

Transforming Some Special Functions Using Integral Operators

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Abstract: The present paper deals with the study of the most general Lauricella functions involving two integral operators, namely Laplace transform and Weyl Fractional Integral transform. Theorems are established to show interesting association between the two transformations. Some known and new results are deduced to establish the applications of the results.

Keywords: Lauricella functions, Horn functions, Laplace transform, Weyl transform and Parseval-Goldstein theorem.

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1. Introduction

“Transformations” and “Representations” are important and ongoing topics of research in special function theory. Enumerable number of researchers have contributed on this topic. Recently, Jain and Pathan¹ have established interesting theorems to interconnect Laplace Transform and Weyl Transform of Kampé de Fériet functions. They have verified their results by specifying the parameters.

Let

$$(1.1) \quad F_{C:D^{\ast}, \dots, D^{(n)}}^{A:B^{\ast}, \dots, B^{(n)}} \left(\begin{matrix} z_1 \\ z_n \end{matrix} \right) = \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) \frac{z_1^{m_1}}{m_1!} \frac{z_2^{m_2}}{m_2!} \dots \frac{z_n^{m_n}}{m_n!}$$

where for convenience,

$$\Omega(m_1, m_2, \dots, m_n) = \frac{\prod_{j=1}^A (a_j)_{m_1 \delta_j^{\ast} + m_2 \delta_j^{\ast} + \dots + m_n \delta_j^{\ast}} \prod_{j=1}^{B^{\ast}} (b_j^{\ast})_{m_1 \delta_j^{\ast}} \dots \prod_{j=1}^{B^{(n)}} (b_j^{(n)})_{m_n \delta_j^{(n)}}}{\prod_{j=1}^C (c_j)_{m_1 \psi_j^{\ast} + m_2 \psi_j^{\ast} + \dots + m_n \psi_j^{\ast}} \prod_{j=1}^{D^{\ast}} (d_j^{\ast})_{m_1 \delta_j^{\ast}} \dots \prod_{j=1}^{D^{(n)}} (d_j^{(n)})_{m_n \delta_j^{(n)}}}$$

Laplace Transform

$$(1.2) \quad L[f(x); s] = \int_0^{\infty} e^{-sx} f(x) dx$$

and Weyl Transform

$$(1.3) \quad W^\mu[f(x); s] = \frac{1}{\Gamma\mu} \int_s^\infty (x-s)^{\mu-1} f(x) dx.$$

By specifying the parameters in (1.1) Kampé de Fériet, $H_A, H_B, H_C, H_1, H_2, H_3, H_4, H_5, H_6, H_7, G_A, G_B, G_C, G_1, G_2, G_3$ etc. can be deduced². In this paper we shall study the effect of the operators in (1.2), (1.3) on (1.1) and establish the co-relations between the two transformations. The main results follow from the theorems in section 2.

2. Co-relation Between Laplace Transform and Weyl Transform

Theorem 1: Let $\Omega(m_1, m_2, \dots, m_n)$ be a sequence of arbitrary complex numbers, $\text{Re}(s) > 0, \text{Re}(\mu) > 0$, and

$$(2.1) \quad h(x; z_1, z_2, \dots, z_n) = \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) \frac{z_1^{m_1}}{m_1!} \frac{z_2^{m_2}}{m_2!} \dots \frac{z_n^{m_n}}{m_n!} x^{m_n + \sigma - 1}$$

then Laplace Transform is

$$(2.2) \quad g(s; z_1, z_2, \dots, z_n) =$$

$$\frac{\Gamma\sigma}{s^\sigma} \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) (\sigma)_{m_n} \frac{z_1^{m_1}}{m_1!} \frac{z_2^{m_2}}{m_2!} \dots \frac{z_{n-1}^{m_{n-1}}}{m_{n-1}!} \left(\frac{z_n}{s}\right)^{m_n}$$

while Weyl Transform is

$$(2.3) \quad W^\mu[x^{-\lambda} g(x; z_1, z_2, \dots, z_n); s] = s^{\mu-\lambda-\sigma} \frac{\Gamma\sigma \Gamma(\lambda-\mu+\sigma)}{\Gamma(\lambda+\sigma)} \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) \times$$

$$\times \frac{(\sigma)_{m_n} (\lambda-\mu+\sigma)_{m_n}}{(\lambda+\sigma)_{m_n}} \frac{z_1^{m_1}}{m_1!} \frac{z_2^{m_2}}{m_2!} \dots \frac{z_{n-1}^{m_{n-1}}}{m_{n-1}!} \left(\frac{z_n}{s}\right)^{m_n}$$

