

On a Couple Stress Rivlin-Ericksen Ferromagnetic Fluid Heating From Below with Varying Gravity and Suspended Particles Through a Porous Medium.

Rahul

Department of computer science
D A V college Muzaffarnagar(U.P.)-251001, INDIA
Email: rahulambuj@gmail.com

Naveen Sharma

Department of Mathematics
D A V college Muzaffarnagar(U.P.)-251001, INDIA
Email: ns2000dav@gmail.com

M. K. Sharma

Department of Mathematics
Chaudhary Charan singh University, Meerut(U.P.)-250004, INDIA
Email: drmukeshsharma@gmail.com

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Abstract: The effect of suspended particles, couple stress and permeability of the medium on the thermal instability of a couple stress Rivlin-Ericksen ferromagnetic fluid with varying gravity fields through a porous medium are investigated. The dispersion relation has been developed and solved analytically using the normal mode approach and linear stability theory. For stationary conventions, it is found that suspended particles always have a destabilizing effect if $\lambda > 0$ and a stabilizing effect if $\lambda < 0$. Couple stress has a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$. Permeability has a stabilizing effect if $\lambda < 0$ and a destabilizing effect if $\lambda > 0$. Magnetization always has a stabilizing effect either $\lambda > 0$ or $\lambda < 0$. Oscillatory modes are not allowed and the principle of exchange of stability is satisfied.

Keywords: Thermal instability, Couple-stress fluid, Rivlin Ericksen fluid, Ferromagnetic fluid, suspended particles.

2010AMS Classification: 76S0.

1. Introduction

Now a days, the non-Newtonian fluids are very attractive due to a large number of applications in modern industry and engineering. The mechanism of lubrications of synovial joints is the most important application of couple stress fluid in non Newtonian fluid due to which in recent time couple stress fluid attracts the attention of researchers in the field of fluid. Stokes¹ has proposed the theory of couple stress fluid in 1966 and formulated by Stokes² in 1984. A detailed theoretical and experimental study on thermal instability on a fluid layer heated from below by applying different hydrodynamic's assumptions has been given by Chandrasekhar³ in 1981. Sharma and Sharma⁴ have discussed a couple stress fluid heated from below in porous medium. Sunil et al⁵ have examined a layer of compressible couple stress fluid with suspended particles in porous medium and found that suspended particles has destabilizing effect on system. Kumar and Mohan⁶ have studied the Thermosolutal convection in rotated couple stress fluid in porous medium. Sharma and Thakur⁷ studied the effect of different fields on couple stress fluid. Sharma et al⁸ have studied the effect of suspended particles on heated a couple stress fluid from below.

The effect of suspended particles on thermal convection of Rivlin-Ericksen elastico viscous fluid has been analyzed by Sharma and Kumar⁹. Aggarwal and Verma^{10, 11} have examined the effect of rotation and magnetic field on Walters'B' elastico-viscous fluid with suspended particles. Rana¹², Rana and Thakur¹³ have discussed the effect of suspended particles on thermal instability in porous and Darcy Brinkman porous medium.

Ferromagnetic fluids are the mixture of very fine nanoscale ferromagnetic particles with the carrier fluid. Ferromagnetic fluids are artificially synthesized fluid. Body couple and Polarization force are two main characteristics of ferromagnetic fluid. Ferromagnetic fluid is used in instrumentation, lubrication, vacuum technology, printing, vibration and in many more scientific applications. A ferrofluid is a liquid which becomes highly magnetized in the presence of a magnetic field. The importance of research on magnetic fluids has been given by Rosenweig¹⁴. Siddheswar^{15,16} has discussed Rayleigh – Benard convection in a ferromagnetic fluid in different situations. Sunil et al¹⁷ investigated the influence of dust particles on ferromagnetic fluid thermal convection. Sunil et al¹⁸ investigated the influence of magnetic field-dependent viscosity on ferromagnetic fluid thermal instability in the presence of dust particles. The effects of dust particles on thermal instability of couple stress ferromagnetic fluids have been studied by Pulkit et.al^{19,20}.

