

Probability Models for the Number of Rural Out-Migration

S. R. J. Singh

Department of Mathematics & Statistics, J.N.K. Vishwavidyalaya,
College of Agriculture, Rewa, M.P.

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Abstract: In this paper, some probability models have been proposed to describe the distribution of rural out-migrants from rural areas of Rewa district of Madhya Pradesh as well as from the rural areas of Varanasi district of Uttar Pradesh. The maximum likelihood estimation technique has been used to estimate the parameters involved in the models and asymptotic variances and covariances of the estimators have also been obtained. Application is made to two sets of demographic data of RDPG-1978 and Demographic Survey of Rewa-Rural 1998. It is found that the probability models derived in this paper is suitably fitted and may be applied in the real situations to test the suitability of total number of migrants at the household level.

1. Introduction

The population movement and its effect on social, economic and demographic aspects has recently occupied a considerable place in social science research. It has a significant implication on regional as well as national planning. Several approaches have been used to analyse the data of migration according to the conceptualization of migration process (Hugo¹, Mehta and Kohli², Bandyopadhyay and Chakraborty³). The studies mentioned above concentrated mainly upon macro-level approach and local and/or regional heterogeneity prevailing in the country has been ignored. The macro-level migration studies have their own importance since this approach describes the aggregate flow or rate of migration and identity factors active for out-migration (Banerjee⁴). On the other hand, micro-level (i.e. at the level of household or individual) studies help to identify the behavioural parameter of the process.

Like many developing countries, two types of households are usually observed in India according to the occurrence of migration. First, the households from where male members aged 15 years and above migrate singly leaving their wives, and children at home, second, male members migrate with their wives, children and the other dependent relatives. It is to be mentioned here that persons migrating from both types of households maintain close tie with remaining members of the households in their place of origin through regular visit and sending remittances. However, first category maintains more tie compared to second category of households.

Several attempts have been made to study the pattern of rural out-migration through the use of probability models (Singh and Yadava⁵, Yadava and Singh⁶, Yadava⁷, Yadava et al.⁸, Iwunor⁹). In 1981, Singh and Yadava proposed negative binomial distribution to describe the trend of rural out migration for male migrants aged 15 years and above. The idea of cluster was incorporated by Yadava and Singh⁶ and found that Thomas distribution is well suited to describe the number of migrants from a household. A probability distribution under such assumptions fitted well the distribution of male migrants aged 15 years and above. However, the above mentioned models do not fit the distribution of total number of migrants including wife and children from a household.

A number of probability models has been worked out to describe the distribution of households according to total number of migrants under different assumptions. Singh¹⁰ proposed a probability model for the total number of migrants under the assumption that there are two types of households such that first one in which only male aged 15 years and above migrate and second one from which the male migrates with their wives and children. On the same line, several authors have proposed models to describe the distribution of households according to the total number of migrants including wife and children Yadava⁷, Yadava et al.⁸, Singh¹¹.

Most of the studies mentioned above have used moment technique or mean-zero frequency method to estimate the parameters involved their proposed models. It is observed that about 75% variation in migration is equated through zero'th cell frequencies since all the non-migrant households counted in this cell. So, only about 25% variation are explained by the estimated parameters when mean-zero frequency method is applied. Considering these limitations into account, the maximum likelihood estimation technique is proposed in this present study to estimate the parameters involved in the models. Needless to mention that the maximum likelihood estimation method has the advantage that the standard error of the estimators can also be obtained and this method measures the total variation of the distribution. The objectives of this study are:

- i) to propose a model to describe the pattern of rural out-migration for male aged 15 years and above.
- ii) to use maximum likelihood technique to estimate the parameters of the model proposed by Yadava⁷ for the total number of migrants.

2. The Probability Distribution for Male Migrants Aged 15 years and Above

A probability model for describing the variation in the number of rural male out-migrants aged 15 years and above from a household has been derived on the basis of following assumptions:

- i) At the survey point, the household is either exposed to the risk of migration or it is not exposed to the migration risk. Let α and $(1-\alpha)$ be the respective probabilities.

ii) The probability of migrating one male from a household is greater than the probability of migrating two males and probability of two males migrating is greater than that of three males from a household and so on. Thus, the pattern of migration from a household is a decreasing function and follows a logarithmic series distribution with parameter λ .

