

Stochastic Analysis of a Two Non-Identical Unit Standby System where Standby Unit Stops Functioning without Failure

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(Received January 08, 2006)

Abstract: The paper deals with the stochastic analysis of a two non-identical unit standby system model. Initially one unit (p-unit) is operative and other (o-unit) is kept as standby. P-unit gets preference in operation as well as in repair over o-unit. Failure time distributions of both units are assumed to be exponential with parameters α_1 and α_2 respectively. It is also assumed that o-unit may stop functioning with constant failure rate d . The repair time distributions of both the units are taken to be general. Using the regenerative point technique various reliability characteristics of interest are obtained.

1. Introduction

Repairable redundant systems have attracted the attention of several researchers¹⁻³ working in the field of reliability theory, due to their frequent use in business and industrial systems, two unit standby system models daily studied by many researchers⁴⁻¹¹ using various concepts such as imperfect switch, degraded operating models, random shocks, allowed downtime, administrative delay etc. They have assumed that a unit stop functioning only when it is totally failed but in practice, situations may arise when the unit stops functioning without any failure as in case of charging, discharging of units in the system stop functioning when their capacity become less as compared to required level. These chargeable units used in the system as standby units, as inverter, or with electricity or UPS with electricity.

Keeping the above facts in view, we analyse a two non-identical units standby system model.

Initially, one unit (p-unit) is operative and other (o-unit) is kept as standby. The o-unit is a charging unit which can stop functioning without failure.

Using regenerative point technique, the following important measures of system effectiveness are obtained:

- (i) Reliability and mean time to system failure (MTSF).
- (ii) Pointwise and steady state availabilities of the system.
- (iii) Expected uptime of the system and expected busy period of the repairman during $(0,t)$.
- (iv) Expected number of repairs during $(0,t)$.
- (v) Expected charging period of the o-unit during $(0,t)$.
- (vi) Net expected profit incurred by the system during $(0,t)$ and is steady state.

2. System Description and Assumptions

- (i) System consists of two non-identical units priority (p) and ordinary (o). The p-unit gets preference in operation as well as in repair over o-unit.
- (ii) Initially system starts from state S_0 where p-unit is operative and o-unit is kept in standby.
- (iii) A single repairman always available with the system.
- (iv) Once the repairman starts the repairing, he will not leave the system till the repair of the failed units is completed.
- (v) A switching device used to put the p-unit into operation is perfect and instantaneous.
- (vi) Failure time distributions of both units are taken to be negative exponential with parameters α_1 and α_2 respectively, whereas repair time distributions are general.
- (vii) Each repaired unit works as good as new.

3. Notations and States of the System

- α_1, α_2 : Constant failure rate of p-unit and o-unit respectively.
- d : Constant discharging rate of the o-unit.
- c : Constant Charging rate of o-unit.
- $g_1(\cdot), G_1(\cdot)$: p.d.f. and c.d.f. of time to repair of p-unit.
- $g_2(\cdot), G_2(\cdot)$: p.d.f. and c.d.f. of time to repair of o-unit.
- $*, \sim$: Symbols for Laplace and Laplace-Stieltjes transforms, i.e.

$$q_{ij}^*(S) = \int e^{-st} q_{ij}(t) dt$$

$$\tilde{Q}_{ij}(S) = \int e^{-st} dQ_{ij}(t)$$

- $\odot, \otimes$: Symbols for ordinary | Stieltjes convolution, i.e.

$$A(t) \odot B(t) = \int_0^t A(u)B(t-u)du.$$

$$A(t) \otimes B(t) = \int_0^t B(t-u)dA(u).$$

To write the states of the system we define the following symbols:

- N_{10}, N_{20} : P and o-out in normal mode and operative.
 N_{2s} : O-unit is in normal mode and standby.
 F_{1r}, F_{1w} : P-unit in failure mode and under repair/waiting for repair.
 F_{2r}, F_{2w} : O-unit in failure mode and under repair/waiting for repair.
 F_{2c}, F_{2cw} : O-unit in failure mode and under charging/waiting for charging.