To keep in view the importance of couple stress Rivlin-Ericksen ferromagnetic fluid we have analysed the effect of suspended particles, Couple Stress and permeability of the medium when a couple stress Rivlin-Ericksen ferromagnetic fluid heated from below with varying gravity through a porous medium.

2. Mathematical Formulation

Consider a thin layer of infinite incompressible, electrically non conducting couple stress Rivlin-Ericksen ferromagnetic fluid bounded between two infinite horizontal planes situated at $z = 0$ & $z = d$ permitted with suspended particles. A variable gravity field $g(0, 0, -g)$ applied along the vertical direction which is taken as Z axis. where $g = \lambda g_0$, g_0 is the value of g at $z=0$, which is always positive & λ can be positive or negative as gravity increases or decreases upwards from its value at $z=0$ (g_0).

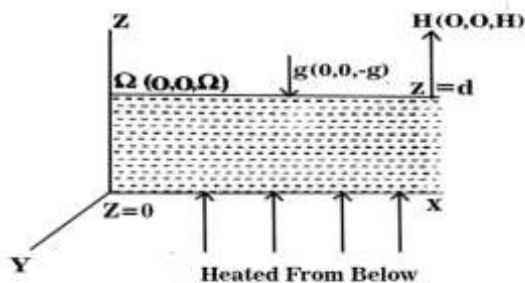


Figure-1

The considered fluid layer is heated from below so that the uniform temperature gradient $\beta = \left(\left| \frac{dT}{dz} \right| \right)$ is maintained. A ferromagnetic fluid reacts so quickly to a magnetic torque that we can simulate the following conditions to maintain,

$$(2.1) \quad M \times H = 0,$$

Where M represents magnetization and H represents the magnetic field intensity.

The Maxwell's equation is also satisfied by ferromagnetic fluid. Here we examine electrically non conducting fluid, therefore the displacement current is negligible. Maxwell's equation becomes,

$$(2.2) \quad \nabla \cdot B = 0, \nabla \times H = 0,$$

The magnetic field intensity H , magnetization M and magnetic induction B are correlated in Chu formulation of electro dynamics such that,

$$(2.3) \quad B = \mu_0 (H + M),$$

Let us assume that magnetization depends on the amplitude of magnetic field and temperature. Let us also assume that magnetization is aligned with magnetic field so that,

$$(2.4) \quad M = \frac{H}{H} M(H, T),$$

Let us take Pressure, Density, Temperature, Coefficient of thermal expansion, Kinematic viscosity, kinematic viscous elasticity, couple stress viscosity, medium porosity, medium permeability, magnetic permeability, thermal diffusivity, resistivity, velocity of fluid, velocity of suspended particles, and number density of suspended particles are denoted by p , ρ , T , α , ν , ν' , μ' , ε , K_1 , μ_e , κ_T , η , $q = (u_1, u_2, u_3)$, $q_s(x, t)$ and $N_s(x, t)$ respectively. $K = 6\pi\mu\eta'$ is the Stokes drag coefficient. η' is the radius of suspended particle.

In presence of rotation, magnetic field and suspended particles the equation of motion, continuity and heat conduction of couple stress Rivlin-Ericksen ferromagnetic fluid through porous medium are

$$(2.5) \quad \frac{1}{\varepsilon} \left[\frac{\partial q}{\partial t} + \frac{1}{\varepsilon} (q \cdot \nabla) q \right] = -\frac{1}{\rho_0} \nabla p + g \left(1 + \frac{\delta \rho}{\rho_0} \right) + \frac{1}{\rho_0} M \nabla H +$$

$$\left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 q - \frac{1}{K_1} \left(\nu + \nu' \frac{\partial q}{\partial t} \right) \frac{KN_S}{\varepsilon \rho_0} (q_s - q),$$

$$(2.6) \quad \nabla \cdot q = 0,$$

$$(2.7) \quad \frac{\partial T}{\partial t} + (q \cdot \nabla) T = \kappa_T \nabla^2 T$$

The equation of state is

$$(2.8) \quad \rho = \rho_0 [1 - \alpha(T - T_0)],$$

Where T_0 and ρ_0 represents the value of temperature and density at $z=0$ respectively. ∇H denotes the magnetic field gradient.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

$$H = |H|, B = |B|, M = |M|,$$

Due to suspended particles in fluid there is a force get generated which is proportional to the difference of velocity of fluid and velocity of suspended particles. Since the force applied by the suspended particles on fluid and the force applied by fluid on suspended particles are equal. So there must be an another force, equal in magnitude but opposite in sign in equation of motion for the suspended particles. The distance between particles are taken to be relatively large with their diameter so inter particles reactions are not counted.

