac{(2m - 3\gamma m^2 + \wedge \gamma t^2)(3t^{2mn} + 4\beta_0^2 R_0^{2n})}{8\pi G(4 - 3\gamma)t^{2mn+2}},$$

$$(3.12) \quad \beta R = \beta_0 t_0^{mn} t^{m(1-n)},$$

$$(3.13) \quad Q = \frac{c^2 \beta_0 R_0^n (2m - 3\gamma m^2 + \wedge \gamma t^2) \sqrt{t^{2mn} + \beta_0^2 R_0^{2n}}}{2\pi G(4 - 3\gamma)mn t^{2mn+2}},$$

$$(3.14) \quad \eta = \frac{c^2 (2m - 3\gamma m^2 + \wedge \gamma t^2) \sqrt{t^{2mn} + \beta_0^2 R_0^{2n}}}{4\pi G(4 - 3\gamma)mn t^{mn+1}},$$

$$(3.15) \quad \Omega_m = \frac{6m(2m-1) - 4\wedge \gamma t^2}{3m^2(4-3\gamma)},$$

$$(3.16) \quad \Omega_r = \frac{(2m - 3\gamma m^2 + \wedge \gamma t^2)(3t^{2mn} + 4\beta_0^2 R_0^{2n})}{3m^2(4 - 3\gamma)t^{2mn}},$$

provided $\gamma \neq 4/3$. From (3.10) and (3.11) the following realistic conditions surface:

$$(3.17) \quad m > \frac{1}{4} \left(1 + \sqrt{\frac{16}{3} \wedge t_0^2} \right), \quad \rho_{r0} \geq \frac{3m(2-3\gamma m) + 3\wedge \gamma t_0^2}{8\pi G(4-3\gamma)t_0^2}.$$

Thus our model forces a lower limit on ρ_{r0} which corresponds to the universe in which CMBR obeys the Hubble flow. In others words an increase in the deviation of CMBR from Hubble flow results into a corresponding increase in ρ_{r0} . Model parameters are γ , m and n . These are to be determined by the cosmological parameters t_0 , ρ_{r0} , $\beta_0 R_0$, R_0 and η_0 . In the absence of $\wedge$, model parameters are related to the cosmological parameters by

$$\frac{2m-1}{2-3\gamma m} = \frac{(3+4\beta_0^2)\rho_{m0}}{6\rho_{r0}}, \quad 3\gamma = 4 - \frac{3m(2m-1)}{4\pi G\rho_{m0}t_0^2}, \quad n = \frac{c^2(2-3\gamma m)}{4\pi G(4-3\gamma)\eta_0 t_0} \sqrt{1+\beta_0^2}.$$

We note that for a physically meaningful model, we get $1/2 < m < 2/3\gamma$. We further note that ρ_m is a monotonically decreasing function of time. For $n > 0$, ρ_r and ρ_m/ρ_r display a reasonable behavior as these monotonically decrease and increase respectively. On the other hand βR increases monotonically for $0 < n < 1$, remains constant for $n = 1$ and decreases monotonically for $n > 1$.

4. Cosmological Models with Non- interacting Radiation and Matter

The matter distribution is assumed to be dust, i.e.

$$(4.1) \quad p_m = 0.$$

The two-fluids are non-interacting which implies that⁹

$$(4.2) \quad \rho_m = \rho_{m0} \left(\frac{R_0}{R} \right)^3.$$

From the equation (2.8), (2.9), (2.10) and (4.1), we obtain a second order non linear differential equation in R only⁹

$$(4.3) \quad \ddot{R} R^2 + \dot{R}^2 R - \frac{4\pi G \rho_{m0} R_0^3}{3} - \frac{2}{3} \Lambda R^2 = 0.$$

The first integral is

$$(4.4) \quad \dot{R}^2 = \frac{\Lambda}{3} R^2 + \frac{8\pi G}{3R} \rho_{m0} R_0^3 + \frac{b}{R^2},$$

b being constant of integration. In the absence of Λ its solution is given by¹⁰

$$(4.5) \quad t - t_1 = \frac{1}{\sqrt{6\pi G \rho_{m0} R_0^3}} \left(R + \frac{3b}{8\pi G \rho_{m0} R_0^3} \right)^{1/2} \left(R - \frac{3b}{4\pi G \rho_{m0} R_0^3} \right),$$

where t_1 is a constant of integration to be evaluated by the initial condition, i.e. $R = 0$ at $t=0$:

$$(4.6) \quad t_1 = \frac{3b^{3/2}}{(4\pi G \rho_{m0} R_0^3)^{3/2}}.$$

b is evaluated by subjecting (4.4) to the conditions at the present epoch:

$$(4.7) \quad b = R_0^4 \left[H_0^2 - \frac{8}{3} \pi G \rho_{m0} \right],$$

H_0 being the value of Hubble's parameters at the present epoch. This completes the evolution of R from (4.5) as a function of t . From equations (2.9), (2.10), (4.2) and (4.4), we obtain the following equation in β and η for $\Lambda = 0$ as

$$(4.8) \quad \frac{\eta \dot{\beta}}{\beta \sqrt{1 + \beta^2}} + \frac{b C^3}{4\pi G R^4} = 0.$$

