

A Probability Model for the Single Male Migrants

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Abstract: This paper is devoted to the derivation of an inflated probability model for describing the distribution of single male migrants at micro-level under certain assumptions. Parameters involved in the model have been estimated and the model has been applied to observed set of data for it suitably.

1. Introduction

Out-migration at micro-level has played decisive role in the development of theories of migration and investigation of factors affecting movement process, is best carried-out with models (Shaw¹, Speare et al.², Dejong et al.³), while studying the rural out-migration, the study of migration is in relation to human grouping has been emphasized.

In recent years, increased attention has been paid to the proposition and derivation of probability models for the movement of human population at micro-level (Yadava⁴). In this regard Singh and Yadava⁵ have proposed the negative binomial distribution to study the trends in rural out-migration at household-level.

The present paper aims to develop inflated probability model for the number of single male-migrants. An estimate of parameters is suggested along with asymptotic expression for their variance.

2. Model

Let the random variable X denotes the number of rural out-migrants from a household. Inflated probability model to describe the variation in the number of male migrants has been obtained on the basis of the following assumptions:

- (i) Let α be the probability that household is exposed to the risk of migration at the survey point and $(1-\alpha)$ be the probability that the household is not exposed to the risk of migration.
- (ii) The k males migrating from a household have more probability than the probability of $(k+1)$ males migrating, $k = 1, 2, \dots, N$. Thus the probability, a decreasing function of X , is assumed to follow truncated geometric distribution.

From the assumptions (i) and (ii), with the help of Johnson and Kotz⁶, the probability model becomes:

$$(1) \quad P[X=0] = 1 - \alpha + \frac{\alpha p}{1 - q^{N+1}}, k=0 \text{ and } P[X=k] = \alpha \left[\frac{pq^k}{1 - q^{N+1}} \right], k=1, 2, \dots, N$$

3. Estimation

Let N be a known quantity. An estimate of it may be the maximum size of the household in the community. If N is taken to be known, then the proposed model (1) involves two unknown parameters α and p only.

Maximum Likelihood Method: Consider a sample consisting of observations of the random variable X with the probability function (1) in which f_0 designates the number of zero observation; f_1 the number of one observation and f the total number of observations. The values chosen as estimates of and are those which maximize the expression:

$$(2) \quad P(x_1, x_2, \dots, x_m, \alpha, p) \\ = \left[\left(1 - \alpha + \frac{\alpha p}{1 - q^{N+1}} \right)^{f_0} \left(\frac{\alpha p q}{1 - q^{N+1}} \right)^{f_1} \left(1 - 1 + \alpha - \frac{\alpha p}{1 - q^{N+1}} - \frac{\alpha p q}{1 - q^{N+1}} \right)^{f - f_0 - f_1} \right]$$

Taking logarithmic of (2), differentiating with respect of α and p in turn, and equating to zero yields the estimating equation:

$$(3) \quad \alpha \left(1 - \frac{p}{1 - q^{N+1}} \right) = \frac{f - f_0}{f} \text{ and}$$

$$(4) \quad \frac{f_0 \left[\frac{f - f_0}{f \left(1 - \frac{p}{1 - q^{N+1}} \right)} \right] \left[\frac{1 - q^{N+1} - p q^N (N+1)}{(1 - q^{N+1})^2} \right]}{\left[1 - \frac{f - f_0}{f} \right]} + f_1 \left[\frac{1}{p} - \frac{1}{p} - \frac{(N+1)q^N}{1 - q^{N+1}} \right] \\ + \frac{f - f_0 - f_1}{(1 - (2p - p^2) - q^{N+1})} \left[- \left\{ \frac{(1 - q^{N+1}) 2q - (2p - p^2)(N+1)q^N}{(1 - q^{N+1})^2} \right\} \right] = 0$$