The equations of motion, continuity for suspended particles for above assumptions, are

$$(2.9) \quad m N_S \left[\frac{\partial q_S}{\partial t} + \frac{1}{\varepsilon} (q_S \cdot \nabla) q_S \right] = K N_S (q - q_S),$$

$$(2.10) \quad \varepsilon \frac{\partial N_S}{\partial t} + \nabla \cdot (N_S \cdot q_S) = 0,$$

Where $m N_S$ is density of suspended particles.

Let C_V , C_S , ρ_p and C_P denote the capacity of the fluid at constant volume, the heat capacity of suspended particles, density of porous material and heat capacity of porous material respectively. If there is thermal equilibrium, then the equation of heat conduction is,

$$(2.11) \quad E \cdot \frac{\partial T}{\partial t} + (q \cdot \nabla) T + \frac{m N_S C_S}{\rho_0 C_V} \left(\varepsilon \frac{\partial}{\partial t} + q_S \cdot \nabla \right) T = \kappa_T \nabla^2 T,$$

Where $E = \varepsilon + (1 - \varepsilon) \frac{\rho_P C_P}{\rho_0 C_V}$ is a constant.

$\kappa_T = \frac{q'}{\rho_0 C_V}$ where q' is effective thermal conductivity of pure fluids.

In general, to complete the system a state equation is needed, which satisfy M in two thermodynamic variables H and T . In present paper we consider magnetization is independent of magnetic field and depend on temperature only. Therefore

$$M = M(T),$$

As the first approximation, we consider that

$$(2.12) \quad M = M_0[1 - \gamma(T - T_0)],$$

Where M_0 is the magnetization at $T = T_0$ and

$$\gamma = \frac{1}{M_0} \left(\frac{\partial M}{\partial T} \right)_H,$$

3. Basic State and Perturbation Equations

In the basic state is supposed to be a rest state of fluid and is given by

$$q = (0,0,0), p = p(z), \rho = \rho(z) = \rho_0(1 + \alpha\beta z), M = M(z) = M_0(1 + \gamma\beta z),$$

$$(3.1) \quad T = T(z) = T_0 - \beta z, q_s = (0,0,0), N_s = N_0$$

To study the character of equilibrium let us apply a little perturbation on the layer of fluid due to which some disturbances takes place in the system. Now we consider that these small disturbances are the function of time and space variable. The perturbed flow may be represented as

$$(3.2) \quad q = (0,0,0) + (u_1, u_2, u_3), T = T(z) + \theta, \rho = \rho(z) + \delta\rho, \\ p = p(z) + \delta p, \\ q_s = (0,0,0) + (l, r, s) \quad M = M(z) + \delta M$$

Where the perturbation in fluid velocity q , velocity of suspended particles q_s , density ρ , pressure p , temperature T and magnetization M are $q(u_1, u_2, u_3)$, $q_s(l, r, s)$, $\delta\rho$, δp , θ , δM respectively. By using (3.2) in (2.5) to (2.12) and linearizing them,

$$(3.3) \quad \frac{1}{\varepsilon} \frac{\partial q}{\partial t} = -\frac{\nabla \delta p}{\rho_0} + \frac{\delta \rho}{\rho_0} g + \frac{\delta M}{\rho_0} \nabla H + \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 q - \frac{1}{K_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) q + \frac{KN_0}{\varepsilon \rho_0} (q_S - q)$$

$$(3.4) \quad \nabla \cdot q = 0$$

$$(3.5) \quad (E + b\varepsilon) \frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{c_p} \right) (u_3 + bs) + \kappa_T \nabla^2 \theta$$

$$(3.6) \quad \delta \rho = -\alpha \rho_0 \theta$$

$$(3.7) \quad m N_0 \left[\frac{\partial q_S}{\partial t} \right] = K N_0 (q - q_S)$$