Eliminating t between (4.4) and (4.8), we obtain

$$(4.9) \quad \frac{a_1 \eta d\beta}{\beta \sqrt{1 + \beta^2}} + \frac{dR}{2 R^3 \sqrt{R + a_2^2}} = 0,$$

with

$$(4.10) \quad a_1 = \frac{2\pi G}{a_2 b^{1/2} c^3}, \quad a_2 = \left(\frac{3b}{8\pi G \rho_{m0} R_0^3} \right)^{1/2}.$$

Some sort of physical input involving β and η is required, however, in the following lines we shall try to integrate (4.8) subject to various hypothetical assumptions involving β or η .

Model (iii) We assume

$$(4.11) \quad \frac{\beta}{\beta_0} = \left(\frac{R}{R_0} \right)^{-n}, \quad n > 0.$$

From the equation (2.7), (2.10), (4.8) and (4.11), we obtain

$$(4.12) \quad \begin{aligned} \text{a)} \quad \rho_r &= \frac{b}{8\pi G} \frac{(3R^{2n} + 4\beta_0^2 R_0^{2n})}{R^{2n+4}}, \\ \text{b)} \quad Q &= \frac{b\beta_0 R_0^n c^2 \sqrt{R^{2n} + \beta_0^2 R_0^{2n}}}{2\pi G R^{2n+4}}, \\ \text{c)} \quad \eta &= \frac{bc^3 \sqrt{3(R^{2n} + \beta_0^2 R_0^{2n})}}{4\pi G n R^{n+2} \sqrt{8\pi G \rho_{m0} R_0^3 R + 3b}}. \end{aligned}$$

We note that ρ_m, ρ_r, Q and η are always monotonically decreasing function of time, whereas ρ_m / ρ_r is monotonically increasing. From (4.11), it is evident that βR increases or decreases monotonically according as $0 < n < 1$ or $n > 1$ respectively and remains constant for $n = 1$.

Model(iv)

$$(4.13) \quad \beta = t^{-n}.$$

From the equations (2.7), (2.10), (4.8) and (4.13), we obtain

$$(4.14) \quad \rho_r = \frac{b}{8\pi G R^4} \frac{(3t^{2n} + 4)}{t^{2n}}, \quad Q = \frac{bc^2}{2\pi G R^4} \frac{\sqrt{1 + t^{2n}}}{t^{2n}}, \quad \eta = \frac{bc^3 t^{1-n} \sqrt{1 + t^{2n}}}{4\pi G n R^4}.$$

ρ_m, ρ_r, Q and η are always monotonically decreasing function of time, whereas ρ_m / ρ_r is monotonically increasing. βR increases or decreases monotonically according as $0 < n < 1$ or $n \geq 1$ respectively.

To study the properties of the model at various stages of evolution, we assume that at the present epoch the cosmological parameters are given by

$$R_0 = 1.229904 \times 10^{28} \text{ cm}, \quad \beta_0 R_0 = 6 \times 10^7 \text{ cm s}^{-1}, \quad H_0 = 1.782473425 \times 10^{-18} \text{ s}^{-1}, \\ \rho_{r0} = 4.47 \times 10^{-34} \text{ g cm}^{-3}, \quad \rho_{m0} = 5.681493957 \times 10^{-30} \text{ g cm}^{-3}.$$

In view of the equations (4.5), (4.7) and (4.13), these values give rise to $b = 5.719281489 \times 10^{72}$, $n = 1.484202062$ and $t_0 = 3.739832126 \times 10^{17} \text{ s}$. To study the properties of the model (iv) at various stages of evolution, we solve the equation (4.5) with the help of numerical methods. For the value of R calculated at corresponding epoch, the equation (4.5) is satisfied upto the sixth places of decimal only. We have worked out various cosmological properties of the corresponding universe in the age interval 10^{15} s to 10^{20} s (Table I). We observe that $\rho_m, \rho_r, \Omega_m, \beta R$, and Ω_t are monotonically decreasing function of time, whereas ρ_m / ρ_r is monotonically increasing.

