

A Probability Model For The Estimation of Level of Fecundability of Migrated Couples

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Abstract: In the recent years the growing interest of the demographers has been to study the fertility pattern of the migrated couples. In this connection the present work is devoted to develop a probability model for estimating the level of fecundability of migrated couples through closed birth interval. The parameters involved in the proposed model have been estimated. Suitability of the model has been tested through the observed set of data.

1. Introduction

There has been a rapid growth of interest in the study of impact of migration on fertility. This interest is seen widely in the developed countries as a large number of studies on migration and fertility have come out. In developing countries like India, such studies could not gain that wide interest. However, migration, specially in developing countries, plays a key role for changing the socio economic and cultural environments of the people living in rural areas and ultimately affects the fertility performance (Agarwala¹, Zachariah², Rele and Kanitkar³, Yadava⁴, Jolly⁵, Bhatia and Sabagh⁶, 1980; Singh et al⁷, Sharma⁸, Prasad⁹, Pathak¹⁰).

Human fertility is mainly governed by fecundability, which is defined as probability that a non-pregnant fecund woman will conceive in one menstrual cycle. age at marriage, rest period (the period in which no conception is possible) and other factors such as sterility (absence of fecundability), use of contraceptive devices, temporary withdrawal from cohabitation etc. These factors vary from female to female in a population. For any female, they may vary over time and be affected by age duration of marriage number of pregnancies or live births already experienced any such other maternal characteristics.

So far as temporary withdrawal from cohabitation is concerned, it occurs mainly due to physical separation of husband from wife. Migration of the male partner may be the one of the reasons of such separation. In this situation female gets less time of exposure for conception and hence, the rest period is affected. Due to this reason the birth interval (the interval between two consecutive births) is affected and fertility performance of the couple.

The effect of such separation on fertility depends, among other factors, on the proportion of wives who are thus separated from their husbands and the period of their separation. Without knowing the exact values of these two factors, Zachariah² estimated by using the 'general fertility rate' that on an average twelve percent births are averted by the separation on husband from wife in Greater Bombay under the assumptions that (i) the wife may be living in the village permanently and the husbands may visit her only once in a year (ii) merely fertility is changed by migration not fecundity.

In recent years increasing attention has been paid to develop models for social and demographic processes including couple fertility. The suggested models include for consideration the point events like conceptions or live births and other for the interval between consecutive events (conception, birth, etc.) as the variable of interest.

Yadava⁴ studied the fertility of the couples whose male partners were migrated without their wives by considering a fertility model under certain reasonable assumptions. By this model he was able to estimate the level of fecundability of migrated couples.

The objective of this paper is to develop a probability model for estimating the level of fecundability of migrated couples through closed birth interval in Section 2 probability model has been developed in Section 3 estimation procedure has been described for estimating the parameters involved in the model. In Section 4 suitability of the model has been tested through observed set of data.

However for completeness, description of procedure for estimation of fecundability is given in section 2

2. A Procedure For The Estimation Of Fecundability (A Model For Closed Birth Interval)

The fecundability 'p' is defined as the probability of conception to a female in one menstrual cycle, if the female is exposed to the risk of conception. In this definition time is discrete and its unit is a month.

Let us consider a group of females of sufficiently large marital duration at least $(i + 1)^{\text{th}}$ ($i \geq 1$) birth and their husbands had been migrating before the i^{th} birth. Let in every year migrated husband visit their wives randomly for a constant period, say for y months. Further it is also assumed that all those females will be susceptible to risk of conception during the visit of their husbands in the second year who were not susceptible in the first year during the visit of their husbands after its birth.

Following Sharma⁸ the probability of conception during the stay of husband when he visits his wife may be taken as:

$$(2.1) \quad \sum_{j=1}^y p q^{j-1}, q = 1 - p,$$

Under the assumptions:

- (i) Let S be the proportion of females exposed to the risk of conception during the visit of their husbands in the first year after the i^{th} birth.
- (ii) It may be observed that some of the parameters such as probability of conception vary from year to year due to variation in the biological and social characteristics. Probability of conception is not constant year to year but it varies with some proportion α_k . Therefore (2.1) becomes in the following form:

$$(2.2) \quad \sum_{k=1}^n \sum_{j=1}^y \alpha_k p_k q_k^{j-1} = p_k^* \text{ (say)}$$

which is the probability of conception in year to year with proportion α_k .

