

Stochastic Behaviour and Profit Function of a System with Two Dissimilar Units Subject to Random Failures Under Head of Line Repair Policy

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Abstract: This paper deals with stochastic behaviour of a system having two dissimilar units in which one is a preferred unit and other is non preferred unit, in standby. The system has three modes: normal, waiting and total failure. A switch is used to operate the standby unit and it works successfully with probability ($\alpha_s = 1 - \alpha_f$). The preferred unit always gets priority over non priority unit when both are in failure mode. The failure and repair rates follow exponential and general distributions respectively. Supplementary variable technique has been employed to obtain various probability states. The steady state behaviour, special cases and numerical examples have been taken.

1. Introduction

Earlier many researchers have studied two identical unit standby system. In 1996 Hyderi and Joorel have also considered two identical unit cold standby redundant system subject to random failure. But in realistic situations both the units are not always happen to be similar. Further it is found that switching devices also plays very important role in the standby systems. Keeping all these in view, in this section author proposes to evaluate the cost benefit function, MTTF, reliability and availability of a two dissimilar units in which one unit is a preferred unit (P) and other non-preferred unit (N) in standby with three states i.e. normal, waiting and fail with supplementary variable technique. The preferred unit always get priority over non-preferred unit when both are in failure mode. Initially P-unit in operative mode while N-unit in standby. When P-unit fails N-unit is switched to operate with a switching device, but there is a possibility that the switching device may not switched on at the time of need with probability ($\alpha_s = 1 - \alpha_f$). In the proposed model all the types of failure are assumed to be exponential while repairs are distributed quite generally. The author has taken head-of-line repair discipline. Transition diagram of the model under this repair policy is shown in Fig. 2. Block diagram of the model is shown in Fig. 1.