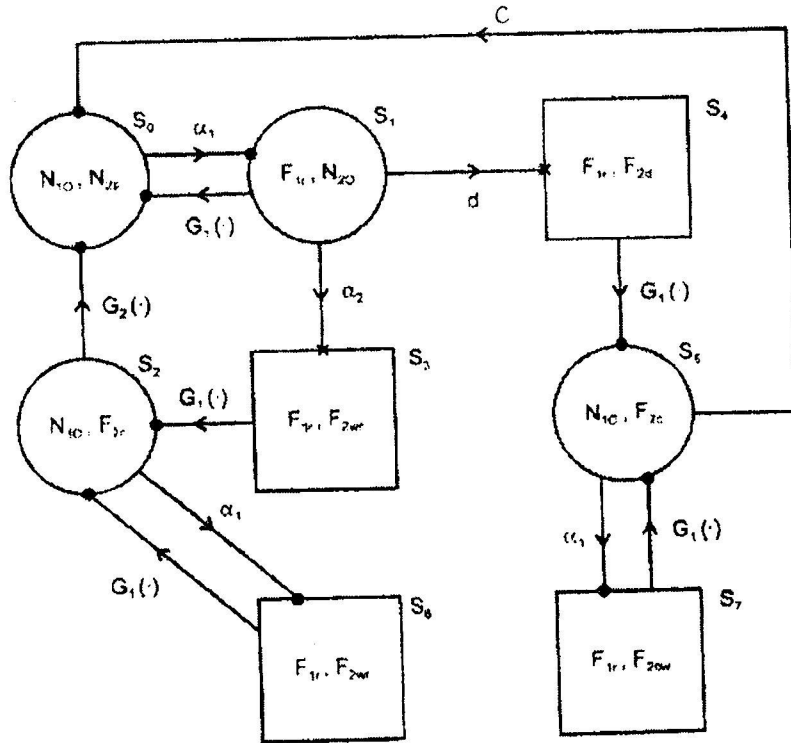


Fig -1

○: Up State

●: Regenerative State

□: Failed State

×: Non-regenerative State

Using the above symbols, the possible states of the system along with transitions of the systems model is shown in Fig. 1. From figure we observe that the epoch of transition from state S_1 to S_4 and S_1 to S_3 are non regenerative.

4. Transition Probabilities and Sojourn Times

The non-zero elements of transition probability matrix, $P = (p_{ij})$, are as follows:

$$p_{01} = 1, \quad p_{10} = \tilde{G}_1(\alpha + d)$$

$$p_{12}^{(3)} = \frac{\alpha_2}{\alpha_2 + d} [1 - \tilde{G}_2(\alpha_2 + d)], \quad p_{15}^{(4)} = \frac{d}{\alpha_2 + d} [1 - \tilde{G}_1(\alpha_2 + d)]$$

$$p_{26} = 1 - \tilde{G}_2(\alpha_1), \quad p_{20} = \tilde{G}_2(\alpha_1), \quad p_{32} = 1,$$

$$p_{45} = 1, \quad p_{50} = \frac{c}{\alpha_1 + c}, \quad p_{57} = \frac{\alpha_1}{\alpha_1 + c},$$

$$p_{62} = 1, \quad p_{75} = 1.$$

We observe the following relations:

$$(4.1) \quad p_{01} = p_{32} = p_{45} = p_{62} = p_{75} = 1,$$

$$(4.2) \quad p_{75} + p_{12}^{(3)} + p_{15}^{(4)} = 1,$$

$$(4.3) \quad p_{20} + p_{26} = 1,$$

$$(4.4) \quad p_{50} + p_{57} = 1,$$

Expected sojourn times in regenerative states $S_i \in E$ are

$$(4.5) \quad \Psi_0 = \int P(T_0 > t) dt = \int e^{-\alpha_1 t} dt = \frac{1}{\alpha_1}.$$