Proof: Applying (1.2) and using the result³ (p.137) in (2.1), we get (2.2). Therefore

$$(2.4) \quad Lh(x; z_1, z_2, \dots, z_n) = g(s; z_1, z_2, \dots, z_n)$$

Again from³ (p.294), we have

$$(2.5) \quad L[x^{\mu-1} (x+a)^{-\lambda}; s] = \Gamma\mu \omega^{(\lambda-\mu-1)/2} a^{(\mu-\lambda-1)/2} e^{as/2} W_{k,m}(as) = \psi(as)$$

where $k = \frac{1-\mu-\lambda}{2}$, $m = \frac{\mu-\lambda}{2}$, $W_{k,m}$ is the usual Whittaker function and we have¹

$$(2.6) \quad L[(x-a)^{\mu-1} x^{-\lambda} H(x-a); s] = e^{-as} \psi(as)$$

where $H(x)$ is Heaviside's function.

Now using Parseval-Goldstien theorem⁴ in (2.4) and (2.6), we get

$$(2.7) \quad \int_0^{\infty} x^{-\lambda} (x-a)^{\mu-1} g(x; z_1, z_2, \dots, z_n) H(x-a) dx = \int_0^{\infty} x^{-\lambda} (x-a)^{\mu-1} g(x; z_1, z_2, \dots, z_n) dx \\ = \int_0^{\infty} e^{-ax} \psi(ax) h(x; z_1, z_2, \dots, z_n) dx$$

which on simplification, with the help of result³ (p.337) establishes (2.3).

General Theorem: Let $\Omega(m_1, m_2, \dots, m_n)$ be a sequence of arbitrary complex numbers, $\text{Re}(s) > 0$, $\text{Re}(\mu) > 0$, and

$$(2.8) \quad h_n(x; z_1, z_2, \dots, z_n)$$

$$= \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) \frac{z_1^{m_1}}{m_1!} \frac{z_2^{m_2}}{m_2!} \dots \frac{z_n^{m_n}}{m_n!} x^{a_1 m_1 + a_2 m_2 + \dots + a_n m_n + \sigma - 1}$$

then the Laplace Transform is

$$(2.9) \quad g_n(s; z_1, z_2, \dots, z_n) = \frac{\Gamma \sigma}{s^{\sigma}} \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) (\sigma)_{a_1 m_1 + \dots + a_n m_n} \times \\ \times \frac{\left(\frac{z_1}{s^{a_1}}\right)^{m_1}}{m_1!} \frac{\left(\frac{z_2}{s^{a_2}}\right)^{m_2}}{m_2!} \dots \frac{\left(\frac{z_n}{s^{a_n}}\right)^{m_n}}{m_n!},$$

while Weyl transform is

$$(2.10) \quad W^{\mu}[x^{-\lambda} g(x; z_1, z_2, \dots, z_n); s] = s^{\mu-\lambda-\sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu - \sigma)}{\Gamma(\lambda + \sigma)} \sum_{m_1, m_2, \dots, m_n=0}^{\infty} \Omega(m_1, m_2, \dots, m_n) \times \\ \times \frac{(\sigma)_{a_1 m_1 + \dots + a_n m_n} (\lambda - \mu + \sigma)_{a_1 m_1 + \dots + a_n m_n}}{(\lambda + \sigma)_{a_1 m_1 + \dots + a_n m_n}} \frac{\left(\frac{z_1}{s^{a_1}}\right)^{m_1}}{m_1!} \frac{\left(\frac{z_2}{s^{a_2}}\right)^{m_2}}{m_2!} \dots \frac{\left(\frac{z_n}{s^{a_n}}\right)^{m_n}}{m_n!}$$

Proof: Following the logic of inductions

$P(1) \forall k [P(k) \rightarrow P(k+1)] \Rightarrow P(n)$, where $P(n)$ is general theorem and using the proof of the theorem (1), we can establish the proof.