$$(3.8) \quad \delta M = -\gamma M_0 \theta$$

With the help of (3.11) eliminate q_S from (3.5)

$$(3.9) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left[\frac{1}{\varepsilon} \frac{\partial q}{\partial t} + \frac{\nabla \delta p}{\rho_0} - \frac{\delta \rho}{\rho_0} g + \frac{\delta M}{\rho_0} \nabla H + \frac{1}{K_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) q \right] + \frac{m N_0}{\varepsilon \rho_0} \frac{\partial q}{\partial t} = \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 q$$

Cartesian form of (3.9) can be expressed as following

$$(3.10) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left[\frac{1}{\varepsilon} \frac{\partial u_1}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial x} (\delta p) + \frac{1}{K_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) q \right] + \frac{m N_0}{\varepsilon \rho_0} \frac{\partial u_1}{\partial t} = \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 u_1$$

$$(3.11) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left[\frac{1}{\varepsilon} \frac{\partial u_2}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial y} (\delta p) + \frac{1}{K_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) u_2 \right] + \frac{m N_0}{\varepsilon \rho_0} \frac{\partial u_2}{\partial t} = \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 u_2$$

$$(3.12) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) \left[\frac{1}{\varepsilon} \frac{\partial u_3}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial z} (\delta p) - \left(\alpha g - \frac{\gamma M_0 \nabla H}{\rho_0} \right) \theta \right] \\ + \frac{m N_0}{\varepsilon \rho_0} \frac{\partial u_3}{\partial t} = \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 u_3,$$

$$(3.13) \quad \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0.$$

Transforming (3.7) in Cartesian form, we obtain,

$$(3.14) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) l = u_1,$$

$$(3.15) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) r = u_2,$$

$$(3.16) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) s = u_3,$$

Using (3.16) we obtain from (3.5)

$$(3.17) \quad \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) \left[(E + b\varepsilon) \frac{\partial}{\partial t} - \kappa_T \nabla^2 \right] \theta = \left(\beta - \frac{g}{c_p} \right) \\ \left(1 + \frac{m}{k} \frac{\partial}{\partial t} + b\right) u_3$$

Operate $\frac{\partial}{\partial x}$ on (3.10) and $\frac{\partial}{\partial y}$ on (3.11) and by adding them we obtain,

$$(3.18) \quad \left[\left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) \left\{ \frac{1}{\varepsilon} \frac{\partial}{\partial t} - \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^2 + \frac{1}{K_1} \left(\nu + \nu' \frac{\partial}{\partial t} \right) \right\} \right] \frac{\partial u_3}{\partial z} \\ + \frac{m N_0}{\varepsilon \rho_0} \frac{\partial}{\partial t}$$

$$= \frac{1}{\rho_0} \left(1 + \frac{m}{k} \frac{\partial}{\partial t}\right) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta p$$

Operate $\left[\nabla^2 - \left(\frac{\partial^2}{\partial z^2} \right) \right]$ on (3.12) and $\frac{\partial}{\partial z}$ on (3.18) and by adding them, we obtain,

$$(3.19) \quad \frac{1}{\varepsilon} \left(1 + \frac{m}{k} \frac{\partial}{\partial t} + \frac{mN_0}{\rho_0} \right) \nabla^2 \frac{\partial u_3}{\partial t} = \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\alpha g - \frac{\gamma M_0 \nabla H}{\rho_0} \right) \theta + \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\nu - \frac{\mu'}{\rho_0} \nabla^2 \right) \nabla^4 - \frac{1}{K_1} \left(1 + \frac{m}{k} \frac{\partial}{\partial t} \right) \left(\nu + \nu' \frac{\partial}{\partial t} \right) \nabla^2 u_3.$$

4. Dispersion Relation

To decompose the disturbances we use Normal Mode method. Let us take perturbation is in the following form,

$$(4.1) \quad (u_3, \theta) = [W(z), \Theta(z)] \cdot \exp(i k_x x + i k_y y + nt)$$