Let x represent the number of male rural out-migrants aged 15 years and above from a household, then under the assumptions (i) and (ii), the probability function of x is given by

$$(1) \quad p(x = k) = 1 - \alpha, \text{ for } k = 0$$

$$= \left[\frac{-\lambda^k}{k \log(1 - \lambda)} \right] \text{ for } k = 1, 2, 3, \dots, 0 < \lambda < 1, 0 < \alpha < 1;$$

The log-series distribution has a long positive tail and the shape of the tail is similar to that of geometric distribution for large values of k . The log-series distribution have the advantage that it has only one parameter instead of two parameters of Negative Binomial Distribution (Chatfield et al.¹²).

Estimation: Consider a sample consisting of N observations of the random variable x with probability function given in expression (1) Suppose that η_k ($k=0, 1, 2, \dots, m$) represents

the number of observations of k 'th cell and $\sum_{k=0}^m \eta_k = N$. The likelihood function for the given

sample $(x_1, x_2, \dots, x_N)$ can be expressed as :

$$(2) \quad L[\alpha, \lambda | (x_1, x_2, \dots, x_N)] = (1 - \alpha)^{n_0} \prod_{k=1}^m \left[\alpha \left(\frac{-\lambda^k}{k \log(1 - \lambda)} \right) \right]^{n_k}$$

$$(3) \quad = \frac{(1 - \alpha)^{n_0} (-\alpha)^{n - n_0} \sum_{k=1}^m n_k x_k}{\left(\prod_{k=1}^m x_k^{n_k} \right) [\log(1 - \lambda)]^{n - n_0}}$$

Taking logarithms of (3) differentiating with respect to α and λ respectively and equating to zero gives the following estimating equations:

$$(4) \quad \frac{\partial \log L}{\partial \alpha} = -\frac{n_0}{1-\alpha} + \frac{n-n_0}{\alpha} = 0$$

$$(5) \quad \frac{\partial \log L}{\partial \lambda} = \frac{\sum_{k=1}^m n_k x_k}{\lambda} + \frac{n-n_0}{(1-\lambda) \log(1-\lambda)} = 0$$

The equation (4) yields the estimator of α as

$$\hat{\alpha} = \frac{n-n_0}{n}$$

The estimating equation for λ is obtained by solving equation (5) as :

$$(6) \quad (1-\lambda) \log(1-\lambda) \sum_{k=1}^m n_k n_k + (n-n_0)\lambda = 0$$

This equation can not be solved analytically, but it can be solved numerically and the numerical solution of (6) is the desired maximum likelihood estimate for λ .

The second partial derivatives of $\log L$ is as follows:

$$(7) \quad \frac{\partial^2 \log L}{\partial \alpha^2} = -\frac{n_0}{(1-\alpha)^2} - \frac{n-n_0}{\alpha^2}$$

$$(8) \quad \frac{\partial^2 \log L}{\partial \lambda^2} = -\frac{\sum_{k=1}^m n_k x_k}{\lambda^2} + \frac{(n-n_0)[1+\log(1-\lambda)]}{[(1-\lambda) \log(1-\lambda)]^2}$$

$$(9) \quad \frac{\partial^2 \log L}{\partial \alpha \partial \lambda} = 0$$

Using the fact that $E(n_0) = n(1-\alpha)$, $E(n-n_0) = n\alpha$

$$E(X_\lambda) = \frac{-\alpha\lambda}{(1-\lambda)\log(1-\lambda)} \text{ and } E\left(\sum_{k=1}^n n_k x_k\right) = -\frac{n\alpha\lambda}{(1-\lambda)\log(1-\lambda)}$$