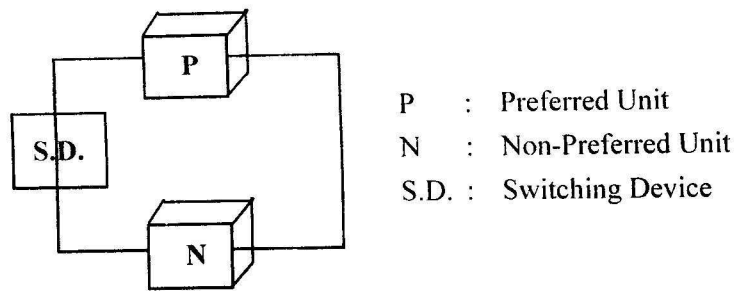


Fig. 1 Block Diagram of System

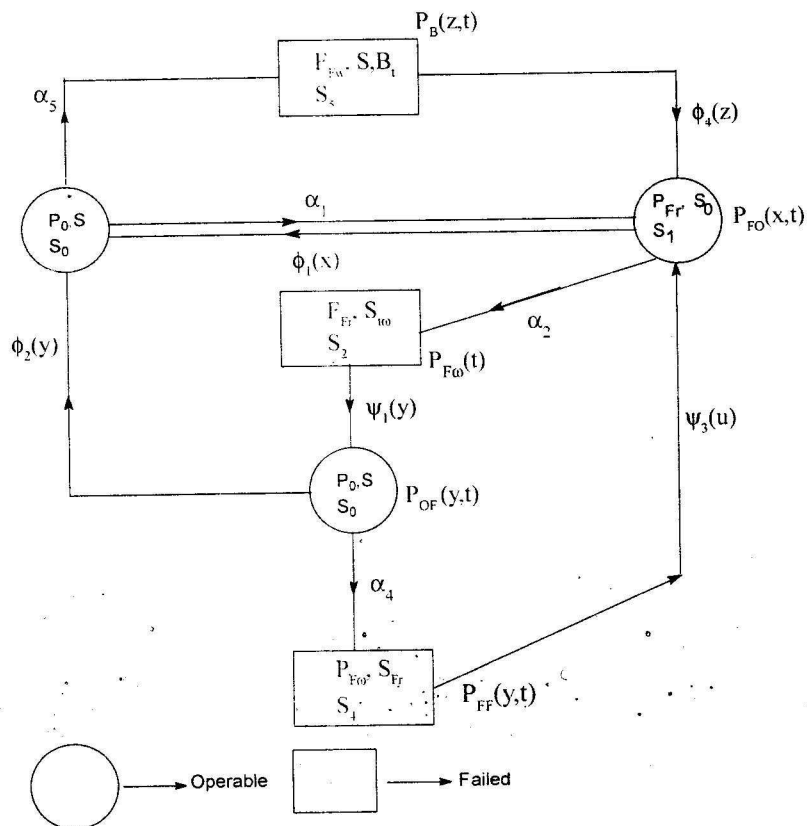


Fig. 2

Assumptions

1. Initially at $t=0$, the system work with normal efficiency.
2. Failures are statistically independent.
3. All repairs follow general distributions.
4. System fails when both the units are in failed state.

Notations

$D / D_x / D_y / D_u / D_z / D_t$	$\frac{d}{dt} / \frac{\partial}{\partial x} / \frac{\partial}{\partial y} / \frac{\partial}{\partial u} / \frac{\partial}{\partial z} / \frac{\partial}{\partial t}$
$\alpha_1, \alpha_2, \alpha_4$	Constant failure rate due to minor/major failure.
α_5	Constant failure rate due to failure of switching devie.
$\phi_1(x)\Delta / \phi_2(y)\Delta / \phi_3(u)\Delta /$ $\phi_4(z)\Delta / \psi_1(y)\Delta$	First order probability that operable/operable/failed/failed/failed system repaired in time interval $(x, x+D)/(y, y+D)/(u, u+D)/$ $(z, z+D)/(y, y+D)$.
$P_{OS}(t)$	$P\{\text{at time } t \text{ the system is in state } S_0\}$.
$P_{FO}(x, t)\Delta$	$P\{\text{the system is in state } S_1 \text{ at time } t \text{ due to failure of P unit}$ and elapsed repair time lies in the interval $(x, x+D)\}$
$P_{F\omega}(y, t)\Delta$	$P\{\text{the system is in state } S_2 \text{ at time } t \text{ due to failure of N unit}$ and elapsed repair time lies in the interval $(y, y+D)\}$.
$P_{OF}(y, t)\Delta$	$P\{\text{the system is in state } S_3 \text{ at time } t \text{ due to repair of p unit}\}$.
$P_{FF}(y, t)\Delta$	$P\{\text{the system is in state } S_4 \text{ at time } t \text{ due to major failure of P}$ unit and elapsed repair time lies in the interval $(y, y+D)$.
$P_B(z, t)\Delta$	$P\{\text{the system is i state } S_5 \text{ due to switch failure and elapsed}$ repair time lies in the interval $(z, z+D)\}$.
$\bar{S}_i^k(s) = k(x)e^{-\int_0^x k(x)dx}$	where $k = \phi_1, \phi_2, \phi_3, \phi_4, \psi_1$

$$\int = \int_0^\infty \text{ unless otherwise stated.}$$

Basic Equations and their laplace Transforms

By elementary probability considerations and continuity arguments, the following difference-differential equations governing the behaviour of the system may be seem to hold good.

$$(1) \quad [D + \alpha_1 + \alpha_5] P_{OS}(t) = \int \phi_1(x) P_{FO}(x, t) dx + \int \phi_2(y) P_{OF}(y, t) dy$$

$$(2) \quad [D_x + D_t + \alpha_2 + \phi_1(x)] P_{FO}(x, t) = 0$$

$$(3) \quad [D_y + D_t + \psi_1(y)] P_{FO}(y, t) = 0$$

$$(4) \quad [D_y + D_t + \alpha_4 + \phi_2(y)] P_{FF}(y, t) = 0$$

$$(5) \quad [D_y + D_t + \psi_2(y)] P_{FF}(y, t) = 0$$

$$(6) \quad [D_z + D_t + \phi_4(z)] P_B(z, t) = 0$$

Boundary Conditions

$$(7) \quad P_{FO}(0, t) = \alpha_1 P_{OS}(t) + \int \phi_4(z) P_B(z, t) dz + \int \psi_2(y) P_{FF}(y, t) dy$$

$$(8) \quad P_{FO}(0, t) = \alpha_2 P_{FO}(x, t) dx$$

$$(9) \quad P_{OF}(0, t) = \int \psi_1(y) P_{FO}(y, t) dy$$

$$(10) \quad P_{FF}(0, t) = \alpha_4 \int P_{OF}(y, t) dy$$

$$(11) \quad P_B(0, t) = \alpha_5 \int P_{OS}(t)$$

Initial Conditions

We assume that the system is initially in the normal state S_0 i.e. $P_{OS}(0) = 1$ and other probabilities are zero.