3. Applications

Case 1: If we take

$$n = 2, A = p, B' = q, B'' = k, C = 1, D' = m, D'' = n, m_1 = r, m_2 = t, \theta'_j = 1, \theta''_j = 1,$$

$$z_1 = w, z_2 = z, b'_j = b_j, \phi'_j = 1, a_1 = u_1, a_2 = u_2, b''_j = c_j, \phi''_j = 1,$$

$$c_j = \alpha_j, \psi'_j = 1, \psi''_j = 1, d'_j = \beta_j, \delta'_j = 1, d''_j = \gamma_j, \delta''_j = 1$$

in equation (2.8), (2.9) and (2.10) then we get

$$h(x; w, z) = \sum_{r,t=0}^{\infty} \frac{\prod_{j=1}^p (a_j)_{r+t} \prod_{j=1}^q (b_j)_r \prod_{j=1}^k (c_j)_t}{\prod_{j=1}^l (\alpha_j)_{r+t} \prod_{j=1}^m (\beta_j)_r \prod_{j=1}^n (\gamma_j)_t} \frac{w^r}{r!} \frac{z^t}{t!} x^{u_1 r + u_2 t + \sigma - 1}$$

then Laplace Transform is

$$g(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{\prod_{j=1}^p (a_j)_{r+t} \prod_{j=1}^q (b_j)_r \prod_{j=1}^k (c_j)_t}{\prod_{j=1}^l (\alpha_j)_{r+t} \prod_{j=1}^m (\beta_j)_r \prod_{j=1}^n (\gamma_j)_t} (\sigma)_{u_1 r + u_2 t} \left(\frac{w}{s^{u_1}} \right)^r \left(\frac{z}{s^{u_2}} \right)^t,$$

while Weyl Transform is

$$W^\mu [x^{-\lambda} g(x; w, z); s] = (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} \frac{\prod_{j=1}^p (a_j)_{r+t} \prod_{j=1}^q (b_j)_r \prod_{j=1}^k (c_j)_t}{\prod_{j=1}^l (\alpha_j)_{r+t} \prod_{j=1}^m (\beta_j)_r \prod_{j=1}^n (\gamma_j)_t} \times$$

$$\times \frac{(\sigma)_{u_1 r + u_2 t} (\lambda - \mu + \sigma)_{u_1 r + u_2 t}}{(\lambda + \sigma)_{u_1 r + u_2 t}} \left(\frac{w}{s^{u_1}} \right)^r \left(\frac{z}{s^{u_2}} \right)^t.$$

If we choose $u_1 = 0$ and $u_2 = 1$ then after some calculation we get the results of Pathan et. al¹.

Case 2: If we take

$$n = 3, A = 3, B' = 0, B'' = 0, B''' = 0, C = 1, D' = 1, D'' = 0, D''' = 0, m_1 = q, m_2 = r, m_3 = t,$$

$$a_1 = \beta, \theta'_1 = 1, \theta''_1 = 1, \theta'''_1 = 0, z_1 = v, z_2 = w, z_3 = z, a_2 = \beta', \theta'_2 = 0, \theta''_2 = 1, \theta'''_2 = 1,$$

$$a_3 = \alpha, \theta'_3 = 1, \theta''_3 = 1, \theta'''_3 = 0, c_1 = \gamma', \psi'_1 = 0, \psi''_1 = 1, \psi'''_1 = 1, d'_1 = \gamma, \delta'_1 = 1,$$

in equation (2.8), (2.9) and (2.10) then we get the transformations for H_A .

$$(3.1) \quad h_A(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1}$$

then Laplace Transform is

$$(3.2) \quad g_A(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_q (\gamma')_{r+t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

and Weyl Transform is

$$(3.3) \quad W^\mu [x^{-\lambda} g_A(x; v, w, z); s] = (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \times \\ \times \frac{(\sigma)_{a_1 q + a_2 r + a_3 t} (\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

Further putting $a_1 = 0$, $a_2 = 0$, $a_3 = 1$ in equation (3.1), (3.2) and in (3.3) then we get

$$h_A(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{t + \sigma - 1}$$

then Laplace Transform is

$$g_A(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_t}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{\left(\frac{z}{s}\right)^t}{t!}$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_A(x; w, z); s] = (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \times \\ \times \frac{(\sigma)_t (\lambda - \mu + \sigma)_t}{(\lambda + \sigma)_t} \frac{v^q}{q!} \frac{w^r}{r!} \frac{\left(\frac{z}{s}\right)^t}{t!}$$

Again putting $a_1 = 0$, $a_2 = 1$, $a_3 = 1$ in equations (3.1), (3.2) and (3.3), we get

$$h_A(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{r+t + \sigma - 1}$$

then Laplace Transform is

$$g_A(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \left(\frac{w}{s} \right)^r \left(\frac{z}{s} \right)^t$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_A(x; v, w, z); s] = (s)^{\mu-\lambda-\sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \times \\ \times \frac{(\sigma)_{r+t} (\lambda - \mu + \sigma)_{r+t}}{(\lambda + \sigma)_{r+t}} \frac{v^q}{q!} \left(\frac{w}{s} \right)^r \left(\frac{z}{s} \right)^t$$