Similarly,

$$(4.6) \quad \Psi_1 = \frac{1 - \tilde{G}_1(\alpha_2 + d)}{\alpha_2 + d},$$

$$(4.7) \quad \Psi_2 = \frac{1 - \tilde{G}_2(\alpha_1)}{\alpha_1},$$

$$(4.8) \quad \Psi_3 = \int \tilde{G}_1(t) dt = \Psi_4,$$

$$(4.9) \quad \Psi_5 = \frac{1}{\alpha_1 + c},$$

$$(4.10) \quad \Psi_6 = \int \tilde{G}_2(t) dt = \Psi_7.$$

5. Reliability Analysis and MTSF

To find reliability of the system, we regard the failed states are absorbing. By simple probabilistic arguments, we have the following recurrence relations:

$$(5.1) \quad R_0(t) = Z_0(t) + q_{01}(t) \odot R_1(t),$$

$$(5.2) \quad R_1(t) = Z_1(t) + q_{10}(t) \odot R_0(t),$$

where

$$Z_0(t) = e^{-\alpha_1 t},$$

$$(5.3) \quad Z_1(t) = e^{-(\alpha_2 + d)t} \tilde{G}_1(t).$$

Taking Laplace transform of (5.1-5.2) and solving for $R_0^*(s)$, we have

$$(5.4) \quad R_0^*(s) = N_1(s) / D_1(s)$$

where

$$(5.5) \quad N_1(s) = Z_0^* - Z_1^* q_{01}^* \text{ and}$$

$$(5.6) \quad D_1(s) = 1 - q_{01}^* q_{10}^*.$$

For brevity, we have omitted the argument 's'.

Taking the inverse Laplace transform of (5.4), we get the expression for system reliability when initially it starts from S_0 .

The MTSF can be obtained by using the formula

$$(5.7) \quad E(T_0) = \int R_0(t) dt = \lim_{s \rightarrow 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)},$$

where

$$N_1(0) = \Psi_0 - \Psi_1 p_{01}$$

and

$$D_1(0) = 1 - p_{01} p_{10}.$$

6. Availability Analysis

Let $A_i(t)$ be the probability that the system is operative at time t when initially it starts from $S_i \in E$. We have the following relations $A_i(t)$'s, ($i=0, 1, 2, 5, 6$)

$$(6.1) \quad A_0(t) = Z_0(t) + q_{01}(t) \odot A_1(t)$$

$$(6.2) \quad A_1(t) = Z_1(t) + q_{10}(t) \odot A_0(t) + q_{12}^{(3)}(t) \odot A_2(t) + q_{15}^{(4)}(t) \odot A_5(t)$$

$$(6.3) \quad A_2(t) = Z_2(t) + q_{20}(t) \odot A_0(t) + q_{26}(t) \odot A_6(t)$$

$$(6.4) \quad A_5(t) = Z_5(t) + q_{50}(t) \odot A_0(t) + q_{57}(t) \odot A_7(t)$$

$$(6.5) \quad A_6(t) = q_{62}(t) \odot A_2(t)$$

$$(6.6) \quad A_7(t) = q_{75}(t) \odot A_5(t)$$

where

$$Z_0(t) = e^{-\alpha_1 t}, \quad Z_1(t) = e^{-(\alpha_2 + d)t} \bar{G}_1(t)$$

$$(6.7) \quad Z_2(t) = e^{-\alpha_1 t} \bar{G}_2(t), \quad Z_5(t) = e^{-(\alpha_1 + c)t}.$$

Taking the Laplace transform of equations (6.1) – (6.6) and solving for $A_0^*(s)$ and omitting the argument 's' for brevity, we obtain:

$$(6.8) \quad A_0^*(s) = \frac{N_2(s)}{D_2(s)}$$

where

$$(6.9) \quad \begin{aligned} N_2(s) = & q_{01}^* Z_1^* (1 - q_{57}^* q_{75}^*) - Z_1^* q_{01}^* q_{26}^* (q_{62}^* - q_{57}^* q_{62}^* q_{75}^*) \\ & + Z_2^* q_{01}^* q_{12}^{(3)*} (1 - q_{57}^* q_{75}^*) + Z_5^* q_{01}^* q_{15}^{(4)*} - Z_5^* q_{01}^* q_{15}^{(4)*} q_{26}^* q_{62}^* \\ & + Z_0^* - Z_0^* q_{57}^* q_{75}^* - Z_0^* q_{26}^* q_{62}^* (1 - q_{57}^* q_{75}^*) \end{aligned}$$

and

$$(6.10) \quad \begin{aligned} D_2(s) = & q_{01}^* q_{10}^* (1 - q_{57}^* q_{75}^*) + q_{01}^* q_{10}^* q_{26}^* q_{62}^* (1 - q_{57}^* q_{75}^*) \\ & - q_{01}^* q_{20}^* q_{12}^{(3)*} (1 - q_{57}^* q_{75}^*) - q_{01}^* q_{15}^{(4)*} q_{50}^* + q_{01}^* q_{15}^* q_{26}^* q_{50}^* q_{62}^* \\ & + (1 - q_{57}^* q_{75}^*) - q_{26}^* q_{62}^* (1 - q_{57}^* q_{75}^*). \end{aligned}$$