Solution of the model

Taking Laplace transforms of equations 1 to 11 and using the boundary and initial conditions, we get

$$(12) \quad \bar{P}_{OS}(s) = \frac{1}{D(s)}$$

$$(13) \quad \bar{P}_{FO}(s) = \frac{A(s)}{D(s)}$$

$$(14) \quad \bar{P}_{F\omega}(s) = \alpha_2 \cdot \frac{1 - \bar{S}_{\psi_1}(s)}{s} \cdot \frac{A(s)}{D(s)}$$

$$(15) \quad \bar{P}_{OF}(s) = \alpha_2 \cdot \bar{S}_{\psi_1}(s) \cdot \frac{1 - \bar{S}_{\phi_1}(s + \alpha_4)}{s + \alpha_4} \cdot \frac{A(s)}{D(s)}$$

$$(16) \quad \bar{P}_{FF}(s) = \alpha_2 \cdot \alpha_4 \cdot \bar{S}_{\psi_1}(s) \cdot \frac{1 - \bar{S}_{\phi_1}(s + \alpha_4)}{s + \alpha_4} \cdot \frac{1 - \bar{S}_{\psi_2}(s)}{s} \cdot \frac{A(s)}{D(s)}$$

$$(17) \quad \bar{P}_B(s) = \alpha_2 \cdot \frac{1 - \bar{S}_{\phi_1}(s)}{s} \cdot \frac{1}{D(s)}$$

The Laplace transforms of the probabilities of the system is in up and down state at time t is given below:

$$(18) \quad \bar{P}_{up}(s) = \left[1 + A(s) + \alpha_2 \bar{S}_{\psi_1}(s) \cdot \frac{1 - \bar{S}_{\phi_2}(s + \alpha_4)}{s + \alpha_4} A(s) \right] \frac{1}{D(s)}$$

$$(19) \quad \bar{P}_{down}(s) = \left[\alpha_2 \cdot \frac{1 - \bar{S}_{\psi_1}}{s} A(s) + \alpha_2 \alpha_4 \bar{S}_{\psi_1}(s) \cdot \frac{1 - \bar{S}_{\phi_2}(s + \alpha_4)}{s + \alpha_4} \cdot \frac{1 - \bar{S}_{\psi_2}(s)}{s} A(s) \right. \\ \left. + \alpha_5 \cdot \frac{1 - \bar{S}_{\phi_4}(s)}{s} \cdot \frac{1}{D(s)} \right]$$

It is worth noticing that

$$\bar{P}_{up}(s) + \bar{P}_{down}(s) = \frac{1}{s}$$

where

$$D(s) = s + \alpha_1 + \alpha_5 - \alpha_1 \bar{S}_{\alpha_1}(s + \alpha_2) - \alpha_5 \bar{S}_{\phi_1}(s) \bar{S}_{\phi_1}(s + \alpha_2) \alpha_2 \alpha_4 A(s) \bar{S}_{\phi_1}(s + \alpha_2)$$

$$(20) \quad \frac{1 - \bar{S}_{\phi_2}(s + \alpha_4)}{s + \alpha_4} - \alpha_2 A(s) \bar{S}_{\psi_1}(s) \bar{S}_{\phi_2}(s + \alpha_4)$$

$$(21) \quad A(s) = \frac{[\alpha_1 + \alpha_5 \bar{S}_{\phi_4}(s)] \frac{1 - \bar{S}_{\phi_2}(s + \alpha_2)}{s + \alpha_2}}{1 - \alpha_2 \alpha_4 \bar{S}_{\psi_1}(s) \bar{S}_{\psi_2}(s) \frac{1 - \bar{S}_{\phi_1}(s + \alpha_2)}{s + \alpha_2} \frac{1 - \bar{S}_{\phi_2}(s + \alpha_4)}{s + \alpha_4}}$$

Ergodic Behaviour of the System

Using Abel's Lemma in Laplace transforms, viz.