The expected values of second partial derivatives is obtained as

$$-E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -\frac{E(n_0)}{(1-\alpha)^2} - \frac{E(n-n_0)}{\alpha^2} = \frac{n}{\alpha(1-\alpha)} = \phi_{11} \text{ (say)}$$

$$-E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -\frac{E\left(\sum_{k=1}^n n_k x_k\right)}{\lambda^2} + \frac{[1+\log(1-\lambda)]E(n-n_0)}{[(1-\lambda)\log(1-\lambda)]^2}$$

$$= -n\alpha\left(\frac{1}{\lambda(1-\lambda)\log(1-\lambda)} + \frac{1+\log(1-\lambda)}{[(1-\lambda)\log(1-\lambda)]^2}\right) = \phi_{22} \text{ (say)}$$

The covariance between α and λ is zero since $E\left(\frac{\partial^2 \log L}{\partial \alpha \partial \lambda}\right) = 0$ and hence the variance of

α and λ can be obtained as

$$v(\hat{\alpha}) = \frac{1}{\phi_{11}} \quad \text{and} \quad v(\hat{\lambda}) = \frac{1}{\phi_{22}}$$

Application: The proposed model is applied to the data collected from rural areas of Rewa district of Madhya Pradesh (M.P.) as well as rural areas of Varanasi district of Uttar Pradesh (U.P.) of India. The data of Rewa rural has been collected under a Demographic survey of Rewa - Rural 1998. The cluster survey sampling methodology has been adopted to select villages. Out of 2200 sample households from 19 cluster, the survey has covered all the 1789 migrants households. A modified definition of household was adopted to cover the information on migration. A household was defined as a dwelling unit where a group of persons usually live together and take food from common kitchen. It, however, includes those who live outside the village but claim the household to be their own. Persons of this category work outside the village and often send remittances. Such persons are called the

migrated members of the household and such households are known as migrant households. Naturally, complete household migration is not included in this study. The Varanasi (Uttar Pradesh) data has been taken from Singh¹⁰.

Table 1: Distribution of observed and expected number of households according to the number of male migrants aged 15 years and above Rewa-Rural (M.P.)

Number of migrants per household	HOUSEHOLD COHORT	
	Observed	Expected
0	1373	1373.00
1	296	303.38
2	79	73.95
3	29	24.02
4	9	
5	3	
6	0	14.63
7	0	
8	0	
Total	1789	1789
χ^2	2.022	
d.f.	2	
$\hat{\alpha}$	0.1765	
$\hat{\lambda}$	0.4875	
$v(\hat{\alpha})$	6.17×10^{-5}	
$v(\hat{\lambda})$	0.00076	
$Cov(\hat{\alpha}\hat{\lambda})$	0.0000	

Table-1 shows the estimated value of the parameters, variances and covariances between the estimates, observed and expected number of households according to the number of migrants in household cohorts of Rewa-Rural (M.P.). The same for Varanasi data in three types of household is portrayed in Table 2.

The estimated value of risk parameter α and λ are 0.1765 and 0.4875 respectively from households (Table 1) which shows the greater chance of migrating out. The estimated value of λ following Table 2 was found 0.51, 0.53 and 0.55 for semi-urban, remote and growth center villages respectively, while the estimated value α was found as 0.11, 0.23 and 0.19 corresponding to the three types of villages. It is observed that the estimated value of λ is nearly constant in three types of villages, while $\hat{\alpha}$ has more variation. The higher value of $\hat{\alpha}$ for remote type of household indicated that such household have a greater chance of migrating out than semi-urban and growth centre types of households.

The value of χ^2 is insignificant at 5% level for all the four household cohorts which indicates that the proposed model is fitted well for all the data sets. This suggests that the proposed probability model is a reasonable approximation to describe the pattern of rural out-migration to the situation at least at micro-level.

Table 2: Distribution of observed and expected number of households according to the number of male migrants aged 15 years and more in three types of households of rural areas of Varanasi district Uttar Pradesh.