Where k_x and k_y are wave number in X and Y direction and k is the resultant disturbances wave number such that,

$$k = \sqrt{[(k_x)^2 + (k_y)^2]},$$

n is the frequency of any arbitrary disturbances which is a complex constant. Now by using relation (4.1) we obtain non dimensional form of (3.17), (3.18) and (3.19)

$$(4.2) \quad (D^2 - a^2) \begin{bmatrix} \left(\frac{\sigma}{\varepsilon} + \frac{M\sigma}{\varepsilon(1+\tau_1\sigma)} + \frac{(1+\sigma F_1)}{P_t} \right) \\ + F(D^2 - a^2)^2 \\ -(D^2 - a^2) \end{bmatrix} W + \frac{\alpha a^2 \lambda d^2}{\nu} \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right) \theta = 0,$$

$$(4.3) \quad [(D^2 - a^2) - \sigma E_1 P_1] \Theta = - \left(\frac{B + \tau_1 \sigma}{1 + \tau_1 \sigma} \right) \frac{\beta d^2}{\kappa_T} W$$

Where we have represent the coordinates x , y and z in new units of length d , time t and (d^2/κ_T) . Let $a = kd$, $\sigma = nd^2/\nu$, $F = \frac{\mu'}{\rho_0 d^2 \nu}$, $F_1 = \frac{\nu'}{d^2}$, $P_t = K_1/d^2$, $P_1 = \nu/\kappa_T$, $P_2 = \nu/\eta$, $\tau = \frac{m}{k}$, $\tau_1 = \frac{m}{d^2}$, $E_1 = (E + b\varepsilon)$, $B = (1 + b)$, $x^* = x/d$, $y^* = y/d$, $z^* = z/d$, $D^* = dD$ and dropping $*$ for convenience.

Where P_1 and P_t denotes Prandlt number and dimensionless medium permeability.

Now eliminate Θ from (4.2) with the help of (4.3) then we get Stability governing equation

$$(4.4) \quad (D^2 - a^2)(D^2 - a^2) - \sigma E_1 P_1 \left[\left(\frac{\sigma}{\varepsilon} + \frac{M\sigma}{\varepsilon(1+\tau_1\sigma)} + \frac{(1+\sigma F_1)}{P_t} \right) + F(D^2 - a^2)^2 - (D^2 - a^2) \right] W - \lambda a^2 R_f \left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma} \right) W = 0,$$

Where

$$R_f = \frac{\alpha\beta d^4}{\nu\kappa_T} \left[g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right] = \frac{\alpha\beta d^4 g_0}{\nu\kappa_T} \left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0} \right] = R \left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0} \right]$$

R_f is the Rayleigh number for ferromagnetic fluid. R is the Rayleigh number for fluid. $T_A = \left(\frac{2\Omega d^2}{\nu} \right)^2$ is the modified Taylor number.

The perturbation in the temperature on boundaries is zero, because the boundaries are kept at constant temperature. So the appropriate condition on boundary is

$$(4.5) \quad W = 0, Z = 0, \Theta = 0 \text{ at } z = 0 \text{ \& } z = 1, DZ = D^2W = D^4W = 0 \text{ at } z = 0 \text{ \& } z = 1.$$

From (4.5), it is clear that all even order derivative of W vanish on boundaries. Therefore proper solution of (4.4) characterizing the lowest mode is,

$$(4.6) \quad W = W_0 \sin \pi z, \quad W_0 \text{ is a constant}$$

By using (4.6) with (4.4), we get,

$$(4.7) \quad R_1 = \frac{1}{\lambda x} (1+x) (1+x + i\sigma_1 E_1 P_1) \left(\frac{1+i\sigma_1 \pi^2 \tau_1}{B+i\sigma_1 \pi^2 \tau_1} \right) \left[\left(\frac{i\sigma_1}{\varepsilon} + \frac{Mi\sigma_1}{\varepsilon(1+i\sigma_1 \pi^2 \tau_1)} + \frac{(1+i\sigma_1 F_3)}{P} \right) + F_2(1+x)^2 + (1+x) \right]$$