Table I

Dust Model(iv): March of ρ_m , ρ_r , ρ_m/ρ_r , Ω_m , Ω_r and βR (the streaming velocity of the galactic matter relative to CMBR) in the time interval (10^{15} s– 10^{20} s), corresponding to $t_0 = 3.739832126 \times 10^{17}$ s, $\beta_0 R_0 = 6 \times 10^7$ cm s $^{-1}$, $\rho_{m0} = 5.681493957 \times 10^{-30}$ g cm $^{-3}$, $\rho_{r0} = 4.47 \times 10^{-34}$ g cm $^{-3}$, $b = 5.719281489 \times 10^{72}$ and $n = 1.484202063$.

S. No	t (s)	R (cm)	ρ_m (g cm $^{-3}$)	ρ_r (g cm $^{-3}$)	ρ_m/ρ_r	βR (cm s $^{-1}$)	Ω_m	Ω_r
1.	10^{15}	2.3780966 $\times 10^{26}$	7.859369424 $\times 10^{-25}$	3.197723978 $\times 10^{-27}$	245.7801073	7.641001619 $\times 10^9$	0.1967303686	0.80043243 $\times 10^{-3}$
2.	10^{16}	1.100616492 $\times 10^{27}$	7.928051081 $\times 10^{-27}$	6.97024936 $\times 10^{-30}$	1137.412834	1.159703485 $\times 10^9$	0.1973573488	0.1735142622 $\times 10^{-3}$
3.	10^{17}	5.105245411 $\times 10^{27}$	7.94376376 $\times 10^{-29}$	1.505656435 $\times 10^{-32}$	5275.947139	1.764107068 $\times 10^8$	0.1974934274	0.374327911 $\times 10^{-4}$
4.	$3.739832126 \times 10^{17}$	1.229904 $\times 10^{28}$	5.681493957 $\times 10^{-30}$	4.47 $\times 10^{-34}$	12710.27731	6×10^7	0.1975153209	0.1553981208 $\times 10^{-4}$
5.	5×10^{17}	1.354944928 $\times 10^{28}$	4.249239993 $\times 10^{-30}$	3.034630404 $\times 10^{-34}$	14002.49595	4.310642443 $\times 10^7$	0.197516756	0.141058249 $\times 10^{-4}$
6.	10^{18}	2.369301824 $\times 10^{28}$	7.94721613 $\times 10^{-31}$	3.24571829 $\times 10^{-35}$	24485.23076	2.684890902 $\times 10^7$	0.1975216573	0.8066971442 $\times 10^{-5}$
7.	10^{19}	1.099697384 $\times 10^{29}$	7.947977241 $\times 10^{-33}$	6.993581025 $\times 10^{-38}$	113646.7457	4.08674353 $\times 10^6$	0.1975291221	1.738097478 $\times 10^{-6}$
8.	10^{20}	5.104307855 $\times 10^{29}$	7.948141879 $\times 10^{-35}$	1.506762972 $\times 10^{-40}$	527497.8233	6.220696173 $\times 10^5$	0.1975304856	0.37446692 $\times 10^{-6}$

Model (v)

$$(4.15) \quad \eta = \frac{1}{R^4}.$$

Equation (4.8) is integrated into

$$(4.16) \quad \beta = \frac{2e^{-b_1 t}}{(1 - e^{-2b_1 t})}, \quad b_1 \equiv \frac{bc^3}{4\pi G},$$

the constant of integration being made to vanish. We note that for $u > 0$, β satisfies the boundary conditions $\beta \rightarrow \infty$ as $t \rightarrow 0$ and $\beta \rightarrow 0$ as $t \rightarrow \infty$. From (2.7), (2.10), (4.8) and (4.16), we get

$$(4.17) \quad \text{a) } \rho_r = \frac{b(3 + 3e^{-4b_1 t} + 10e^{-2b_1 t})}{8\pi G R^4 (1 - e^{-2b_1 t})^2}, \quad \text{b) } Q = \frac{bc^2 e^{-b_1 t} (1 + e^{-2b_1 t})}{\pi G R^4 (1 - e^{-2b_1 t})^2}.$$

ρ_m , ρ_r , Q and η are always monotonically decreasing function of time, whereas ρ_m / ρ_r is monotonically increasing. βR is indeterminate as $t \rightarrow 0$ and $t \rightarrow \infty$.

Model (vi) $\eta = \text{constant}$.