Also putting $a_1 = 1, a_2 = 1, a_3 = 1$ in equations (3.1), (3.2) and (3.3), we get

$$h_A(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{q+r+t+\sigma-1}$$

then Laplace Transform is

$$g_A(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_{q+r+t}}{(\gamma)_q (\gamma')_{r+t}} \left(\frac{v}{s} \right)^q \left(\frac{w}{s} \right)^r \left(\frac{z}{s} \right)^t$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_A(x; v, w, z); s] = (s)^{\mu-\lambda-\sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_q (\gamma')_{r+t}} \times \\ \times \frac{(\sigma)_{q+r+t} (\lambda - \mu + \sigma)_{q+r+t}}{(\lambda + \sigma)_{q+r+t}} \left(\frac{v}{s} \right)^q \left(\frac{w}{s} \right)^r \left(\frac{z}{s} \right)^t.$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 3: If we take

$$n = 3, A = 3, B' = 0, B'' = 0, B''' = 0, C = 0, D' = 1, D'' = 1, D''' = 1, m_1 = q, m_2 = r, m_3 = t, \\ a_1 = \beta, \theta_1' = 1, \theta_1'' = 0, \theta_1''' = 0, z_1 = v, z_2 = w, z_3 = z, a_2 = \beta', \theta_2' = 0, \theta_2'' = 1, \theta_2''' = 1, \\ a_3 = \alpha, \theta_3' = 1, \theta_3'' = 1, \theta_3''' = 0, d_1' = \gamma_1, \delta_1' = 1, d_1'' = \gamma_2, \delta_1'' = 1, d_1''' = \gamma_3, \delta_1''' = 1$$

in equations (2.8), (2.9) and (2.10), we get the transformations for H_B

$$h_B(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma_1)_q (\gamma_2)_r (\gamma_3)_t} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1},$$

then Laplace Transform is

$$g_B(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma_1)_q (\gamma_2)_r (\gamma_3)_t} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_B(x; v, w, z); s] = s^{\mu-\lambda-\sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma_1)_q (\gamma_2)_r (\gamma_3)_t} \times \\ \times \frac{(\sigma)_{a_1 q + a_2 r + a_3 t} (\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}.$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 4: If we take

$$n = 3, A = 3, B' = 0, B'' = 0, B''' = 0, C = 1, D' = 0, D'' = 0, D''' = 0, m_1 = q, m_2 = r, m_3 = t, \\ a_1 = \beta, \theta_1' = 1, \theta_1'' = 1, \theta_1''' = 0, z_1 = v, z_2 = w, z_3 = z, a_2 = \beta', \theta_2' = 0, \theta_2'' = 1, \theta_2''' = 1, \\ a_3 = \alpha, \theta_3' = 1, \theta_3'' = 0, \theta_3''' = 1, c_1 = \gamma, \psi_1' = 1, \psi_1'' = 1, \psi_1''' = 1$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_C .

$$h_C(x; v, w, z) = \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_{q+r+t}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1},$$

then Laplace Transform is

$$g_C(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t} (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_{q+r+t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_C(x; v, w, z); s] = (s)^{\mu-\lambda-\sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q,r,t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r+t}}{(\gamma)_{q+r+t}} \times \\ \times \frac{(\sigma)_{a_1 q + a_2 r + a_3 t} (\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}.$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 5: If we take

$$\begin{aligned} n &= 2, A = 2, B' = 0, B'' = 1, C = 0, D' = 1, D'' = 0, m_1 = r, m_2 = t, \\ a_1 &= \alpha, \theta_1' = 1, \theta_1'' = -1, z_1 = w, z_2 = z, a_2 = \beta, \theta_2' = 1, \theta_2'' = 1, \\ b_1'' &= \gamma, \phi_1'' = 1, d_1' = \delta, \delta_1' = 1 \end{aligned}$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_1 ,

$$h_1(x; w, z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_{r+t}(\gamma)_t}{(\delta)_r} \frac{(w)^r}{r!} \frac{(z)^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then Laplace Transform is