Where

$$x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}, F_2 = \pi^2 F, P = \pi^2 P_t,$$

$$F_3 = \pi^2 F_1, R_1 = \frac{Rf}{\pi^4}$$

5. Analytical Discussion

5.1 Stationary Convection: at stationary convection, when stability sets, the marginal state will be characterized by $\sigma_1 = 0$. Put $\sigma_1 = 0$ in (38), we get,

$$(5.1) \quad R_1 = \frac{(1+x)^2}{B\lambda x} \left[\{ F_2(1+x)^2 + (1+x) + \frac{1}{P} \} \right],$$

Equation (5.1) shows the modified Rayleigh number R_1 as a function of B , F_2 , P and x parameters. Clearly viscoelastic parameter F_3 disappears with σ_1 . It shows that for stationary convections Rivlin-Ericksen fluid behaves like an ordinary Newtonian Fluid. To examine the effect of suspended particles, couple stress, permeability and magnetization we have to study the analytical behaviour of

$$\frac{dR_1}{dB}, \frac{dR_1}{dF_2}, \frac{dR_1}{dP} \text{ and } \frac{dR_1}{dM_0}.$$

$$(5.2) \quad \frac{dR_1}{dB} = -\frac{(1+x)^2}{B^2\lambda x} \left[\{ F_2(1+x)^2 + (1+x) + \frac{1}{P} \} \right],$$

(5.2) shows that suspended particles has a stabilizing effect on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of varying gravity field through porous medium if $\lambda < 0$ (when gravity decreases upwards from its value at $z = 0$) and a destabilizing effect if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$).

$$(5.3) \quad \frac{dR_1}{dF_2} = \frac{(1+x)^4}{B\lambda x}$$

(5.3) shows that couple stress has stabilizing effect on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of varying gravity field and suspended particles through porous medium if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$) and a destabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at $z = 0$).

$$(5.4) \quad \frac{dR_1}{dP} = -\frac{(1+x)^2}{BP^2\lambda x}$$

(5.4) shows that permeability has a stabilizing effect on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of varying gravity field and suspended particles through porous medium if $\lambda < 0$, (when gravity decreases upwards from its value at $z = 0$) and a destabilizing effect if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$).

Replace R_1 by R_f / π^4 and R_f by $R \left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0} \right]$

From (43)

$$(5.5) \quad R = \frac{\frac{\pi^4(1+x)^2}{B\lambda x} \left[\left\{ F_2(1+x)^2 + (1+x) + \frac{1}{P} \right\} \right]}{\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0} \right]}$$

$$(5.6) \quad \frac{dR}{dM_0} = \frac{\frac{\pi^4(1+x)^2}{B\lambda x} \left[\left\{ F_2(1+x)^2 + (1+x) + \frac{1}{P} \right\} \right] \left(\frac{\gamma \nabla}{\rho_0 \alpha} \right)}{\left[1 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda g_0} \right]^2}$$

(5.6) shows that magnetization has stabilizing effect on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of varying gravity field and suspended particles through porous medium for each value of λ either $\lambda > 0$, or $\lambda < 0$ (either gravity increases upwards from its value at $z = 0$ or gravity decreases upwards from its value at $z = 0$).

5.2 Stability of the system & Oscillatory Modes: Multiplying (33) by the conjugate of W i.e. W^* and integrate over the range of z and making use of (34) together with boundary condition (36), we obtain,

$$(5.7) \quad \left[\left(\frac{\sigma}{\varepsilon} + \frac{M\sigma}{\varepsilon(1+\tau_1\sigma)} + \frac{(1+\sigma F_1)}{P_t} \right) \right] I_1 + I_2 + F I_3 - \frac{\alpha \alpha^2 \lambda \kappa_T}{\beta \nu} \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right) \left(\frac{1 + \sigma^* \tau_1}{B + \sigma^* \tau_1} \right) [I_4 + \sigma^* E_1 P_1 I_5] = 0$$