It is a reasonable assumption in the present epoch where R is large. Equation (4.9) is easily integrable in an epoch where η is constant. The resulting integral is

$$(4.18) \quad \ln \frac{\beta}{1 + \sqrt{1 + \beta^2}} - \frac{1}{a_1 \eta} \left[c_2 + \frac{1}{a_2 R} \sqrt{\frac{1}{R^2} + \frac{1}{a_2^2 R}} - \frac{3}{4a_2} \left(\left(\frac{1}{R} + \frac{1}{2a_2^2} \right) \sqrt{\frac{1}{R^2} + \frac{1}{a_2^2 R}} \right) + \frac{3}{16a_2^5} \ln \left(\frac{1}{R} + \frac{1}{2a_2^2} + \sqrt{\left(\frac{1}{R} + \frac{1}{2a_2^2} \right)^2 + \left(\frac{1}{R^2} + \frac{1}{a_2^2 R} \right)} \right) \right] = 0,$$

where c_2 is constant of integration. Solving for β we obtain

$$(4.19) \quad \beta = \frac{2e^w}{1 - e^{2w}},$$

$$(4.20) \quad w \equiv \frac{1}{a_1 \eta} \left[c_2 + \frac{1}{a_2 R} \sqrt{\frac{1}{R^2} + \frac{1}{a_2^2 R}} - \frac{3}{4a_2} \left(\left(\frac{1}{R} + \frac{1}{2a_2^2} \right) \sqrt{\frac{1}{R^2} + \frac{1}{a_2^2 R}} \right) + \frac{3}{16a_2^5} \ln \left(\frac{1}{R} + \frac{1}{2a_2^2} + \sqrt{\left(\frac{1}{R} + \frac{1}{2a_2^2} \right)^2 + \left(\frac{1}{R^2} + \frac{1}{a_2^2 R} \right)} \right) \right].$$

From the equations (2.7), (2.10), (4.8) and (4.19), one obtains

$$(4.21) \quad \text{a) } \rho_r = \frac{b(3 + 3e^{4w} + 10e^{2w})}{8\pi G R^4 (1 - e^{2w})^2}, \quad \text{b) } Q = \frac{bc^2 e^w (1 + e^{2w})}{\pi G R^4 (1 - e^{2w})^2}.$$

We put the equation (4.9) in the following form

$$(4.22) \quad a_1 \eta \frac{d\beta}{\beta \sqrt{1 + \beta^2}} + \frac{d\phi}{(\phi^2 - a_2^2)^3} = 0, \quad \phi \equiv \sqrt{R + a_2^2}.$$

Since ϕ is large, the last equation is simplified by retaining terms of first order of smallness in the second member as:

$$(4.23) \quad \frac{\eta a_1 d\beta}{\beta \sqrt{1+\beta^2}} + \frac{3}{16a_2^5} \left[\frac{1}{\varphi - a_2} + \frac{1}{\varphi + a_2} \right] d\varphi = 0.$$

The resulting integral is

$$(4.24) \quad \beta = \frac{2S}{1-S^2}, \quad S \equiv b_1 \left(\frac{\varphi + a_2}{\varphi - a_2} \right)^{\frac{3}{16\eta a_1 a_2^5}},$$

b_1 being constant of integration. From equations (2.7), (2.10), (4.8) and (4.24), we obtain

$$(4.25) \quad \text{a) } \rho_r = \frac{b(3+3S^4+10S^2)}{8\pi G(1-S^2)^2 R^4}, \quad \text{b) } Q = \frac{bc^2 S(1+S^2)}{\pi G(1-S^2)^2 R^4}.$$

We note that $\beta \rightarrow 0$ as $R \rightarrow \infty$ and β is indeterminate as $R \rightarrow 0$. $\rho_m, \rho_r, \beta R$ and Q are monotonically decreasing function of time. The ratio ρ_m / ρ_r is a monotonically increasing function of time.

5. Conclusion

We present tilted, two-fluid cosmic models where the matter field displays Hubble expansion and the radiation field is represented by a viscous fluid indicating a deviation from the Hubble flow. This deviation from the Hubble flow in FRW space-time models the observed dipole anisotropy in CMBR which is assumed to arise from the uniform streaming motion of observable galaxies relative to CMBR. Parameters appearing in the models are determined from the currently available data of cosmological parameters. In each of the models, obeying the γ -law equation of state, all the cosmological parameters behave reasonably. One of the models put a lower limit on the ρ_{r0} which corresponds to CMBR in Hubble expansion. The model gives rise to a critical value of ρ_{r0} which corresponds to uniform streaming motion throughout the cosmic evolution.

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