$$g_1(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_{r+t}(\gamma)_t}{(\delta)_r} \frac{(\sigma)_{a_1 r + a_2 t}}{r!} \left(\frac{w}{s^{a_1}} \right)^r \left(\frac{z}{s^{a_2}} \right)^t$$

and Weyl's Transform is

$$\begin{aligned} W^\mu [x^{-\lambda} g_1(x; w, z); s] &= (s)^{\mu - \lambda - \sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_{r+t}(\gamma)_t}{(\delta)_r} \times \\ &\times \frac{(\sigma)_{a_1 r + a_2 t}(\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}} \right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}} \right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 6: If we take

$$\begin{aligned} n &= 2, A = 1, B' = 1, B'' = 2, C = 0, D' = 1, D'' = 0, m_1 = r, m_2 = t, \\ a_1 &= \alpha, \theta_1' = 1, \theta_1'' = -1, z_1 = w, z_2 = z, b_1' = \beta, \phi_1' = 1, b_1'' = \gamma, \\ \phi_1'' &= 1, b_2'' = \delta, \phi_2'' = 1, d_1' = \varepsilon, \delta_1' = 1 \end{aligned}$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_2

$$h_2(x; w, z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_r(\gamma)_t(\delta)_t}{(\varepsilon)_r} \frac{(w)^r}{r!} \frac{(z)^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_2(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_r(\gamma)_t(\delta)_t}{(\varepsilon)_r} \frac{(\sigma)_{a_1 r + a_2 t}}{r!} \left(\frac{w}{s^{a_1}} \right)^r \left(\frac{z}{s^{a_2}} \right)^t$$

and Weyl Transform is

$$W^\mu[x^{-\lambda}g_2(x;w,z);s] = (s)^{\mu-\lambda-\sigma} \frac{\Gamma(\sigma)\Gamma(\lambda-\mu+\sigma)}{\Gamma(\lambda+\sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{r-t}(\beta)_r(\gamma)_t(\delta)_t}{(\varepsilon)_r} \times \\ \times \frac{(\sigma)_{a_1r+a_2t}(\lambda-\mu+\sigma)_{a_1r+a_2t}}{(\lambda+\sigma)_{a_1r+a_2t}} \left(\frac{w}{s^{a_1}}\right)^r \left(\frac{z}{s^{a_2}}\right)^t.$$

Some more deductions are possible by setting a_1, a_2 .

Case 7: If we take

$$n=2, A=1, B'=0, B''=1, C=1, D'=0, D''=0, m_1=r, m_2=t, \\ a_1=\alpha, \theta_1'=2, \theta_1''=1, z_1=w, z_2=z, b_1''=\beta, \phi_1'=1, c_1=\gamma, \psi_1'=1, \psi_1''=1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_3

$$h_3(x;w,z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t}(\beta)_t}{(\gamma)_{r+t}} \frac{(w)^r}{r!} \frac{(z)^t}{t!} x^{a_1r+a_2t+\sigma-1},$$

and then the Laplace Transform is

$$g_3(s;w,z) = \frac{\Gamma\sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t}(\beta)_t(\sigma)_{a_1r+a_2t}}{(\gamma)_{r+t}} \left(\frac{w}{s^{a_1}}\right)^r \left(\frac{z}{s^{a_2}}\right)^t$$

and Weyl Transform is

$$W^\mu[x^{-\lambda}g_3(x;w,z);s] = s^{\mu-\lambda-\sigma} \frac{\Gamma\sigma\Gamma(\lambda-\mu+\sigma)}{\Gamma(\lambda+\sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t}(\beta)_t(\sigma)_{a_1r+a_2t}}{(\gamma)_{r+t}} \times \\ \times \frac{(\lambda-\mu+\sigma)_{a_1r+a_2t}}{(\lambda+\sigma)_{a_1r+a_2t}} \left(\frac{w}{s^{a_1}}\right)^r \left(\frac{z}{s^{a_2}}\right)^t.$$

Some more deductions are possible by setting a_1, a_2 .