The steady state availability, when the system starts from S_i , is obtained as follows:

$$(6.11) \quad A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_2(0)}{D_2(0)},$$

where

$$N_2(0) = p_{20} p_{50} (\Psi_0 + \Psi_1) + \Psi_2 p_{12}^{(3)} p_{26} + \Psi_5 p_{15}^{(4)} p_{26},$$

and

$$D_2(0) = p_{20} p_{50} \Psi_0 + p_{01} p_{20} p_{50} \Psi_1 + p_{12}^{(3)} p_{50} \Psi_2 + p_{15}^{(4)} p_{50} \Psi_5 + p_{15}^{(4)} p_{20} p_{57} \Psi_7.$$

7. Busy Period Analysis

Let $B_i(t)$ be the probability that the repairman is busy in repair of failed p-unit when system initially starts from $S_i \in E$. Using simple probabilistic arguments, the value of $B_0(t)$ can easily be obtained in terms of its Laplace transform as follows:

$$(7.1) \quad B_0^*(s) = N_3(s) / D_2(s)$$

where

$$(7.2) \quad N_3(s) = \left[q_{01}^* (Z_1^* + q_{13}^* Z_3^* + q_{14}^* Z_4^*) (1 - q_{57}^* q_{75}^*) \right] \\ - \left[(Z_1^* + q_{13}^* Z_3^* + q_{14}^* Z_4^*) (q_{62}^* - q_{57}^* q_{62}^* q_{75}^*) q_{01}^* q_{26}^* \right] + \left[(Z_2^* q_{01}^* q_{12}^{(3)*}) (1 - q_{57}^* q_{75}^*) \right] \\ + Z_6^* q_{01}^* q_{12}^{(3)*} q_{26}^* (1 - q_{57}^* q_{75}^*) + q_{01}^* q_{13}^{(4)*} q_{75}^* Z_7^* + q_{01}^* q_{13}^{(4)*} (q_{15}^* q_{62}^* Z_7^*).$$

In the long run, the probability that the repairman is busy, is given by

$$(7.3) \quad B_0 = \frac{N_3(0)}{D_2(0)}$$

where

$$N_3(0) = (\Psi_1 + p_{13}\Psi_3 + p_{14}\Psi_4) p_{20} p_{50} + p_{12}^{(3)} p_{50} (\Psi_2 + p_{26}\Psi_6) \\ + p_{15}^{(4)} \Psi_7 (1 - p_{26} p_{57}).$$

The expected busy period of the repairman during $(0, t)$ is given by

$$\mu_b(t) = \int_0^t B_0(u) du.$$

So that

$$(7.4) \quad \mu_b^*(s) = B_0^*(s) / s.$$

8. Charging Period Analysis

Let $C_i(t)$ be the probabilities that the o-unit is under charging when the system initially starts from $S_i \in E$. Using simple probabilistic arguments the system integral equations for $C_i(t)$ ($i = 0, 1, 2, 5, 6, 7$) can easily be developed and by the technique of Laplace transform, the value of $C_0(t)$ in terms of their Laplace transform can be found as

$$(8.1) \quad C_0^*(s) = \frac{N_4(s)}{D_2(s)}$$

where

$$N_4(s) = Z_5^* q_{01}^* q_{12}^{(3)*} q_{26}^* (1 - q_{57}^* q_{75}^*),$$

The steady state probability C_0 that the o-unit is under charging is given by

$$(8.2) \quad C_0 = \frac{N_4(0)}{D_2'(0)},$$

where

$$N_4(0) = \Psi_5 p_{12}^{(3)} p_{26} p_{50}.$$

The expected charging period of the o-unit during $(0, t)$ is given by

$$\mu_c(t) = \int_0^t C_0(t) dt$$

So that,

$$(8.3) \quad \mu_c^*(s) = C_0^*(s) / s.$$

9. Profit Function Analysis

Now we find the profit function of the system by considering mean uptime of the system, expected busy period of the repairman and expected charging period o-unit.