Where

$$I_1 = \int (|DW|^2 + a^2|W|^2) dz$$

$$I_2 = \int (|D^2W|^2 + a^4|W|^2 + 2a^2|DW|^2) dz$$

$$I_3 = \int (|D^3W|^2 + 3a^2|D^2W|^2 + 3a^4|DW|^2 + a^6|W|^2) dz$$

$$I_4 = \int (|D\Theta|^2 + a^2|\Theta|^2) dz$$

$$I_5 = \int (|\Theta|^2) dz$$

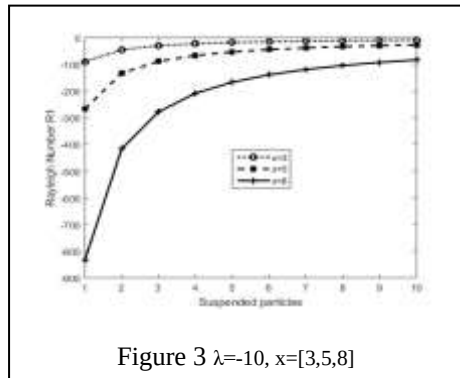
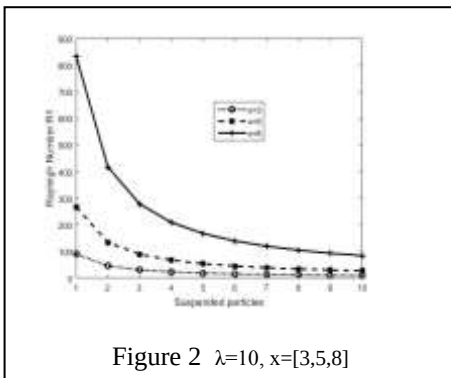
All above specified integrals I_1 - I_5 are positive definite. Put $\sigma = i \sigma_i$ in (45), equating imaginary part, we obtain,

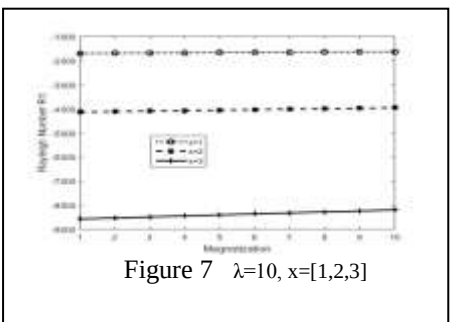
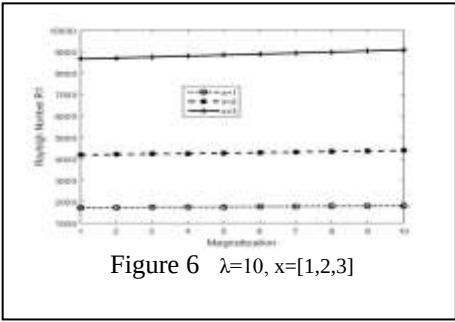
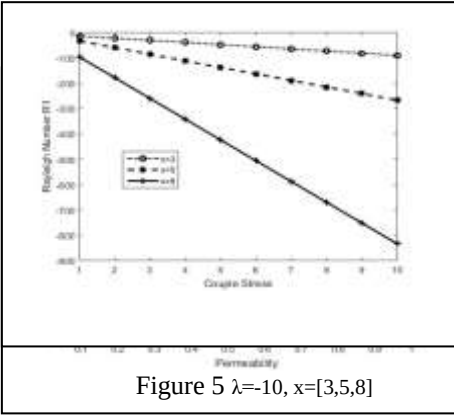
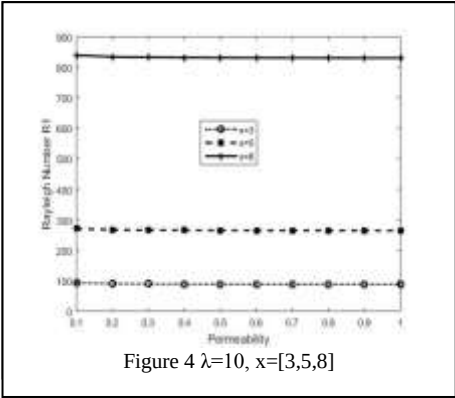
$$(5.8) \quad \sigma_i \left[\left(\frac{1}{\varepsilon} + \frac{M}{\varepsilon(1+\tau_1^2\sigma_i^2)} + \frac{F_1}{P_t} \right) I_1 + \frac{\alpha a^2 \lambda \kappa_T}{\beta \nu} \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right) \left\{ \left(\frac{\tau_1 (B-1)}{B^2 + \tau_1^2 \sigma_i^2} \right) I_4 + \left(\frac{B + \tau_1^2 \sigma_i^2}{B^2 + \tau_1^2 \sigma_i^2} \right) E_1 P_1 I_5 \right\} \right] = 0,$$