$$\lim_{s \rightarrow 0} s \bar{f}(s) = \lim_{t \rightarrow \infty} f(t) = f \quad (\text{say})$$

provided that the limit on the right hand side exists, the time independent probabilities are obtained as follows:

$$(22) \quad P_{OS} = \frac{1}{D'(0)}$$

$$(23) \quad P_{FO} = \frac{A(0)}{D'(0)}$$

$$(24) \quad P_{F\omega} = \alpha_2 M^{\psi_1} \frac{A(0)}{D'(0)}$$

$$(25) \quad P_{OF} = \alpha_2 \frac{1 - \bar{S}_{\phi_2}(\alpha_4)}{\alpha_4} \frac{A(0)}{D'(0)}$$

$$(26) \quad P_{FF} = \alpha_2 [1 - \bar{S}_{\phi_1}(\alpha_4)] M^{\psi_2} \frac{A(0)}{D'(0)}$$

$$(27) \quad P_B = \alpha_5 M^{\phi_4} \cdot \frac{A(0)}{D'(0)}$$

Also

$$(28) \quad P_{up} = \left[1 + A(0) + \alpha_2 \frac{1 - \bar{S}_{\phi_2}(\alpha_4)}{\alpha_4} A(0) \right] \cdot \frac{1}{D'(0)}$$

$$(29) \quad P_{down} = 1 - P_{up}$$

where

$$D'(0) = \left[\frac{d}{ds} D(s) \right]_{s=0} \text{ and } M^0(\theta = \psi_1, \psi_2, \phi_1, \phi_2, \phi_4) = -\bar{S}^0(0)$$

Special Cases

(i) Constant Repair Rates:

When repairs follow exponential time distribution.

$$\text{Setting} \quad \bar{S}^\theta(s) = \frac{\theta}{s + \theta}$$

where $\theta = \psi_1, \psi_2, \phi_1, \phi_2, \phi_4$ in relation (12) to (18), one

$$(30) \quad \bar{P}_{OS}(s) = \frac{1}{E(s)}$$

$$(31) \quad \bar{P}_{FO}(s) = \frac{A_I(s)}{E(s)}$$

$$(32) \quad \bar{P}_{F\omega}(s) = \frac{\alpha_2}{s + \psi_1} \cdot \frac{A_I(s)}{E(s)}$$

$$(33) \quad \bar{P}_{OF}(s) = \frac{\alpha_2 \psi_1}{s + \psi_1} \cdot \frac{1}{s + \alpha_2 + \phi_2} \cdot \frac{A_I(s)}{E(s)}$$

$$(34) \quad \bar{P}_{FF}(s) = \alpha_2 \alpha_4 \frac{\psi_1}{s + \psi_1} \cdot \frac{1}{s + \alpha_2 + \phi_2} \cdot \frac{1}{s + \psi_2} \cdot \frac{A_1(s)}{E(s)}$$

$$(35) \quad \bar{P}_B(s) = \alpha_5 \cdot \frac{1}{s + \phi_1} \cdot \frac{1}{E(s)}$$

$$(36) \quad \bar{P}_{up}(s) = \left[1 + A_1(s) + \frac{\alpha_2 \psi_1}{s + \psi_1} \cdot \frac{1}{s + \alpha_4 + \phi_2} \cdot A_1(s) \right] \frac{1}{E(s)}$$

where

$$E(s) = s + \alpha_1 + \alpha_2 + \alpha_5 - \frac{\alpha_1 \phi_1}{s + \alpha_2 + \phi_1} - \alpha_5 \frac{\phi_4}{s + \phi_4} \frac{\phi_1}{s + \alpha_2 + \phi_1} - \alpha_2 \alpha_4 A_1(s) \frac{\psi_1}{s + \psi_1}$$

$$(37) \quad \frac{\phi_1}{s + \alpha_2 + \phi_1} \cdot \frac{\phi_1}{s + \alpha_2 + \phi_2} - \alpha_2 A_1(s) \frac{\psi_1}{s + \psi_1} \cdot \frac{\phi_2}{s + \alpha_4 + \phi_2}$$

$$(38) \quad A_1(s) = \frac{\left[\alpha_1 + \alpha_5 \frac{\phi_4}{s + \phi_4} \right] \frac{1}{s + \alpha_2 + \phi_1}}{1 - \alpha_2 \alpha_4 \frac{\psi_1}{s + \psi_1} \frac{\psi_2}{s + \psi_2} \frac{1}{s + \alpha_2 + \phi_1} \frac{1}{s + \alpha_2 + \phi_2}}$$