So the proportion conceived during the first year after i^{th} birth will be the product of proportion of females susceptible to risk of conception and probability of conception in a year i.e. $S p_k^*$

How the proportion of females not susceptible to the risk of conception during first year is $(1-S)$ and thus the proportion of females susceptible to the risk of conception at time of visit of their husband during the second year after i^{th} birth will be equal to the proportion of females not susceptible to risk of conception during first year plus proportion of females not conceived (out of the susceptible females of first year) during first year, that is -

$$(2.3) \quad (1-S) + (S - S p_k^*) = (1 - S p_k^*).$$

Therefore, the proportion of females conceived during the visit of their husbands in the second year after i^{th} birth is equal to $(1 - S p_k^*) p_k^*$ and so the proportion of females not conceived during the second year will be susceptible to risk of conception during the visit of their husbands in the third year after i^{th} birth, and this proportion is:

$$(2.4) \quad (1 - S p_k^*) - (1 - S p_k^*) p_k^* = (1 - S p_k^*) (1 - p_k^*).$$

So the proportion of females conceived during the third year is :

$$(2.5) \quad (1 - S p_k^*) (1 - p_k^*)^2.$$

Similarly the proportion of females conceived during the visit of husbands in the fourth year after i^{th} birth is

$$(2.6) \quad (1 - Sp_k^*)(1 - p_k^*)^2 p_k^*$$

and in fifth year is

$$(2.7) \quad (1 - Sp_k^*)(1 - Sp_k^*)^3 p_k^* \text{ and so on.}$$

Let us now discuss the theoretical closed birth intervals for the females conceived during the visits of their husbands in different years after the i^{th} birth under the assumption that every conception will result in live birth. Since some females may conceive in the beginning of the year, some may conceive in the middle and others conceive in the end of the year, depending upon risk of susceptibility of the females and time of visits of their husbands. If we assume that number of females conceived during a year are uniformly distributed then it may be considered that on an average females may conceive in the middle of a year. Under this assumption, the closed birth intervals for the conceptions occurring in different year are given as:

Year of conception after i^{th} birth	Length of closed birth intervals
I. First Year	1.25 years (conception in 6 th month + 9 months of gestation period)
II. Second Year	1.25 year (conception in the second year's 6 th month + 9 months of gestation)
....	
....	

In general if conception occurs in the X_i^{th} year after i^{th} birth then the length of close birth interval is equal to

$(X_i + 0.25)$ years, for $X_i = 1, 2, 3, 4, \dots$

Thus the theoretical average length of close birth interval (which is expected to be observed population) is given by

$$(2.8) \quad 1.25Sp_k^* + 2.25(1 - Sp_k^*)p_k^* + 3.25(1 - Sp_k^*)(1 - p_k^*)p_k^* + 4.25(1 - Sp_k^*)(1 - p_k^*)^2 p_k^* + \dots$$

which after simplification comes out to be

$$(2.9) \quad 1.25Sp_k^* + \frac{(1 - Sp_k^*)(2.25 - 1.25q_k^*)}{p_k^*}$$

where

$$q_k^* = 1 - p_k^*,$$

To estimate of the fecundability can be obtained by equating the observed average length of closed birth interval to its theoretical expression given by (2.9)

Let X , denote the length of closed birth interval. So it may be considered as random variable and it may take values 1.25, 2.25, 3.25, ... then its probability man function, based on the above assumption is given by

$$(2.10) \quad g(x) = \begin{cases} Sp_k^*, & \text{for } x = 1.25 \\ (1 - Sp_k^*)(1 - p_k^*)^{x-2.25} p_k^* & \\ \text{for } x = 2.25, 3.25, \dots \end{cases}$$

$$\text{where } p_k^* = \sum_{k=1}^n \sum_j \alpha_k p_k q_k^{j-1}, \quad k = 1, 2, \dots, n$$

For the sake of simplicity consider the two groups of probability of conception with higher probability of conception (p_1^*) and other with low probability of conception (p_2^*) with usual proportion α and $(1-\alpha)$ respectively.

Then above model can be expressed as

$$(2.11) \quad g(x) = \begin{cases} \alpha Sp_1^* + (1-\alpha) Sp_2^*, & \text{for } x = 1.25 \\ \alpha \left\{ (1 - Sp_1^*)(1 - p_1^*)^{x-2.25} p_1^* + (1-\alpha) \left\{ (1 - Sp_2^*)(1 - p_2^*)^{x-2.25} p_2^* \right\} \right\} & \\ \text{for } x = 2.25, 3.25, \dots \end{cases}$$

3. Estimation

The model has three parameters after supposing the value of $\alpha = 0.4$ and 0.5 . The parameters are estimated by equating proportion of zeroth cell with their corresponding theoretical values, first and second order moments, first and second to their observed values, which result in the following equations:

$$(3.1) \quad f_0/f = \alpha Sp_1^* + (1-\alpha) Sp_2^*$$

$$(3.2) \quad E(X) = \alpha \left\{ 1.25 Sp_1^* + \frac{(1 - Sp_1^*)(2.25 - 1.25 q_1^*)}{p_1^*} \right\}$$

$$+ (1 - \alpha) \left\{ 1.25 S p_2^* + \frac{(1 - S p_2^* (2.25 - 1.25 q_2^*))}{p_2^*} \right\}$$