Case 8: If we take

$$n=2, A=1, B'=0, B''=1, C=0, D'=1, D''=1, m_1=r, m_2=t, \\ a_1=\alpha, \theta_1'=2, \theta_1''=1, z_1=w, z_2=z, b_1''=\beta, \phi_1'=1, d_1'=\gamma, \delta_1'=1, d_1''=\delta, \delta_1''=1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_4

$$h_4(x;w,z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t}(\beta)_t}{(\gamma)_r(\delta)_t} \frac{(w)^r}{r!} \frac{(z)^t}{t!} x^{a_1r+a_2t+\sigma-1},$$

and then the Laplace Transform is

$$g_4(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t} (\beta)_t (\sigma)_{a_1 r + a_2 t}}{(\gamma)_r (\delta)_t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$\begin{aligned} W^\mu [x^{-\lambda} g_4(x; w, z); s] &= s^{\mu-\lambda-\sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t} (\beta)_t (\sigma)_{a_1 r + a_2 t}}{(\gamma)_r (\delta)_t} \times \\ &\times \frac{(\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 9: If we take

$$\begin{aligned} n &= 2, A = 2, B' = 0, B'' = 0, C = 0, D' = 0, D'' = 1, m_1 = r, m_2 = t, \\ a_1 &= \alpha, \theta_1' = 2, \theta_1'' = 1, z_1 = w, z_2 = z, a_2 = \beta, \theta_2' = -1, \theta_2'' = 1, d_1'' = \gamma, \delta_1'' = 1, \end{aligned}$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_5

$$h_5(x; w, z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t} (\beta)_{t-r}}{(\gamma)_t} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_5(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t} (\beta)_{t-r} (\sigma)_{a_1 r + a_2 t}}{(\gamma)_t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$\begin{aligned} W^\mu [x^{-\lambda} g_5(x; w, z); s] &= (s)^{\mu-\lambda-\sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r+t} (\beta)_{t-r} (\sigma)_{a_1 r + a_2 t}}{(\gamma)_t} \\ &\times \frac{(\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 10: If we take

$$n = 2, A = 2, B' = 0, B'' = 1, C = 0, D' = 0, D'' = 0, m_1 = r, m_2 = t,$$

$$a_1 = \alpha, \theta_1' = 2, \theta_1'' = -1, z_1 = w, z_2 = z, a_2 = \beta, \theta_2' = -1, \theta_2'' = 1, b_1'' = \gamma, \phi_1'' = 1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_6

$$h_6(x; w, z) = \sum_{r,t=0}^{\infty} (\alpha)_{2r-t} (\beta)_{t-r} (\gamma)_t \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_6(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} (\alpha)_{2r-t} (\beta)_{t-r} (\gamma)_t (\sigma)_{a_1 r + a_2 t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!},$$

and Weyl Transform is

$$\begin{aligned} W^\mu[x^{-\lambda} g_6(x; w, z); s] &= (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} (\alpha)_{2r-t} (\beta)_{t-r} (\gamma)_t \times \\ &\times \frac{(\sigma)_{a_1 r + a_2 t} (\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 11: If we take

$$n = 2, A = 1, B' = 0, B'' = 2, C = 0, D' = 1, D'' = 0, m_1 = r, m_2 = t,$$

$$a_1 = \alpha, \theta_1' = 2, \theta_1'' = -1, z_1 = w, z_2 = z, b_1'' = \beta, \phi_1'' = 1, b_2'' = \gamma, \phi_2'' = 1, d_1' = \delta, \delta_1' = 1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for H_7

$$h_7(x; w, z) = \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r-t} (\beta)_t (\gamma)_t}{(\delta)_r} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_7(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r-t} (\beta)_t (\gamma)_t (\sigma)_{a_1 r + a_2 t}}{(\delta)_r} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$W^\mu[x^{-\lambda} g_7(x; w, z); s] = (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} \frac{(\alpha)_{2r-t} (\beta)_t (\gamma)_t}{(\delta)_r} \times$$

$$\times \frac{(\sigma)_{a_1 r + a_2 t} (\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}.$$

Some more deductions are possible by setting a_1, a_2 .

Case 12: If we take

$$\begin{aligned} n &= 3, A = 2, B' = 0, B'' = 1, B''' = 0, C = 1, D' = 0, D'' = 0, D''' = 0, m_1 = q, m_2 = r, m_3 = t, \\ a_1 &= \alpha, \theta_1' = -1, \theta_1'' = 1, \theta_1''' = 1, z_1 = v, z_2 = w, z_3 = z, a_2 = \beta, \theta_2' = 1, \theta_2'' = 0, \theta_2''' = 1, \\ c_1 &= \gamma, \psi_1' = -1, \psi_1'' = 1, \psi_1''' = 1, b_1' = \beta', \phi_1'' = 1, \end{aligned}$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_A

$$h_A(x; v, w, z) = \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta)_{q+t} (\beta')_r}{(\gamma)_{r+t-q}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1},$$