The asymptotic variance of (α, p) is obtained by inverting the information matrix whose elements are negatives of the second order derivatives of logarithmic of the likelihood function. The second order derivatives of L follows as:

$$(5) \quad \frac{\partial^2 L}{\partial \alpha^2} = -f_0 \left[\frac{\left(1 - \frac{p}{1-q^{N+1}} \right)^2}{1 - \alpha \left(1 - \frac{p}{1-q^{N+1}} \right)} \right] - \frac{f - f_0 - f_1}{\alpha^2} - \frac{f_1}{\alpha^2}$$

$$(6) \quad \frac{\partial^2 L}{\partial p^2} = - \frac{\alpha f_0 q^N \{ q \{ (N+1)p + N \} - 2(N+1)(p^2 - Npq) \}}{(1-q^{N+1})^3} \\ + f_1 \left[\frac{(N+1)q^{N+1}(N-q^{N+1})}{(1-q^{N+1})^2} - \frac{p^2 + q^2}{p^2 q^2} \right] \\ + (f - f_0 - f_1) \left(2 - \frac{p}{1-q^{N+1}} \right) \left[(N+1)q^N \left(2 - \frac{p}{1-q^{N+1}} \right) \right. \\ \left. \left(-N + p + \frac{p}{1-q^{N+1}} \right) - 2 \frac{p}{1-q^{N+1}} \right]$$

$$(7) \quad \frac{\partial^2 L}{\partial \alpha \partial p} = \frac{\partial^2 L}{\partial p \partial \alpha} = f_0 \frac{\left[\frac{1 - q^{N+1} - pq^N(N+1)}{(1-q^{N+1})^2} \right]}{\left[1 - \alpha \left(1 - \frac{p}{1-q^{N+1}} \right) \right]^2}$$

since

$$(8) \quad E[f_0] = f \left[1 - \alpha + \frac{\alpha p}{1-q^{N+1}} \right]$$

$$(9) \quad E[f_1] = f \alpha \frac{p}{1-q^{N+1}}$$

and

$$(10) \quad E[f - f_0 - f_1] = f \alpha \left(1 - \frac{p}{1-q^{N+1}} - \frac{pq}{1-q^{N+1}} \right)$$

where E denotes for the expectation. The elements of information matrix follow from (5), (6) and (7)

$$(11) \quad \phi_{11} = E \left[-\frac{\partial^2 L}{\partial \alpha^2} \right] \Big| f = \left[\left(1 - \frac{\alpha}{1-q^{N+1}} \right)^2 + \frac{\left(1 - \frac{pq}{1-q^{N+1}} \right)}{\alpha} \right]$$

$$(12) \quad \phi_{22} = E \left[-\frac{\partial^2 L}{\partial p^2} \right] \Big| f = - \frac{\alpha \left(1 - \alpha + \frac{\alpha p}{1-q^{N+1}} \right) q^N \left[q \{ (N+1)p + N \} - 2(N+1)(p^2 - Npq) \right]}{(1-q^{N+1})^3}$$

$$+ \frac{\alpha p}{1-q^{N+1}} \left\{ \frac{(N+1)q^{N+1}(N-q^{N+1})}{(1-q^{N+1})^2} - \left(\frac{p^2 + q^2}{p^2 q^2} \right) \right\}$$

$$+ \alpha \left(1 - \frac{p}{1-q^{N+1}} - \frac{pq}{1-q^{N+1}} \right) \left(2 - \frac{p}{1-q^{N+1}} \right)$$

$$\left\{ (N+1)q^N \left(2 - \frac{p}{1-q^{N+1}} \right) \left(-N + p + \frac{p}{1-q^{N+1}} \right) - \frac{2p}{1-q^{N+1}} \right\} \Big| f$$

$$(13) \quad \phi_{12} = E \left(-\frac{\partial^2 L}{\partial \alpha \partial p} \right) \Big| f = \phi_{21} = E \left(-\frac{\partial^2 L}{\partial p \partial \alpha} \right)$$

$$= - \left(1 - \alpha + \frac{\alpha p}{1-q^{N+1}} \right) \frac{\left[\frac{1-q^{N+1} - pq^N(N+1)}{(1-q^{N+1})^2} \right]}{\left[1 - \alpha \left(1 - \frac{p}{1-q^{N+1}} \right) \right]^2}$$