and

$$(3.3) \quad E(X^2) = \alpha \left\{ (1.25)^2 S p_1^* + (1 - S p_1^*) \left[\frac{5.06}{p_1^*} + \frac{q_1^* (0.44 + 1.56 q_1^*)}{p_1^{*2}} \right] \right\} \\ + (1 - \alpha) \left\{ (1.25)^2 S p_2^* + (1 - S p_2^*) \left[\frac{5.06}{p_2^*} + \frac{q_2^* (0.44 + 1.56 q_2^*)}{p_2^{*2}} \right] \right\}$$

where

f_0 = the observed value of zeroth cells

f = total observed values

$E(X)$ = observed mean

$E(X^2)$ = second order raw moment

4. Application

The proposed model is applied to data of 'RDPG-A Survey 1978' by taking mid values of the observed intervals given in Tables 1 and 2, which gives the value of X for first close birth interval, the proposed model is applied to these data. The observed and estimated values of the frequencies (number of migrated couples) according to the length of first closed birth interval given in Tables 1 and 2. From Table 3, the estimated values of p_1^* and p_2^* are 0.5361 and 0.3992 respectively when $\alpha = 0.4$ and estimated value of S i.e. proportion of females exposed to the risk of conception is 0.1832, and the values of p_1^* and p_2^* are 0.3999, 0.5096 respectively when $\alpha = 0.5$ and estimated value of S is 0.1867.

The values of χ^2 are calculated and are given in corresponding tables. The χ^2 values are insignificant at 5% level of significance. It means that the observed pattern of first close birth interval is described well by the proposed model.

Table1: Observed and expected distribution of females according to time from first birth to second birth whose first birth is after migration [when $\alpha = 0.4$]

Average in years	Time in years	Number of couples	
		Observed	Expected
1.25	0.75 - 1.75	9	8.82
2.25	1.75 - 2.75	38	44.03
3.25	2.75 - 3.75	25	23.65
4.25	3.75 - 4.75	15	12.91
5.25	4.75 - 5.75	9	7.15
6.25	≥ 5.75	10	9.44
Total		106	106.00

$$\hat{S} \quad 0.1832$$

$$\hat{p}_1^* \quad 0.5361$$

$$\hat{p}_2^* \quad 0.3992$$

$$\chi^2 \quad 1.7568$$

$$\text{d.f.} \quad 2$$

Table 2: Observed and expected distribution of females according to time from first birth to second birth whose first birth is after migration [when $\alpha = 0.5$]

Average in years	Time in years	Number of couples	
		Observed	Expected
1.25	0.75 - 1.75	9	8.99
2.25	1.75 - 2.75	38	44.05
3.25	2.75 - 3.75	25	23.75
4.25	3.75 - 4.75	15	12.94
5.25	4.75 - 5.75	9	7.12
6.25	≥ 5.75	10	9.15
Total		106	106.00

$\hat{S}$	0.1867
$\hat{p}_1^*$	0.3999
$\hat{p}_2^*$	0.5096
χ^2	1.7975
d.f.	2

Table 2: Estimated values of the parameters

	$\alpha = 0.4$	$\alpha = 0.5$
$\hat{S}$	0.1832	0.1867
$\hat{p}_1^*$	0.5361	0.3999
$\hat{p}_2^*$	0.3992	0.5096

Reference

1. S.N. Agarwala: Socio-economic and Demographic characteristics of the rural Migrants and non-migrants, *Journal of the Institute of Economic Research*, 3(2) (1968).
2. K.C. Zachariah: *Migration in Greater Bombay*, Asia Publishing House, Bombay 1968.
3. J.R. Rele and T. Kanitkar: Residence Background and Fertility in Greater Bombay, *Population Studies*, 28 (1974) 299-308.
4. K.N.S. Yadava: Some Aspects of Out-migration and its effect on fertility, Ph.D. Thesis in Statistics, Banaras Hindu University, Varanasi, India 1977.
5. K.G. Jolly : Differential Fertility Behaviour by Education and Residence Status in Delhi Metropolis, *Demography India*, 9 (1 & 2) (1980) 75-81.
6. C. Jagdish Bhatia and Georges Sabagh. Migration and Fertility in India, *Demography India*, 9 (182) (1980), 54-74.
7. S.N. Singh, R.C. Yadava and K.N.S. Yadava: A Study on the Fertility of Migrants, *Health and Population-Perspectives and Issues*, 4(1) (1981) 59-65.
8. L. Sharma : A Study of the Pattern of Out-Migration from Rural Areas, Ph.D. Thesis in Statistics, Banaras Hindu University, Varanasi, India 1984.
9. R. Prasad: Interrelation between Migration and Fertility in India States, Paper Presented at the National Seminar on Migration Research in Context of Development, August 9-11, (1986), B.H.U., Varanasi India.
10. K.B. Pathak: A Probability distribution for the number of Conceptions, *Sankhya*, 28B (1966) 213-218.