and then the Laplace and Weyl Transforms are respectively

$$\begin{aligned} g_A(s; v, w, z) &= \frac{\Gamma \sigma}{s^\sigma} \sum_{r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta)_{q+t} (\beta')_r (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_{r+t-q}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}, \\ W^\mu [x^{-\lambda} g_A(x; v, w, z); s] &= (s)^{\mu - \lambda - \sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta)_{q+t} (\beta')_r}{(\gamma)_{r+t-q}} \times \\ &\quad \times \frac{(\sigma)_{a_1 q + a_2 r + a_3 t} (\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!} \end{aligned}$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 13: If we take

$$\begin{aligned} n &= 3, A = 1, B' = 1, B'' = 1, B''' = 1, C = 1, D' = 0, D'' = 0, D''' = 0, m_1 = q, m_2 = r, m_3 = t, \\ a_1 &= \alpha, \theta_1' = -1, \theta_1'' = 1, \theta_1''' = 1, z_1 = v, z_2 = w, z_3 = z, c_1 = \gamma, \\ \psi_1' &= -1, \psi_1'' = 1, \psi_1''' = 1, b_1' = \beta_1, \phi_1' = 1, b_1'' = \beta_2, \phi_1'' = 1, b_1''' = \beta_3, \phi_1''' = 1, \end{aligned}$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_B

$$h_B(x; v, w, z) = \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta_1)_q (\beta_2)_r (\beta_3)_t}{(\gamma)_{r+t-q}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1},$$

and then the Laplace and Weyl Transforms are respectively

$$g_B(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta_1)_q (\beta_2)_r (\beta_3)_t (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_{r+t-q}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

$$W^\mu [x^{-\lambda} g_B(x; v, w, z); s] = (s)^{\mu-\lambda-\sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{r+t-q} (\beta_1)_q (\beta_2)_r (\beta_3)_t}{(\gamma)_{r+t-q}} \times$$

$$\times \frac{(\sigma)_{a_1 q + a_2 r + a_3 t} (\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}.$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 14: If we take

$$n = 3, A = 3, B' = 0, B'' = 0, B''' = 0, C = 1, D' = 0, D'' = 0, D''' = 0, m_1 = q, m_2 = r, m_3 = t,$$

$$a_1 = \alpha, \theta_1' = 1, \theta_1'' = 0, \theta_1''' = 1, z_1 = v, z_2 = w, z_3 = z, a_2 = \beta, \theta_2' = 1, \theta_2'' = 1, \theta_2''' = 0,$$

$$a_3 = \beta', \theta_3' = 0, \theta_3'' = 1, \theta_3''' = -1, c_1 = \gamma, \psi_1' = 1, \psi_1'' = 1, \psi_1''' = -1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_C

$$h_C(x; v, w, z) = \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r-t} (\sigma)_{a_1 q + a_2 r + a_3 t + \sigma - 1}}{(\gamma)_{r+t-q}} \frac{v^q}{q!} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 q + a_2 r + a_3 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_C(s; v, w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r-t} (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_{r+t-q}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}$$

and Weyl Transform is

$$W^\mu [x^{-\lambda} g_C(x; v, w, z); s]$$

$$= (s)^{\mu-\lambda-\sigma} \frac{\Gamma(\sigma) \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{q, r, t=0}^{\infty} \frac{(\alpha)_{q+t} (\beta)_{q+r} (\beta')_{r-t} (\sigma)_{a_1 q + a_2 r + a_3 t}}{(\gamma)_{r+t-q}}$$

$$\frac{(\lambda - \mu + \sigma)_{a_1 q + a_2 r + a_3 t}}{(\lambda + \sigma)_{a_1 q + a_2 r + a_3 t}} \frac{\left(\frac{v}{s^{a_1}}\right)^q}{q!} \frac{\left(\frac{w}{s^{a_2}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_3}}\right)^t}{t!}.$$

Some more deductions are possible by setting a_1, a_2, a_3 .