Numerical Analysis

Availability Analysis: Setting $\phi_1 = \psi_1 = \phi_2 = \phi_3 = \phi_4 = 1$, $\alpha_1 = 0, \alpha_2 = 0.15$, $\alpha_4 = 0.2, \alpha_5 = 0.25$ in equation (36) and (37), we get the inverse Laplace transform of $\bar{P}_{up}(s)$

$$(39) \quad P_{up}(t) = 0.8371e^{-0.0006t} + 8.8631e^{-1.3147t} + 0.0069e^{-1.1337t} - 0.5171e^{-0.9805t} \\ - 8.3518e^{-1.2103t} \cos(0.4984)t + 0.8658e^{-1.2103t} \sin(0.4984)t$$

Putting $t = 0, 1, 2, \dots$ in equation (39) one can compute Table 1 and Fig. 4.

Cost Effectiveness of the System

Assuming that the service facility is always available and remains busy for time t during interval $(0, t]$. Let C_1 and C_2 be revenue cost per unit time and service cost per unit time respectively. The profit function $G(t)$ in the interval $(0, t]$ is given as follows:

$$(40) \quad G(t) = C_1 \int_0^t P_{up}(t) dt - C_2 t = C_1 [-1395.1667e^{-0.0006t} - 6.7415e^{-1.3147t} - 0.0061e^{-1.1337t} + 0.5274e^{-0.9805t} - 3.0418e^{-1.2103t} \sin(0.4984)t + 5.648e^{-1.2103t} \cos(0.4984)t + 1395.7389] C_2 t$$

Setting $C_1=1$, $t=0,1,2,\dots$ in equation (40), the expected profit for $C_2=0.1, 0.3$ and 0.5 are shown in Table 2 and Fig. 5.

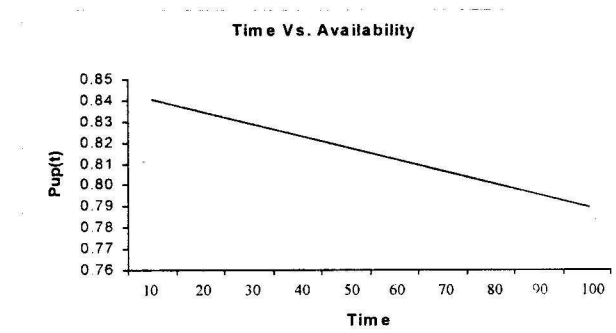


Fig. 4

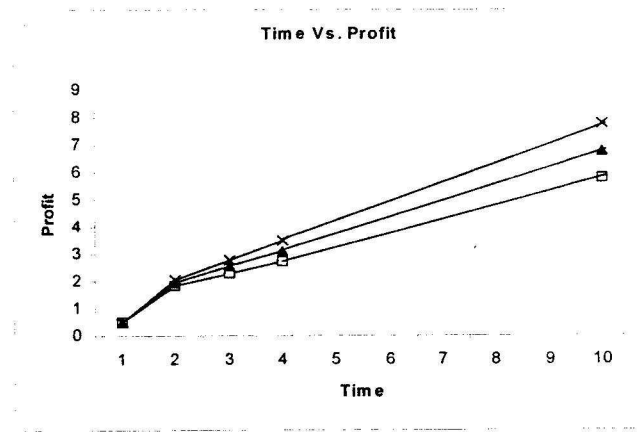


Fig.5

References

1. L.R. Goel and Praveen Gupta: Stochastic analysis of a two unit parallel system with partial and catastrophic failures and preventive maintenance. *Microelectron. Reliab*, **24(5)** (1984) 881-883.
2. J. Singh and P. Goyal: Availability analysis of standby complex system having imperfect switch over device. *Microelectron. Reliab*, **35** (1996) 285-288.
3. S.K. Singh; R.P. Singh and R.B. Singh: Profit evaluation in two unit cold standby system having two type of independent repair facilities, *IJOMAS*, **8** (1992) 277-288.