From (46) the quantity is in bracket will be positive definite if $g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda}$ and $B > 1$. It means $\sigma_i = 0$, modes are non oscillatory or oscillatory modes are not allowed and principle of exchange of stability is satisfied if $g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda}$ and $B > 1$

6. Numerical Computations

Dispersion equation (4.4) is also analysed numerically. The critical Rayleigh number R_l & Rayleigh number for ferromagnetic fluid is calculated for different values of Couple stress F_2 , suspended particles B , permeability P and Magnetization M_0 parameters.





In figure 2, a graph has been plotted between the critical Rayleigh number R_1 and the suspended particles parameter B for $x = [3, 5, 8]$, which shows R_1 decreases with increase in B for $\lambda > 0$. So suspended particles have a destabilizing effect on the system for $\lambda > 0$.

In figure 3, a graph has been plotted between the critical Rayleigh number R_1 and the suspended particles parameter B for $x = [3, 5, 8]$, which shows R_1 increases with increase in B for $\lambda < 0$. So suspended particles have a stabilizing effect on the system for $\lambda < 0$.

In figure 4, a graph has been plotted between the critical Rayleigh number R_1 and the couple stress parameter F_2 for $x = [3, 5, 8]$, and $\lambda = 10 > 0$, which shows R_1 increases with increase in F_2 . So couple stress has a stabilizing effect on the system for $\lambda > 0$.

In figure 5, a graph has been plotted between the critical Rayleigh number R_1 and the couple stress parameter F_2 for $x = [3, 5, 8]$, and $\lambda = -10 < 0$, which shows R_1 decreases with increase in F_2 . So couple stress has a destabilizing effect on the system for $\lambda < 0$.

In figure 6, a graph has been plotted between the critical Rayleigh number R_1 and the permeability of the medium P for $x = [3, 5, 8]$ and $\lambda = 10 > 0$,

which shows R_1 decreases with increase in P . So permeability has a destabilizing effect on the system for $\lambda > 0$.

In figure 7, a graph has been plotted between the critical Rayleigh number R_1 and the permeability of the medium P for $x = [3, 5, 8]$ and $\lambda = -10 < 0$, which shows R_1 increases with increase in P . So permeability has a stabilizing effect on the system for $\lambda < 0$.

In figure 8, a graph has been plotted between the Rayleigh number R and the magnetization of ferromagnetic fluid M_0 for $x = [3, 5, 8]$, and $\lambda = 10 > 0$, which shows R increases with increase in M_0 . So, magnetization has a stabilizing effect on the system for $\lambda > 0$.

In figure 9, a graph has been plotted between Rayleigh number R and the magnetization of ferromagnetic fluid M_0 for $x = [3, 5, 8]$, and $\lambda = -10 < 0$, which shows R increases with increase in M_0 . So, magnetization has stabilizing effect on the system for $\lambda < 0$.

7. Conclusion

In the present paper, the effect of different parameters such as suspended particles, couple stress, viscoelasticity, permeability and magnetization has been examined on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of a variable gravity field through a porous medium.

Results of investigation are as follows:-

- (1) Suspended particles always have a destabilizing effect if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$) and a stabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at $z = 0$).
- (2) Couple stress has a stabilizing effect on thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in presence of varying gravity field and suspended particles through porous medium if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$) and a destabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at $z = 0$).
- (3) Permeability has a destabilizing effect on the thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid in the presence of suspended particles, varying gravity through a porous medium if $\lambda > 0$ (when gravity increases upwards from its value at $z = 0$) and a stabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at $z = 0$).
- (4) Magnetization always has a stabilizing effect on considered fluid layer.

- (5) Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid for stationary convection.
- (6) Modes are non oscillatory or oscillatory modes are not allowed and principle of exchange of stability is satisfied if $g_0 > \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda}$ and $B > 1$.

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