Case 15: If we take

$$n = 2, A = 3, B' = 0, B'' = 0, C = 0, D' = 0, D'' = 0, m_1 = r, m_2 = t,$$

$$a_1 = \alpha, \theta_1' = 1, \theta_1'' = 1, z_1 = w, z_2 = z, a_2 = \beta, \theta_2' = -1, \theta_2'' = 1,$$

$$a_3 = \beta', \theta_3' = 1, \theta_3'' = -1, ,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_1

$$h_1(x; w, z) = \sum_{r,t=0}^{\infty} (\alpha)_{r+t} (\beta)_{t-r} (\beta')_{r-t} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_1(s; w, z) = \frac{\Gamma \sigma}{s^\sigma} \sum_{r,t=0}^{\infty} (\alpha)_{r+t} (\beta)_{t-r} (\beta')_{r-t} (\sigma)_{a_1 r + a_2 t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$\begin{aligned} W^\mu[x^{-\lambda} g_1(x; w, z); s] \\ = (s)^{\mu - \lambda - \sigma} \frac{\Gamma \sigma \Gamma(\lambda - \mu + \sigma)}{\Gamma(\lambda + \sigma)} \sum_{r,t=0}^{\infty} (\alpha)_{r+t} (\beta)_{t-r} (\beta')_{r-t} \\ \frac{(\sigma)_{a_1 r + a_2 t} (\lambda - \mu + \sigma)_{a_1 r + a_2 t}}{(\lambda + \sigma)_{a_1 r + a_2 t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 16: If we take

$$n = 2, A = 2, B' = 1, B'' = 1, C = 0, D' = 0, D'' = 0, m_1 = r, m_2 = t,$$

$$a_1 = \beta, \theta_1' = -1, \theta_1'' = 1, z_1 = w, z_2 = z, a_2 = \beta', \theta_2' = 1, \theta_2'' = -1,$$

$$b_1' = \alpha, \phi_1' = 1, b_1'' = \alpha', \phi_1'' = 1, ,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_2

$$h_2(x; w, z) = \sum_{r,t=0}^{\infty} (\alpha)_r (\alpha')_t (\beta)_{t-r} (\beta')_{r-t} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1 r + a_2 t + \sigma - 1},$$

and then the Laplace Transform is

$$g_2(s; w, z) = \frac{\Gamma\sigma}{s^\sigma} \sum_{r,t=0}^{\infty} (\alpha)_r (\alpha')_t (\beta)_{t-r} (\beta')_{r-t} (\sigma)_{a_1r+a_2t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$\begin{aligned} W^\mu[x^{-\lambda} g_2(x; w, z); s] \\ = (s)^{\mu-\lambda-\sigma} \frac{\Gamma\sigma \Gamma(\lambda-\mu+\sigma)}{\Gamma(\lambda+\sigma)} \sum_{r,t=0}^{\infty} (\alpha)_r (\alpha')_t (\beta)_{t-r} (\beta')_{r-t} \\ \frac{(\sigma)_{a_1r+a_2t} (\lambda-\mu+\sigma)_{a_1r+a_2t}}{(\lambda+\sigma)_{a_1r+a_2t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

Case 17: If we take

$$n = 2, A = 2, B' = 0, B'' = 0, C = 0, D' = 0, D'' = 0, m_1 = r, m_2 = t,$$

$$a_1 = \alpha, \theta_1' = -1, \theta_1'' = 2, z_1 = w, z_2 = z, a_2 = \alpha', \theta_2' = 2, \theta_2'' = -1,$$

in equations (2.8), (2.9) and (2.10) then we get the transformations for G_3

$$h_3(x; w, z) = \sum_{r,t=0}^{\infty} (\alpha)_{2t-r} (\alpha')_{2r-t} \frac{w^r}{r!} \frac{z^t}{t!} x^{a_1r+a_2t+\sigma-1},$$

and then the Laplace Transform is

$$g_3(s; w, z) = \frac{\Gamma\sigma}{s^\sigma} \sum_{r,t=0}^{\infty} (\alpha)_{2t-r} (\alpha')_{2r-t} (\sigma)_{a_1r+a_2t} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}$$

and Weyl Transform is

$$\begin{aligned} W^\mu[x^{-\lambda} g_3(x; w, z); s] \\ = s^{\mu-\lambda-\sigma} \frac{\Gamma\sigma \Gamma(\lambda-\mu+\sigma)}{\Gamma(\lambda+\sigma)} \sum_{r,t=0}^{\infty} (\alpha)_{2t-r} (\alpha')_{2r-t} \\ \frac{(\sigma)_{a_1r+a_2t} (\lambda-\mu+\sigma)_{a_1r+a_2t}}{(\lambda+\sigma)_{a_1r+a_2t}} \frac{\left(\frac{w}{s^{a_1}}\right)^r}{r!} \frac{\left(\frac{z}{s^{a_2}}\right)^t}{t!}. \end{aligned}$$

Some more deductions are possible by setting a_1, a_2 .

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