$$\begin{aligned} P(t) &= \text{Expected total revenue in } (0, t) - \text{Expected total repair cost in } (0, t] \\ &\quad - \text{Expected cost of charging the o-unit in } (0, t] \end{aligned}$$

$$(9.1) \quad = K_0 \mu_{up}(t) - K_1 \mu_b(t) - K_2 \mu_c(t).$$

The expected profit per unit time in steady state is given by

$$(9.2) \quad P(t) = K_0 A_0 - K_1 B_0 - K_2 C_0$$

where K_0 is the revenue per unit uptime and K_1 and K_2 are the amounts paid to the repairman per unit time when he is busy in repairing the failed unit, per unit charging cost respectively.

10. Particular Case

When the repair time distribution is negative exponential, i.e.

$$(10.1) \quad \left[\begin{aligned} G_1(t) &= 1 - e^{-\beta_1(t)} \\ G_2(t) &= 1 - e^{-\beta_2(t)} \end{aligned} \right]$$

Then the charged transition probability and mean sojourn time are as follows:

$$\begin{aligned}
p_{10} &= \frac{\beta_1}{\beta_1 + \alpha_2 + d}, & p_{12}^{(3)} &= \frac{\alpha_2}{\alpha_2 + d + \beta_1}, & p_{15}^{(4)} &= \frac{d}{\alpha_2 + d + \beta_1} \\
p_{20} &= \frac{\beta_2}{\alpha_1 + \beta_2}, & p_{26} &= \frac{\alpha_1}{\alpha_1 + \beta_2}, & \Psi_1 &= \frac{1}{\beta_1 + \alpha_2 + d}, \\
\Psi_2 &= \frac{1}{\alpha_1 + \beta_1}, & \Psi_3 &= \beta_1, & \Psi_4 = \Psi_6 = \Psi_7 &= \frac{1}{\beta_1}
\end{aligned}$$

11. Graphical Study of the System Behaviour

For more concrete study of the system behaviour, we plot curves for mean time to system failure (MTSF) and profit function with respect to α_1 (failure rate) of p-unit for different values of $\beta_1 = (0.075, 0.050, 0.025)$. Keeping other parameters fixed, the curves so obtained are shown in Fig. 2 and 3 respectively.

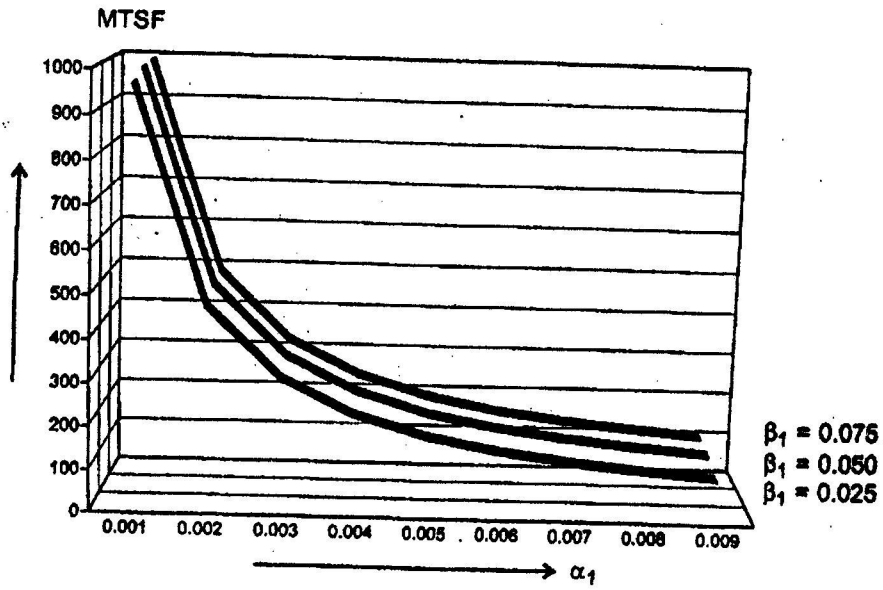


Fig 2: Behaviour of MTSF with respect to α_1

Fig. 2 shows that the variation in MTSF with respect to α_1 for different values of $\beta_1 = 0.025, 0.050$ and 0.075 . From the Figure, it is observed that initially MTSF decreases rapidly and tends to vanish as α_1 increases while it increases with the increase in β_1 .