Number of migrants per household	Types of Household					
	Semi-urban		Remote		Growth Centre	
	Observed	Expected	Observed	Expected	Observed	Expected
0	1042	1042.00	872	872.00	978	978.00
1	95	92.31	176	184.91	154	158.78
2	19	23.49	59	48.84	47	43.48
3	10	7.97	18	17.20	18	15.87
4	2		6		9	
5	2		4		1	
6	0	5.23	0	12.04	0	11.87
7	1		0		0	
8	0		0		1	
Total	1171	1171	1135	1135	1208	1208
χ^2	1.463		2.925		0.778	
d.f.	2		2		2	
$\hat{\beta}$	0.1102		0.2317		0.1904	
$\hat{\xi}$	0.5090		0.5283		0.5476	
$\hat{p}$						
$v(\hat{\alpha})$	8.37×10^{-5}		0.000157		0.000128	
$v(\hat{\lambda})$	0.002379		0.001131		0.001248	
$Cov(\hat{\alpha}, \hat{\lambda})$	0.0000		0.0000		0.0000	

3. The Probability Distribution for Total Number of Migrants

Yadava⁷ proposed a probability model to describe the distribution of households according to total number of migrants and used mean-zero frequency method to estimate the parameters. He applied the proposed model to the Varanasi data, a district of Northern Uttar Pradesh, India. The maximum likelihood estimation technique is proposed here to estimate the parameters and expression for the asymptotic variances and covariances of the estimated parameters have been obtained. The application of the model is attempted here for Rewa Rural (M.P.) data as well as Varanasi (U.P.) data using maximum likelihood method.

In brief, Yadava⁷ model along with assumptions is given below:

- At the survey point, let β be the proportion of households which poses at-least one migrants.
- Out of β proportion of households, let ξ be the proportion of households which poses only one migrant at the survey point.
- Out of $(1-\xi)\beta$ proportion of households, let π be the proportion of households from which only males ≥ 15 years migrate and $(1-\pi)$ be the proportion of households which poses both types of migrants (male ≥ 15 years as well as males with their families).

iv. The number of migrants from a household follows a mixture of two displaced geometric distribution with π proportion of households from which only males aged 15 years migrates and $(1-\pi)$ be the proportion of households from which both type of migration occur.

v. Let p_1 and p_2 be the probability of migration of a person from π and $(1-\pi)$ proportions of households respectively.

Based on above assumptions, the probability distribution for the total number of migrants, x , is given by

$$\begin{aligned}
 p(x = k) &= 1 - \beta \quad \text{if } k = 0 \\
 &= \xi\beta \quad \text{if } k = 1 \\
 (10) \quad &= (1 - \xi)\beta \{ \pi p_1 q_1^{k-2} + (1 - \pi) p_2 q_2^{k-2} \} \quad \text{if } k = 2, 3,
 \end{aligned}$$

The model (10) involves five parameters ($\beta, \delta, \pi, p_1, p_2$) which is difficult to estimate from the observed data. If it is assume that $p_1 = p_2 = p$ (say), i.e. probability of migration from both types of households is same, the model becomes as:

$$\begin{aligned}
 (11) \quad p(x = k) &= 1 - \beta \quad \text{for } k = 0 \\
 &= \xi\beta \quad \text{for } k = 1 \\
 &= (1 - \xi)\beta p q^{k-2} \quad \text{for } k = 2, 3,
 \end{aligned}$$

Estimation of Parameters and Asymptotic Variances & Covariances

The model (11) involves three parameters ξ, β and p to be estimated from the observed distribution of total number of migrants from households. Let $x_1, x_2, \dots, x_N$ denote a random sample of size N from the population (11). Further, suppose that n_k ($k=0, 1, 2, \dots, m$) be the

number of observations corresponding to the value of k and $\sum_{k=0}^m n_k = N$. The likelihood function for the given sample can be expressed as

$$(12) \quad L = \prod_{k=0}^m [p(x=k)]^{x_k} = (1-\beta)^{x_0} (\xi\beta)^{x_1} \prod_{k=2}^m [(1-\xi)\beta p q^{k-2}]^{x_k}$$

$$= (1-\beta)^{x_2} \xi^{x_1} \beta^{x-x_1} (1-\xi)^{x-x_2-x_1} p^{x-x_2-x_1} q^{\sum_{k=2}^m (k-2)x_k}$$