Therefore, by inverting the information matrix, the expression for asymptotic variances of the estimators can be obtained as

$$(14) \quad V(\tilde{\alpha}) = \frac{1}{f} \left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - \phi_{12}^2} \right]$$

$$(15) \quad V(\tilde{p}) = \frac{1}{f} \left[\frac{\phi_{22}}{\phi_{11}\phi_{22} - \phi_{12}^2} \right]$$

Method of Moments: Since the proposed model contains only two unknown parameters to be estimated, we estimate them by equating zeroth cell theoretic frequency to the observed frequency of the respective cell, and theoretical mean equal to observed mean as follows:

$$(16) \quad 1 - \alpha + \frac{\alpha p}{1 - q^{N+1}} = \frac{f_0}{f}$$

and

$$(17) \quad \alpha \left(\frac{1}{p} - \frac{1 + Nq^N}{1 - q^{N+1}} \right) = E(x).$$

Hence, the moment estimates of α and p can be obtained by using the above expressions (16) and (17).

4. Application

The proposed model (1) has been applied to the migration data taken from the survey "Rural Development and Population Growth (R.D.P.G)- 1978" conducted by the centre of population studies Varanasi, India.

From the table II, we find that the values of $\tilde{p}$ is nearly constant in all three types of villages, while values of the estimated values of $\tilde{\alpha}$ show more variation. It may be said that most of the variation in the migration pattern are due to the risk of migration. We also see from the table II that the observed values of χ^2 are insignificant for the set of data at 5 percent level of significance by both the methods.

From table I, it may be concluded that the proposed model for the number of rural out-migrants under consideration is a reasonable approximation to the situation at least at micro-level. Thus it is more useful in calculating the various probabilities of single male out-migrants connected with the process of migration from a household and also for predictions to the migration rates in a specified population.

Table 1: Distribution of observed and expected number of households according to the single male migrants from a household in three types of villages.

No. of Migrants	Types of Villages								
	Semi-Urban			Remote			Growth Center		
	Obs.	Expected		Obs.	Expected		Obs.	Expected	
		Method of Moments	M.L.E.		Method of Moments	M.L.E.		Method of Moments	M.L.E.
0	1032	1032.00	1032.00	871	871.00	871.00	972	972.00	972.00
1	95	88.73	86.62	176	176.52	174.58	154	151.06	147.68
2	19	27.77	28.32	59	58.02	58.67	47	51.96	52.87
3	10	8.69	9.26	18	19.12	19.70	18	17.87	18.93
4	2	2.72	4.80	6	6.27	6.62	9	6.15	6.78
5	2			4	3.07	3.43	1	2.11	3.74
6	0			0			0		
7	1			0			0		
8	0			0			0		
Total	1161	1161.00	1161.00	1134	1134.00	1134.00	1202	1202.00	1202.00

Table 2: Estimated value of the parameters and Chi squares.

Parameter	Types of Villages					
	Semi-Urban		Remote		Growth Center	
	Expected		Expected		Expected	
	Method of Moments	M.L.E.	Method of Moments	M.L.E.	Method of Moments	M.L.E.
$\hat{\alpha}$	0.355	0.339	0.705	0.690	0.557	0.535
$V(\hat{\alpha})$	—	0.000676	—	0.0001	—	0.000225
$\hat{p}$	0.687	0.673	0.671	0.664	0.656	0.642
$V(\hat{p})$	—	0.000064	—	0.000144	—	0.000169
χ^2	3.782	3.945	0.377	0.314	2.541	2.323
d.f.	3	3	4	4	4	4

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