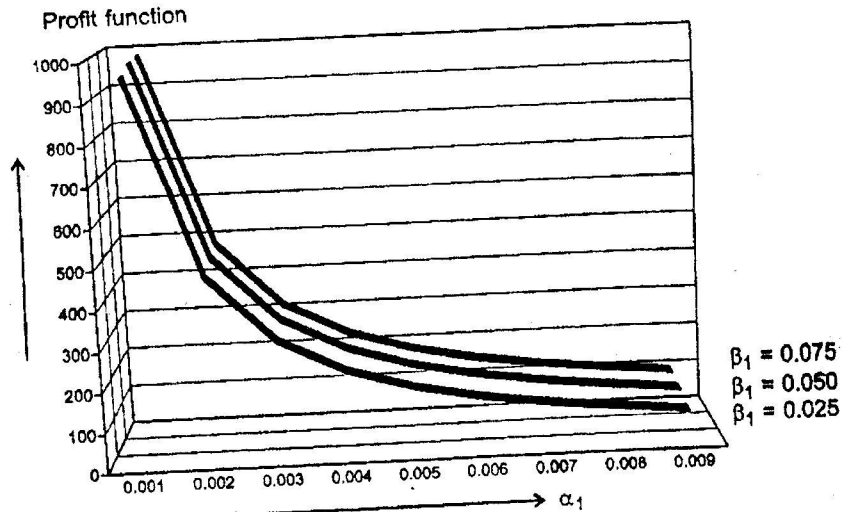


Fig 3: Behaviour of profit function with respect to α_1

From Fig. 3 it is observed that profit function decreases as α_1 increases when $\beta_1 = 0.025, 0.050$ and 0.075 and $K_0 = 1500, K_1 = 300, K_2 = 150$. From the graph, it is also observed that profit function is an increasing function of β_1 (repair rate).

Acknowledgement

One of the authors Mr. Dhanpal Singh is thankful to C.C.S. University, for providing financial assistance during this work. Authors are thankful to Prof. Rakesh Gupta, Department of Statistics, C.C.S. University, Meerut for having fruitful discussions during the preparation of this paper.

References

1. G.S. Agrafotis and M.Z. Tsoukalas, Reliability analysis and optimization applications of a two unit standby redundant system with spare units, *Microelectron, Reliab*, **34(5)** (1994) 1469-1476.
2. L.R. Goel and R. Gupta, Analysis of a two unit standby system with three modes and imperfect switching devices, *Microelectron, Reliab*, **2493** (1984) 425-429.
3. R. Gupta and A. Chaudhary, Stochastic analysis of priority unit standby system with repair machine failure, *Int. Jr. Sys. Sci.*, **26(12)** (1995) 2435-2440.
4. E. Balagurusamy, *Reliability Engineering*, (1984) Tata McGraw-Hill Publishing Co. Ltd., New Delhi.
5. D.K. Chow, Reliability of some redundant systems with repair, *IEEE Trans. Reliab.*, **R-22** (1973) 223-228.
6. B.S. Dhillon, *Human Reliability with human factors*, (1986), Proc. Press Inc., New York.

7. V. Goyal and K. Murari, Cost analysis of a two-unit standby system with two types of repairman, *Microelectron, Reliab.*, **24** (1984) 849-855.
8. R. Gupta, Analysis of a two unit cold standby system with degradation and linearly increasing failure rates, *Int. Jr. Sys. Sci.*, **26(12)** (1991) 2329-2338.
9. J.P.S. Joorel and P.D. Hyderi, Behaviour of a priority standby redundant system with different types of repair facilities, *Microelectron Reliab*, **35(5)** (1995) 833-836.
10. D. Pandey and B.S. Kalra, System with deteriorated performance and degraded failures under circumstantial and common cause failures, *International Journal of Mmanagement and Systems*, **19(2)** (2003).
11. N. Ravichandran, *Stochastic Method in Reliability Theory*. Wiley Eastern Limited, India.