Taking logarithms of (12) and differentiating with respect to β , ξ and p respectively and equating to zero, we get

$$(13) \quad \frac{\delta \log L}{\delta \beta} = -\frac{n_0}{1-\beta} + \frac{n-n_0}{\beta} = 0$$

$$(14) \quad \frac{\delta \log L}{\delta \xi} = \frac{n_1}{\xi} - \frac{n-n_0-n_1}{1-\xi} = 0$$

$$(15) \quad \frac{\delta \log L}{\delta p} = \frac{n-n_0-n_1}{p} - \frac{\sum_{k=3}^m (k-2)n_k}{1-p} = 0$$

Solving equations (13), (14) and (15) the estimators of β , ξ and p can easily be obtained as

$$\hat{\beta} = \frac{n-n_0}{n}, \hat{\xi} = \frac{n_1}{n-n_0} \text{ and } \hat{p} = \frac{n-n_0-n_1}{(n-n_0-n_1) + \sum_{k=3}^m (k-2)n_k}$$

The second partial derivations of $\log L$ can be obtained as :

$$(16) \quad \frac{\delta^2 \log L}{\delta \beta^2} = -\frac{n_0}{(1-\beta)^2} - \frac{n-n_0}{\beta^2}$$

$$(17) \quad \frac{\delta^2 \log L}{\delta \xi^2} = \frac{n_1}{\xi^2} - \frac{n-n_0-n_1}{(1-\xi)^2}$$

$$(18) \quad \frac{\delta^2 \log L}{\delta p^2} = -\frac{n-n_0-n_1}{p^2} - \frac{\sum_{k=3}^m (k-2)n_k}{(1-p)^2}$$

$$(19) \quad \frac{\delta^2 \log L}{\delta \beta \delta \xi} = \frac{\delta^2 \log L}{\delta \beta \delta p} = \frac{\delta^2 \log L}{\delta \xi \delta p} = 0$$

Here, $E(n_0) = n(1-\beta)$, $E(n_1) = n\xi\beta$, $E(n_k) = n(1-\xi)\beta pq^{k-2}$ for $k = 2, 3, \dots, m$.

Again $E(n-n_0) = np$, $E(n-n_0-n_1) = n\beta(1-\xi)$

$$E\left[\sum_{k=3}^m (k-2)n_k\right] = E[n_3 + 2n_4 + 3n_5 + \dots + (m-2)n_m]$$

$$= n(1-\xi)\beta pq[1 + 2q + 3q^2 + \dots + (m-2)q^{m-2}]$$

$$(20) \quad = n(1-\xi)\beta q \left[\frac{1-q^{m-2}}{p} - (m-2)q^{m-2} \right] \text{ for small } m \dots\dots$$

$$(21) \quad = \frac{n(1-\theta)\beta q}{p} \text{ for largem.}$$

Using the above facts, the expected value of the second partial derivatives can be obtained as :

$$(22) \quad -E\left(\frac{\delta^2 \log L}{\delta \beta^2}\right) = \frac{E(n_0)}{(1-\beta)^2} - \frac{E(n-n_0)}{\beta^2} = \frac{n}{\beta(1-\beta)} = \phi_{11}(\text{say})$$

$$(23) \quad -E\left(\frac{\delta^2 \log L}{\delta \xi^2}\right) = \frac{E(n_1)}{\xi^2} + \frac{E(n-n_0-n_1)}{(1-\xi)^2} = \frac{n\beta}{\xi(1-\xi)} = \phi_{22}(\text{say})$$

$$-E\left(\frac{\delta^2 \log L}{\delta p^2}\right) = \frac{E(n-n_0-n_1)}{p^2} + \frac{E\left[\sum_{k=3}^m (k-2)n_k\right]}{(1-p)^2}$$

$$= \frac{n\beta(1-\xi)q + n(1-\xi)\beta p[1 - q^{m-2} - (m-2)pq^{m-2}]}{p^2 q}$$

$$(24) \quad = \phi_{33}(a) \text{ (say), for small } m.$$

$$(25) \quad \text{and } -E\left(\frac{\delta^2 \log L}{\delta p^2}\right) = \frac{n\beta(1-\xi)}{p^2 p} = \phi_{33}(b) \text{ (say), for large } m.$$

The covariances between the estimators becomes zero since

$$E\left(\frac{\delta^2 \log L}{\delta \beta \delta \xi}\right) = E\left(\frac{\delta^2 \log L}{\delta \beta \delta p}\right) = E\left(\frac{\delta^2 \log L}{\delta \xi \delta p}\right) = 0 \text{ and hence the asymptotic variances}$$

of the estimators can be obtained as :

$$V(\hat{\beta}) = \frac{1}{\phi_{11}}, \quad V(\hat{\xi}) = \frac{1}{\phi_{22}}, \text{ and}$$

$$V(\hat{p}) = \frac{1}{\phi_{11}(a)} \text{ when } m \text{ is small}$$

$$= \frac{1}{\phi_{22}(b)} \text{ when } m \text{ is large.}$$

Application: The above probability distribution for describing the total number of migrants from a household is fitted using the estimators $\hat{\beta}$, $\hat{\xi}$ and $\hat{p}$ to the data collected from rural areas of Rewa district of Madhya Pradesh (M.P.) as well as the data used by Yadava⁷. The observed number of households according to the total number of migrants in the household cohort of Rewa-Rural (M.P.) the expected number of households, the variances and covariances between the estimates are presented in Table 3. The corresponding results for the three types of villages of the data used by Yadava⁷ are presented in Table 4. Table 3 shows that the proportion of households having only one migrants ($\hat{\xi}$) is higher. This may be due to higher cost on travel and higher cost of living at the place of destination along with families.

The higher value of ($\hat{\beta}$) in household cohort indicated that a higher rate of migration was observed in Rewa-Rural migrants households. moreover, the average number of migrants were found 0.3339 for migrant households.

The chi-square values following Table 3 were found insignificant which confirms that the model fits the data set reasonably well. It is interesting that the estimated values of the

parameters using mean zero frequency method (Table 4). however, the vaiances and covariances of the estimated parameters are portrayed in the table as mean zero frequency method failed to produce the expression for variances and covariances.

Table 3: Distribution of observed and expected number of househlds according to the total number of migrants in household cohorts of the rural areas of Rewa district

Total Number of migrants per household	Household Cohort	
	Observed	Expected
0	1373	1373.00
1	292	292.00
2	67	56.17
3	37	33.95
4	17	20.52
5	6	12.40
6	7	7.50
7	3	
8	5	
Total	1789	11.46
χ^2	7.35	
d.f.	5	
$\hat{\beta}$	0.182873	
$\hat{\xi}$	0.672811	
$\hat{p}$	0.39554	
$v(\hat{\beta})$	6.29×10^{-5}	
$v(\hat{\xi})$	0.000507	
$v(\hat{p})$	0.000712	
covariances	0.0000	
Average no. of migrants for households	0.3339	

Table 4: Distribution of observed and expected number of households according to the total number of migrants in three types of villages of Varanasi, (U.P.) India*

Number of migrants per household	Types of Household					
	Semi-urban		Remote		Growth Centre	
	Observed	Expected	Observed	Expected	Observed	Expected
0	1032	1032.00	871.00	871.00	972	972.00
1	58	58.00	139	139.00	124	124.00
2	23	20.16	52	42.00	32	33.84
3	6	14.44	15	27.78	25	23.04
4	10	10.34	14	18.36	12	15.68
5	8	7.40	11	12.14	10	10.67
6	7	5.30	10	8.03	5	7.27
7	1	13.36	6	15.85	5	15.50
8	16		16		17	
Total	1161	1161	1134	1134	1202	1202
χ^2	6.93		12.27		9.73	
d.f.	4		4		4	
$\hat{\beta}$	0.1111		0.2319		0.1913	
$\hat{\xi}$	0.4496		0.5285		0.5391	
$\hat{p}$	0.2840		0.3388		0.3193	
$v(\hat{\beta})$	8.51×10^{-5}		0.00157		0.000129	
$v(\hat{\xi})$	0.001918		0.000948		0.001081	
$v(\hat{p})$	0.000907		0.00067		0.000722	
covariances	0.0000		0.0000		0.0000	
Average no. of migrants for households	0.3264		0.5547		0.4675	

*Source: Singh¹⁰ and Yadava⁷

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